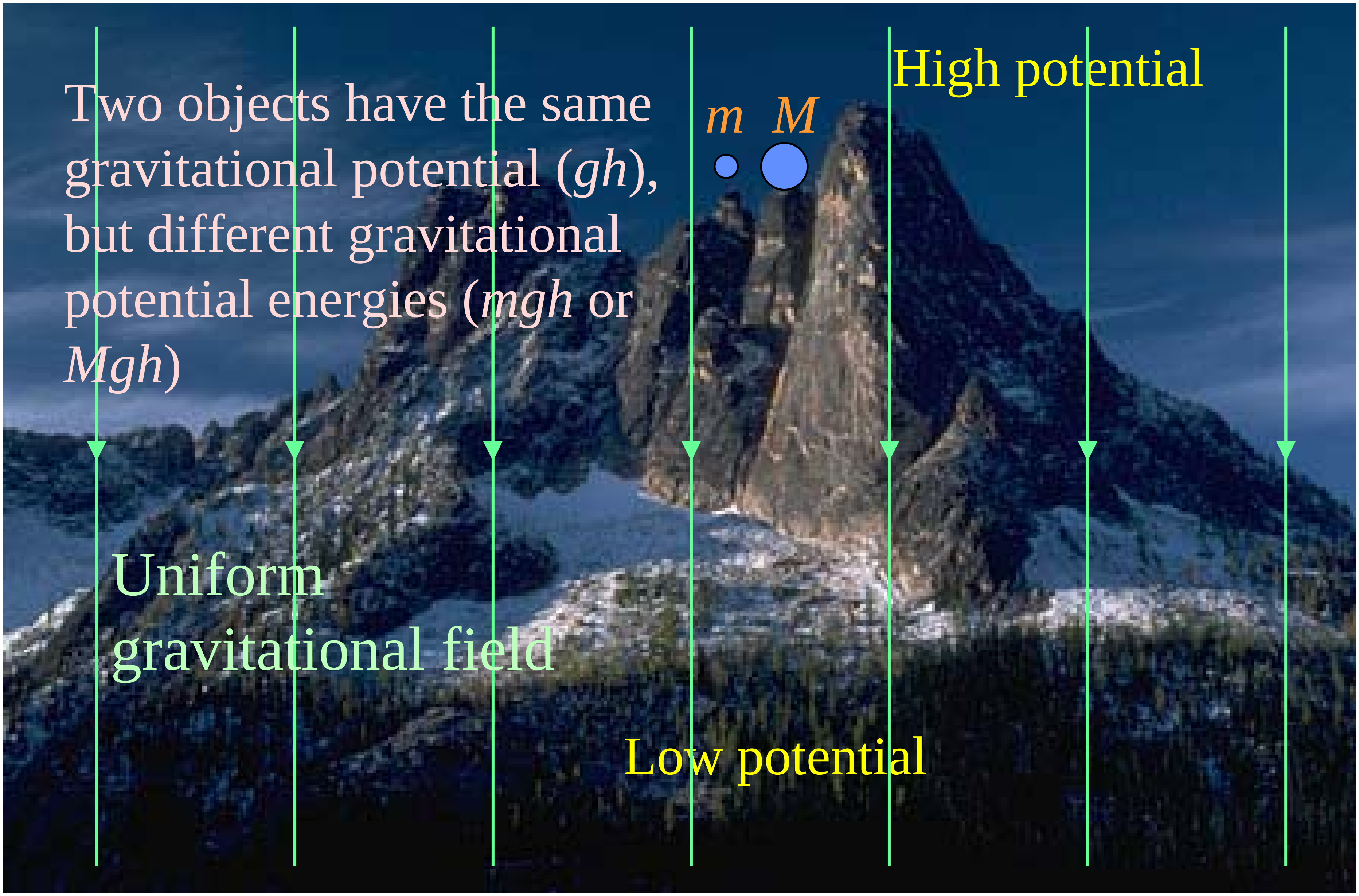
Chapter 17

Electric Potential Energy and the Electric Potential

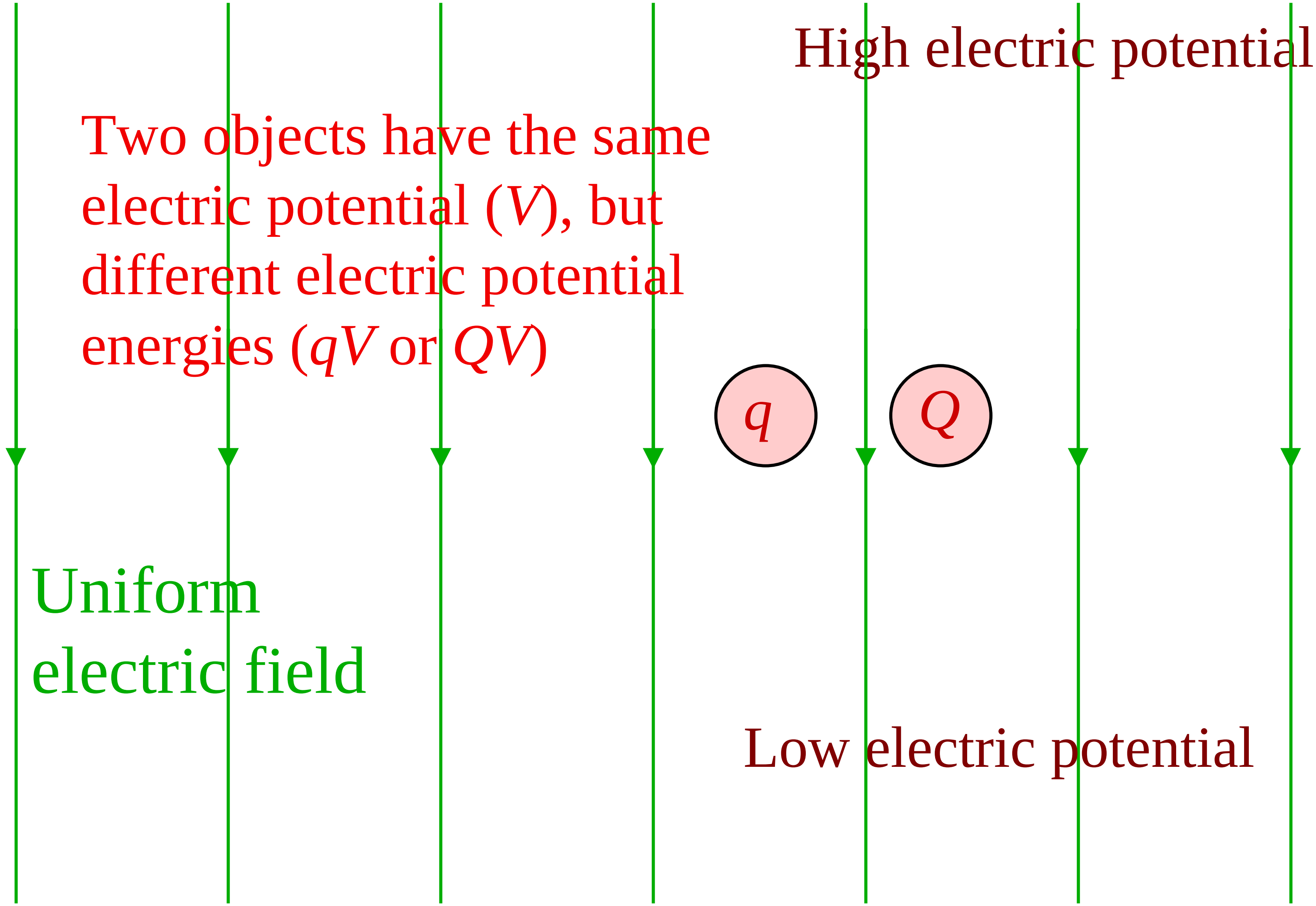
Consider gravity near the surface of the Earth

- The gravitational field is "uniform." This means it always points in the same direction with the same magnitude. This explains why *g* is a constant near the surface of the earth.
- We could consider the gravitational potential energy *(mgh)* as made of two parts.
  (1) mass which depends on the object, and
  (2) the "gravitational potential" ($V_G = gh$) which only depends on the elevation. We could then write the gravitational potential energy as: $U_G = mgh = mV_G$
- Every elevation has the same gravitational potential regardless of mass, but the potential energy depends on mass.


