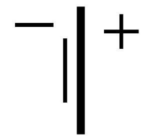
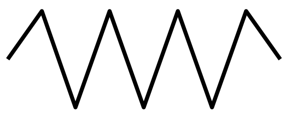
Chapter 18

Circuit Elements, Independent Voltage Sources, and Capacitors

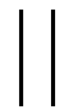
Ideal Wire



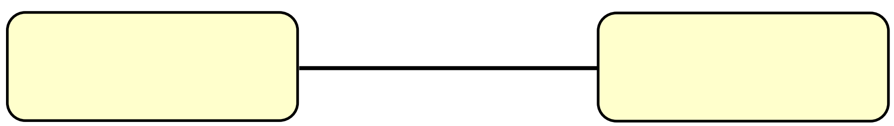
Ideal Battery



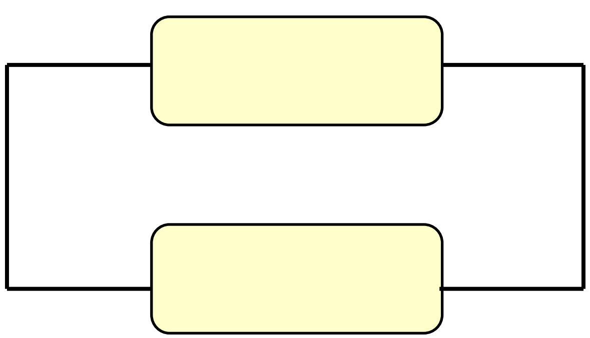
Ideal Resistor



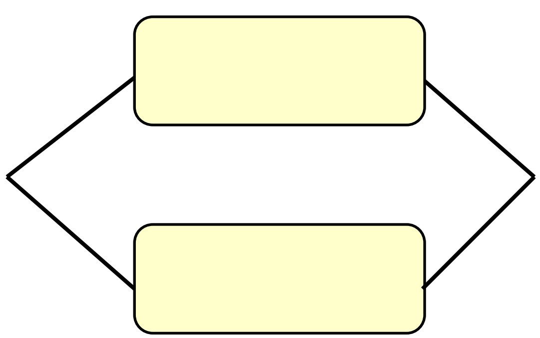
Ideal Capacitor



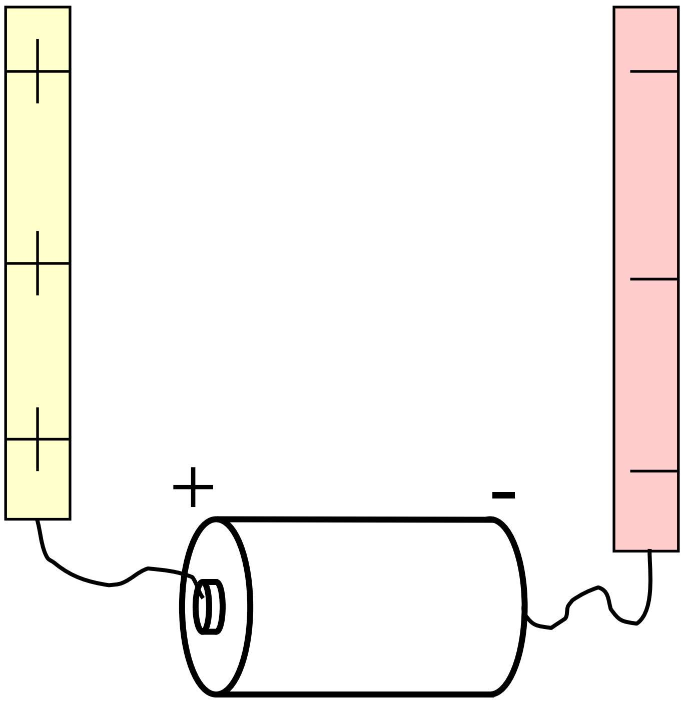
Series



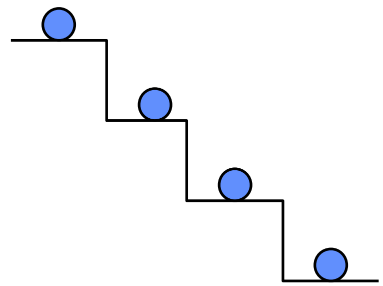
Parallel



An ideal battery provides a constant potential difference

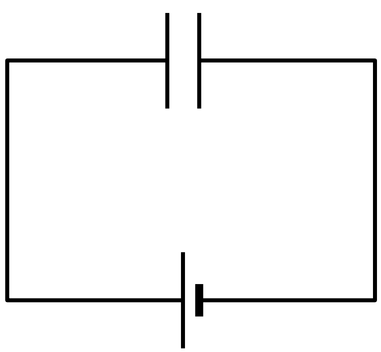


High Potential