High potential
Two objects have the same gravitational potential (*gh*), but different gravitational potential energies (*mgh* or *Mgh*)
m
M
Uniform gravitational field
Low potential

Now consider an Electric field

- The electric potential (*V*) depends only on the position of a charge in the electric field. Every position has the same electric potential.
- The electric potential energy (U_E) depends on two things:
  (1) the charge of the object (*q*), and
  (2) the electric potential.
- $\Delta U_E = q\Delta V$
- Electric potential is a scalar
- SI units of electric potential is joules/coulombs which is given the name volts (V)



Positive charge plays the role of mass using this “gravitational analogy”.

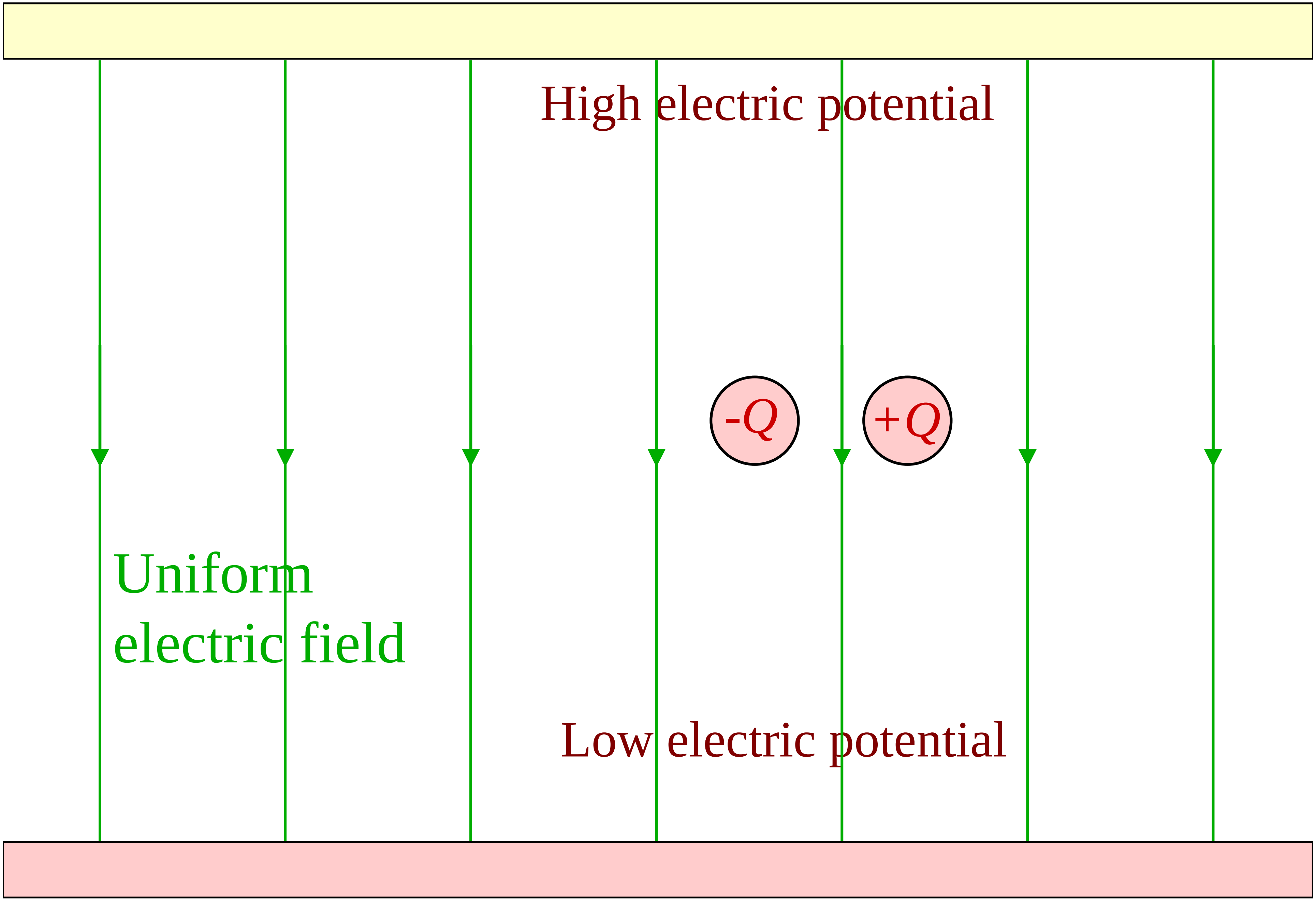
Electric fields, Electric potential and Work