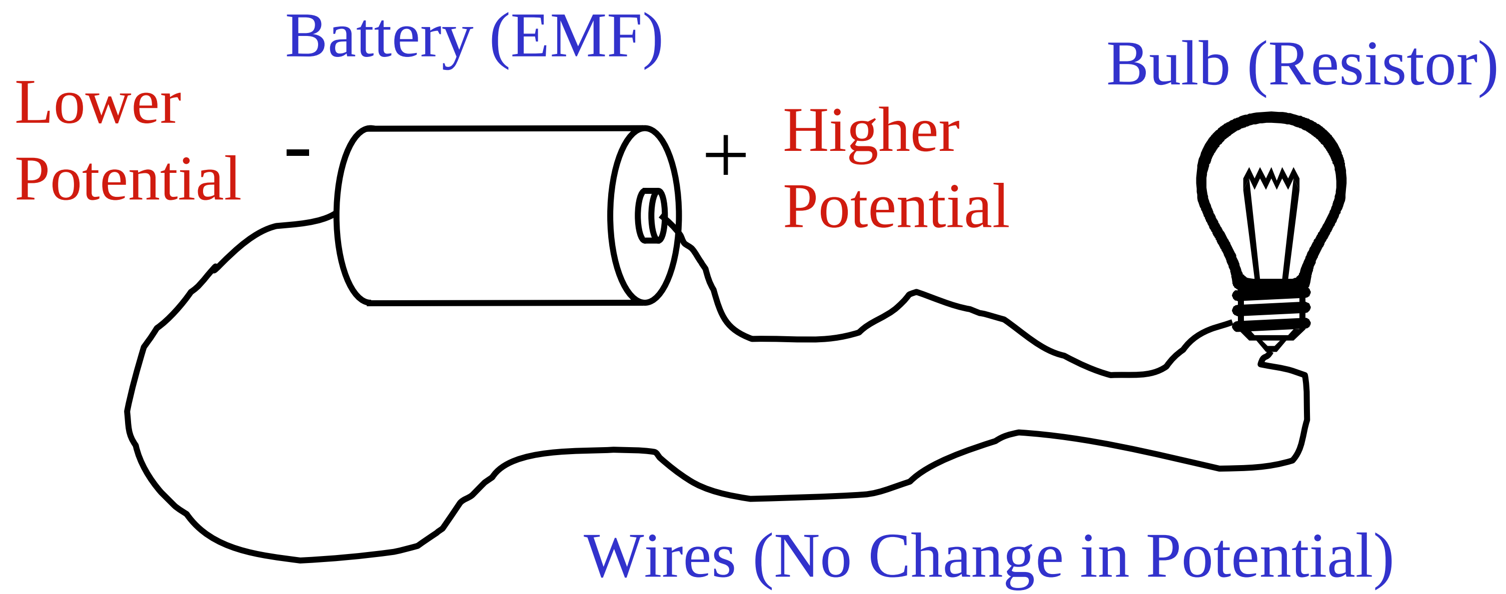
Low Potential

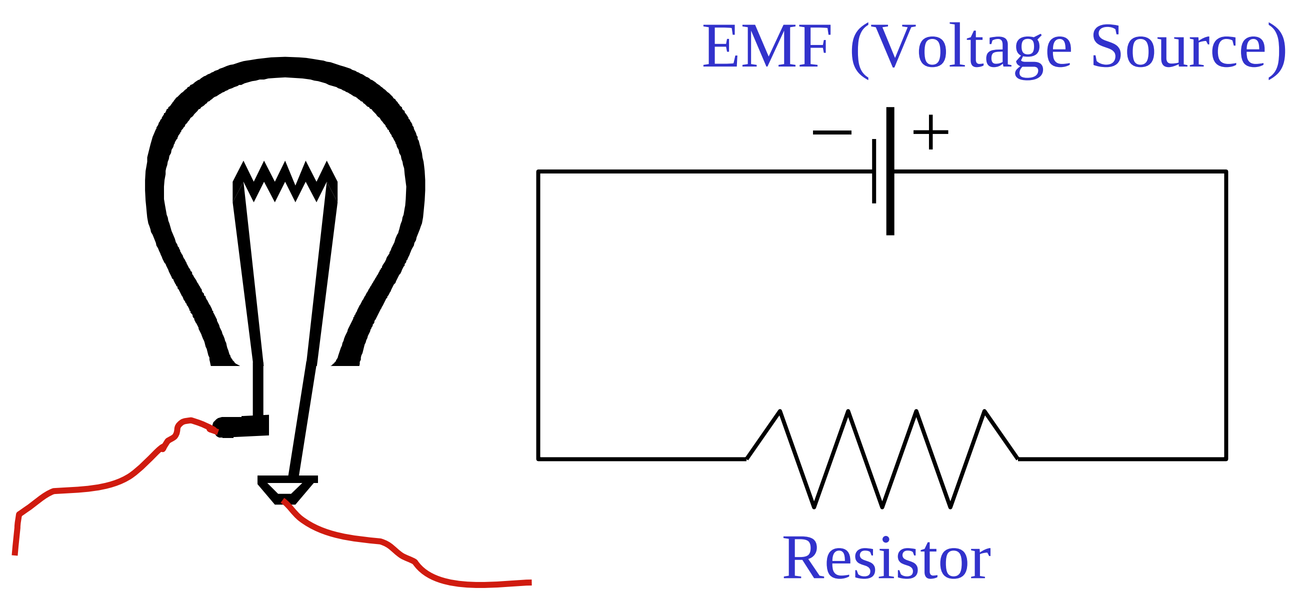
Capacitor



Battery

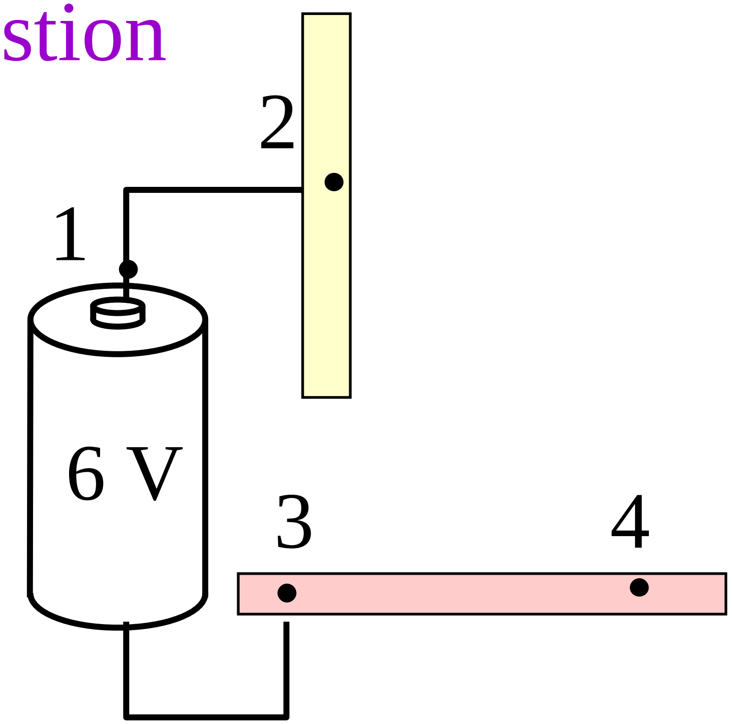
Charge moves until there is no voltage difference


Battery (EMF)
Lower
Potential
-
+
Higher
Potential
Bulb (Resistor)
Wires (No Change in Potential)


EMF (Voltage Source)
−
+
Resistor

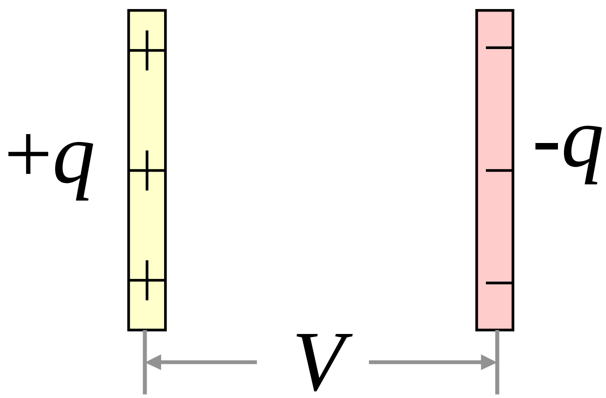
Interactive Question

Consider the figure on the right. What will be the magnitude of the potential difference between the indicated points?



	<u>1,2</u>	<u>1,3</u>	<u>3,4</u>
A)	V_{12}=6 V	V_{13}=6 V	V_{34}=6 V
B)	V_{12}=6 V	V_{13}=0 V	V_{34}=6 V
C)	V_{12}=0 V	V_{13}= 6 V	V_{34}= 0 V
D)	$0<V_{12}<6$ V	V_{13}= 6 V	$0<V_{34}<6$ V
E)	$0<V_{12}<6$ V	$0<V_{13}<6$ V	$0<V_{34}<6$ V

Capacitors



$q=CV$

- Two conductors carrying equal but opposite charge
- The capacitance *C* is changed only by changing the physical characteristics of the capacitor.
- The SI unit for capacitance is coulombs/volts = farads (F)
- *V* remains constant if the capacitor is attached to a battery.
- *Q* remains constant when the charged conductors are disconnected from all wires or *isolated*.

Capacitance

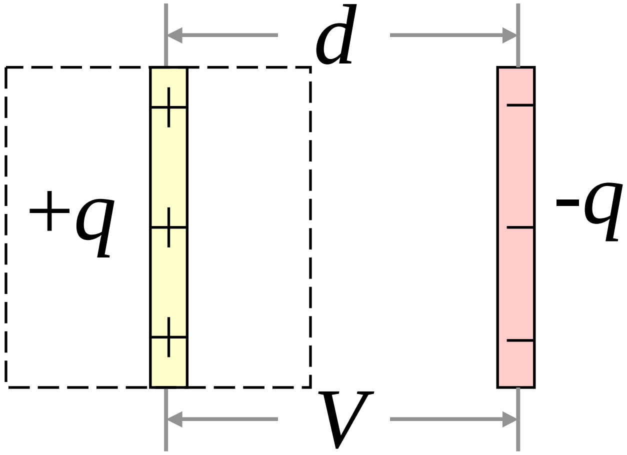
Even a single conductor is said to have a capacitance.

Consider a spherical conductor with radius R

$V = kq/R$

$C = q/V = R/k = 4\pi\varepsilon_0 R$

Parallel Plate Capacitor



$q = CV$

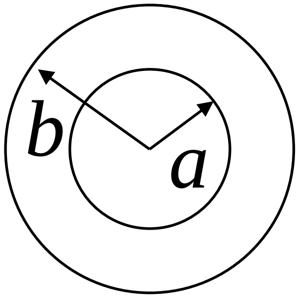
A: Area of plate

$$\varepsilon_0 \int \mathbf{E} \cdot d\mathbf{A} = q$$
$$\varepsilon_0 EA = q$$

$$\Delta V = V = -\int \mathbf{E} \cdot d\mathbf{s}$$
$$= Ed$$

$$\varepsilon_0 VA/d = q$$
$$C = \varepsilon_0 A/d$$

Cylindrical Capacitor



Gauss's Law

$$\varepsilon_0 \int \mathbf{E} \cdot d\mathbf{A} = q$$
$$\varepsilon_0 EA = q$$
$$\varepsilon_0 E\, 2\pi r L = q$$
$$E = q/(2\pi\varepsilon_0 rL)$$

Relationship of E to V

$$V = -\int_a^b \mathbf{E} \cdot d\mathbf{s}$$
$$= q/(2\pi\varepsilon_0 L) \int dr/r$$
$$= q/(2\pi\varepsilon_0 L) \ln(b/a)$$