1. Electric fields are conservative fields. (They must be conservative for potential energy to be defined.)
2. $W_{\mathrm{ext}} = -W_E = \Delta U_E = -\int \mathbf{F}\cdot d\mathbf{s} = -\int q\mathbf{E}\cdot d\mathbf{s}$

   $\Delta U_E/q = \Delta V = -\int \mathbf{E}\cdot d\mathbf{s}$

   Notice:
   - This equation is similar in form to $\mathbf{F}/q = \mathbf{E}$
   - There is a relationship between V and $\mathbf{E}$
3. We only measure **changes** in electric potential and potential energy, not absolute values.

   Yet, $q\Delta V = \Delta U_E$ is often written simply as $qV = U_E$
4. The sign of the charge matters. Negative particles have an increase of potential energy when they move to a lower potential.


High electric potential
-Q
+Q
Uniform
electric field
Low electric potential

Interactive Question

Which statement best explains why it is possible to define an *electrostatic potential* in a region of space that contains an electrostatic field.

A) Work must be done to bring two positive charges closer together.

B) Like charges repel one another and unlike charges attract one another.

C) A positive charge will gain kinetic energy as it approaches a negative charge.

D) The work required to bring two charges together is independent of the path taken.

E) A negative charge will gain kinetic energy as it moves away from another negative charge.

Interactive Question

Which is true concerning the work done by an external force in moving an electron at constant speed between two points in an electrostatic field?

A) It is always zero.
B) It is always positive.
C) It is always negative.
D) It depends on the total distance covered.
E) It depends only on the displacement of the electron.

Electric Potential for a Uniform Electric Field

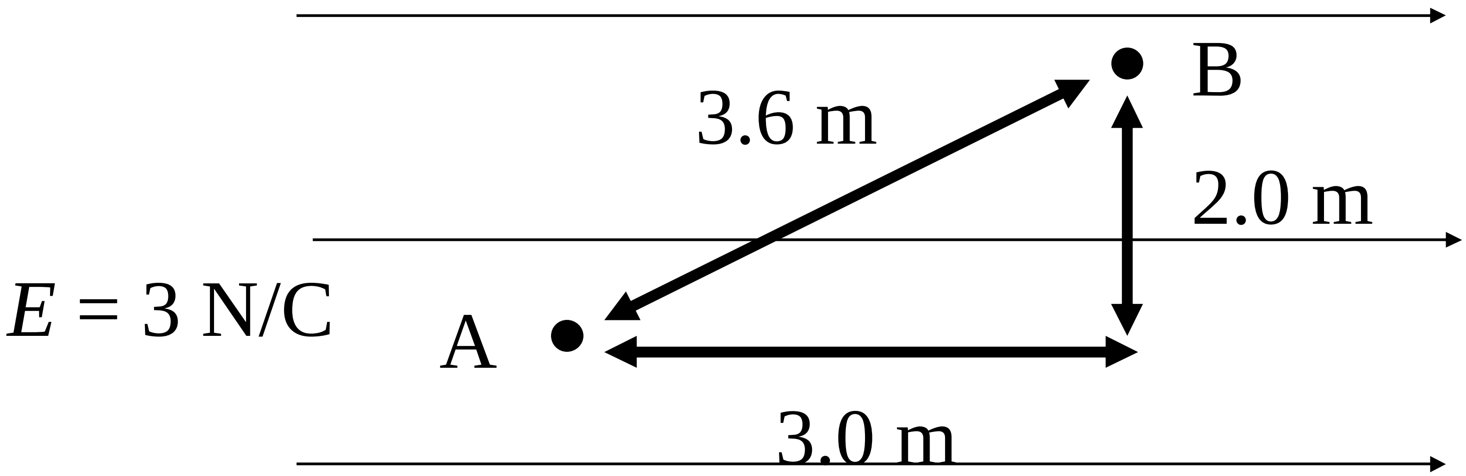
$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s} = -\mathbf{E} \cdot \int d\mathbf{s} = -\mathbf{E} \cdot \mathbf{s}$$