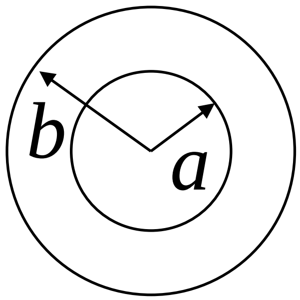
Solve for C

$$q = CV$$
$$= V 2\pi\varepsilon_0 L/\ln(b/a)$$
$$C = 2\pi\varepsilon_0 L/\ln(b/a)$$

Spherical Capacitor

Gauss's Law

$$\varepsilon_0 \int \mathbf{E}\cdot d\mathbf{A} = q$$
$$\varepsilon_0 EA = q$$
$$\varepsilon_0 E\, 4\pi r^2 = q$$
$$E = q/(4\pi\varepsilon_0 r^2)$$



Relationship of E to V

$$V = -\int_a^b \mathbf{E}\cdot d\mathbf{s}$$
$$= -q/(4\pi\varepsilon_0) \int dr/r^2$$
$$= -q/(4\pi\varepsilon_0)(1/b - 1/a)$$
$$= q/(4\pi\varepsilon_0)\{(b-a)/ab\}$$

Solve for C

$$q = CV$$
$$= 4\pi\varepsilon_0 ab/(b-a)V$$
$$C = 4\pi\varepsilon_0 ab/(b-a)$$

Interactive Question

A parallel plate capacitor with plates of area A and plate separation d is charged so that the potential difference between the plates is V. If the capacitor is then isolated and its plate separation is halved to $d/2$, what happens to the *capacitance*?

A) It decreases by a factor of 4
B) It is halved
C) It is doubled
D) It is increased by a factor of 4
E) It is unchanged

Interactive Question

A parallel plate capacitor with plates of area *A* and plate separation *d* is charged so that the potential difference between the plates is *V*. If the capacitor is then isolated and its plate separation is halved to *d*/2, what happens to the *potential difference* between the plates?

A) It decreases by a factor of 4
B) It is halved
C) It is doubled
D) It is increased by a factor of 4
E) It is unchanged

Interactive Question

The plates of a parallel plate capacitor are maintained with constant voltage by a battery as the plate separation is doubled. What happens to the amount of charge on the plates?

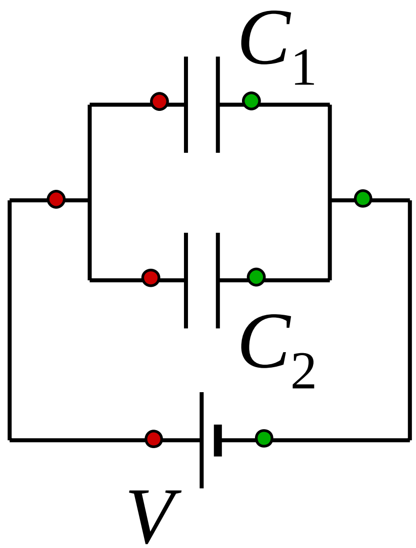
A) It decreases by one-half
B) It decreases by one-fourth
C) It stays the same
D) It increases by two times
E) It increases by four times

Interactive Question

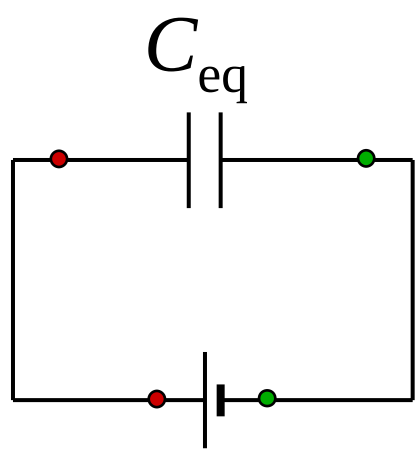
The plates of a parallel plate capacitor are maintained with constant voltage by a battery as the plate separation is doubled. What happens to the electric field between the plates?

A) It decreases by one-half
B) It decreases by one-fourth
C) It stays the same
D) It increases by two times
E) It increases by four times

Capacitors in Parallel

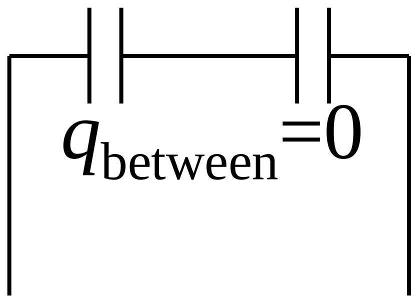


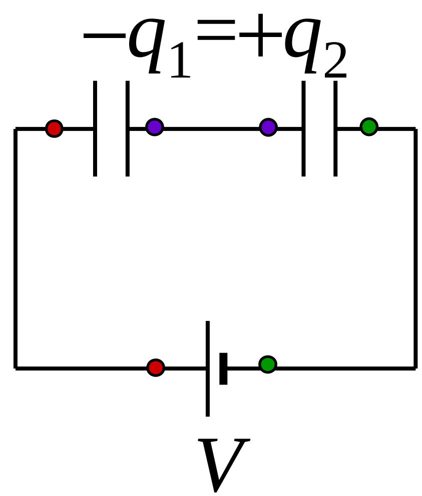
$$q = q_1 + q_2 = C_1V_1 + C_2V_2 = C_1V + C_2V = V(C_1 + C_2) = C_{\mathrm{eq}}V$$



$$C_{\mathrm{eq}} = C_1 + C_2$$
$$C_{\mathrm{eq}} = \sum C_i$$

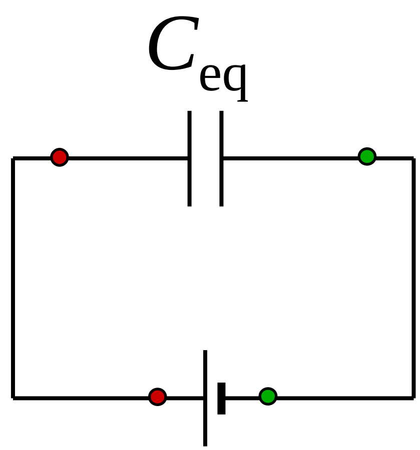
Capacitors in Series





$V = V_1+V_2 = q_1/C_1+q_2/C_2 = q/C_1+q/C_2 = q(1/C_1+1/C_2)$

$V= V/C_{\text{eq}}$

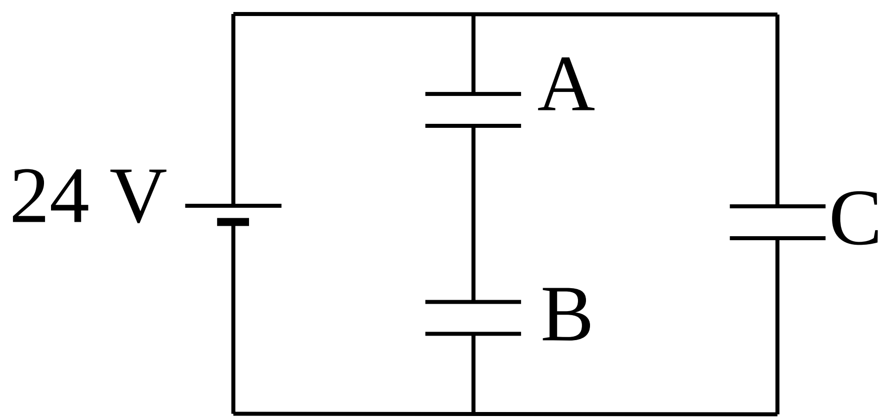


$1/C_{\text{eq}}= 1/C_1+1/C_2$

$1/C_{\text{eq}}= \sum 1/C_i$

Interactive Question

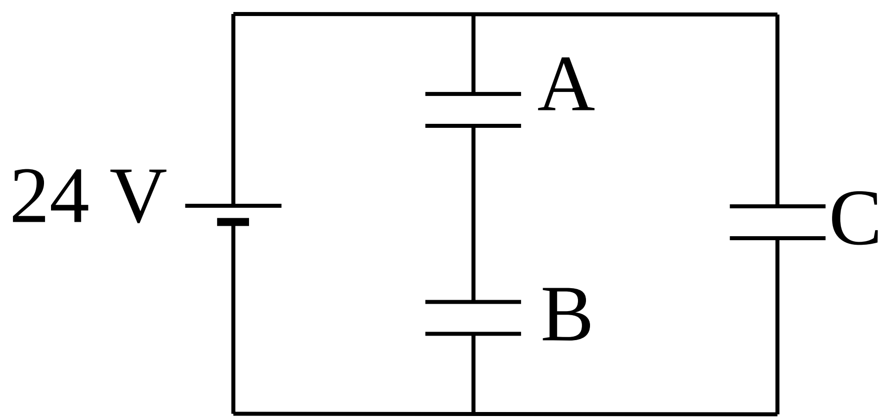
Three capacitors are connected to a 24 Volt battery as shown. Which capacitor(s) have the same charge stored on them?



A) A and B
B) A and C
C) B and C
D) A, B, and C
E) It depends on the individual capacitance

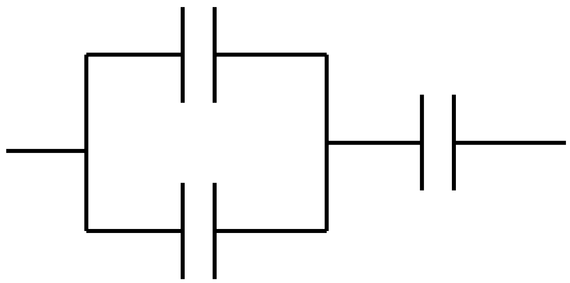
Interactive Question

Three capacitors are connected to a 24 Volt battery as shown. Which individual capacitor(s) have a potential difference of 24 V across the capacitor?



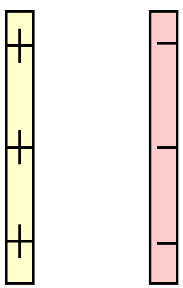
A) A
B) B
C) C
D) A, and B
E) A, B, and C

Problem: Three capacitors with equal capacitance, *C*, are set up as shown. What is the equivalent capacitance of this arrangement?



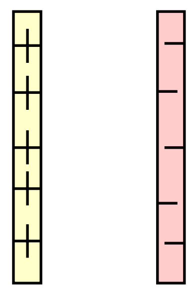
<u>Problem:</u> A 2.50 μF capacitor is charged to 1000 V and a 6.80 μF capacitor is charged to 650 V. The positive plates are connected to each other and the negative plates are connected to each other. What will be the potential difference across each and the charge on each?

$C_1 = 2.50\times10^{-6}$ F



$V_1 = 1000$ V

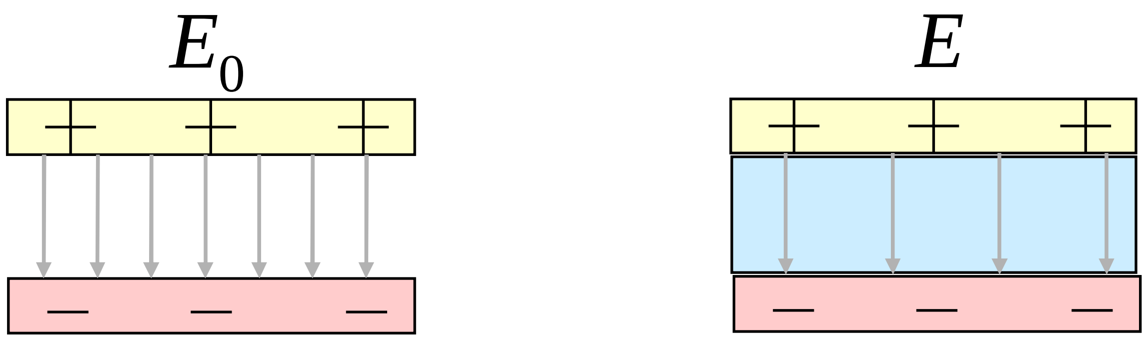
$C_2 = 6.80\times10^{-6}$ F



$V_2 = 650$ V

Dielectrics in Capacitors

Dielectrics increase the capacitance by lowering the electric field in a capacitor for the same amount of charge.



$\kappa = E_0/E$ where κ is the dielectric constant

With the dielectric, the capacitance of a parallel plate capacitor becomes $C = \kappa\varepsilon_0 A/d$

Problem: A parallel plate capacitor has plates that are 2.0 cm by 3.0 cm that are separated by 1.0 mm. The maximum electric field in air, called the dielectric strength of air, is 3.00×10^6 V/m.

(a) What is the maximum charge that can be placed on this capacitor?

Problem: A parallel plate capacitor has plates that are 2.0 cm by 3.0 cm that are separated by 1.0 mm. The maximum electric field in air, called the dielectric strength of air, is 3.00×10^6 V/m.

(b) Paper with a dielectric constant of $\kappa = 3.7$ and a dielectric strength of 16×10^6 V/m is placed between the plates. How much charge can the capacitor hold now?

Interactive Question

A dielectric (with constant κ) is inserted between the plates of a capacitor attached to battery with potential V. What statement below is true?

A) the capacitance is reduced by a factor κ
B) the charge on the plates is reduced by a factor of κ
C) the charge on the plates is increased by a factor of κ
D) the electric field between the plates is reduced by a factor of κ
E) the potential difference between the plates is increased by a factor of κ

Interactive Question

A parallel plate capacitor is charged at a potential V. A dielectric with $\kappa=4$ is inserted between the plates while the potential remains constant. What happens to the amount of charged stored on the plates?

A) It decreases by a factor of 4
B) It decreases by a factor of 2
C) It stays the same
D) It increases by a factor of 2
E) It increases by a factor of 4

The Effect of Dielectrics on Electrostatic Equations

In all the electrostatic equations we have studied, if we replace air with a dielectric material, we simply substitute $\varepsilon = \varepsilon_0 \kappa$ for ε_0.

For example, Gauss's law should be written

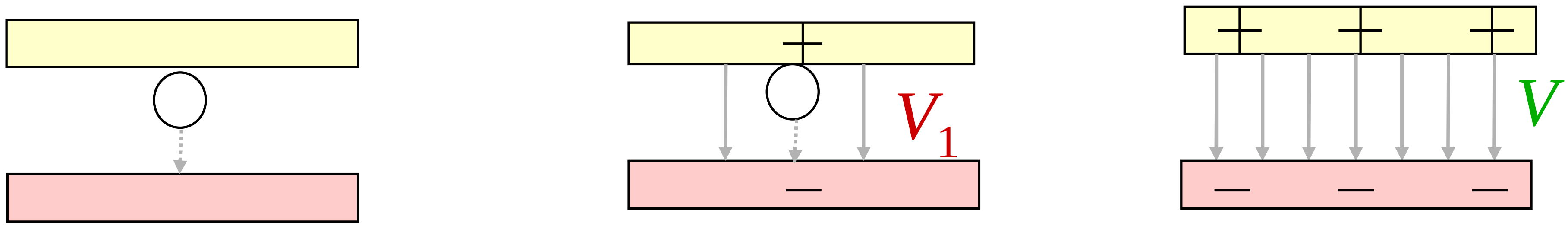
$$\oint \mathbf{E} \cdot d\mathbf{A} = q_{\text{encl}} / \varepsilon_0 \kappa$$

If a point charge is placed inside a dielectric, then Coloumb's law becomes:

$$\mathbf{E} = \hat{\mathbf{r}}\, q / (4\pi \varepsilon_0 \kappa r^2)$$

Energy Stored in a Capacitor

Consider charging a capacitor:



$W = \Delta U_E = \int V dq' = (1/C) \int q' \, dq' = q^2/(2C)$

Since $q = CV$

$U_E = q^2/(2C) = CV^2/2 = qV/2$

The energy density (energy per volume is)

$u_E = U_E/Ad = CV^2/2Ad = (1/2)(\varepsilon_0 A/d)(V^2/Ad)$

$= (1/2)(\varepsilon_0 V^2/d^2)$

$u_E = (1/2)\varepsilon_0 E^2$

This is a general result

Problem: An electronic flash works by storing energy in a capacitor and releasing the energy very quickly. Suppose an electronic flash has a 750 μF capacitor and a potential of 330V.

(a) What is the energy stored?

(b) Assuming the flash lasts for 5×10^{-3} s, what is the power used?

Interactive Question

A parallel plate capacitor with plates of area A and plate separation d is charged so that the potential difference between the plates is V. If the capacitor is then isolated and its plate separation is halved to $d/2$, what happens to the *energy stored* in the capacitor?

A) It decreases by a factor of 4
B) It is halved
C) It is doubled
D) It is increased by a factor of 4
E) It is unchanged