Let's say the field is along the *x* axis:

$$\Delta V = -E_x \Delta x$$

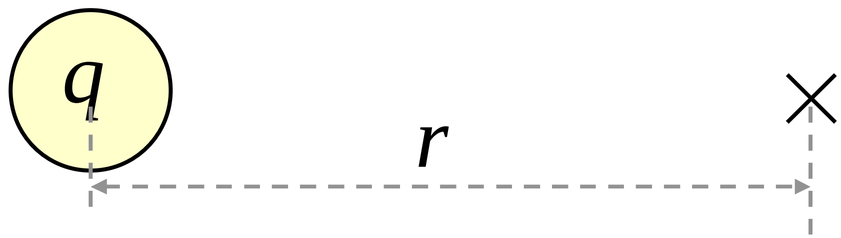
Interactive Question

A 1 μC point charge is moved from point A to B in the uniform electric field shown. Which statement is true concerning the potential energy of the point charge?



A) It increases by 6 μJ.
B) It decreases by 6 μJ.
C) It decreases by 9 μJ.
D) It increases by 10.8 μJ
E) It decreases by 10.8 μJ

Electric Potential for a Point Charge



$$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s} = -\int \frac{kq}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{r}$$

$$= -\int \frac{kq}{r^2} dr$$

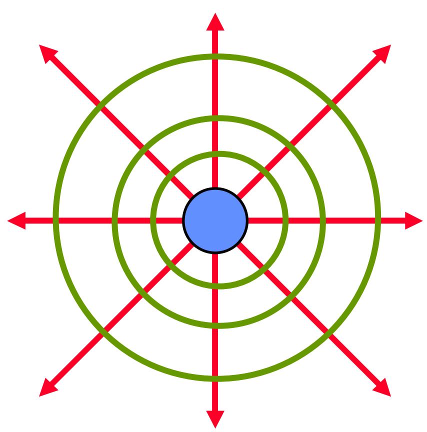
$$= -kq \int \frac{1}{r^2} dr = \frac{kq}{r} + V_0$$

We usually define the potential at ∞ to be 0, so V_0=0,

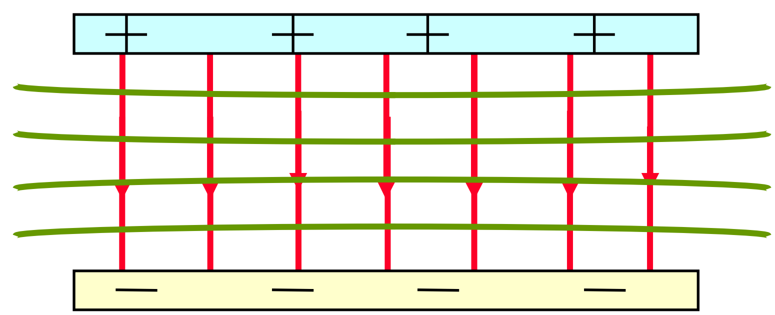
$$V = kq/r$$

Equipotential Surfaces

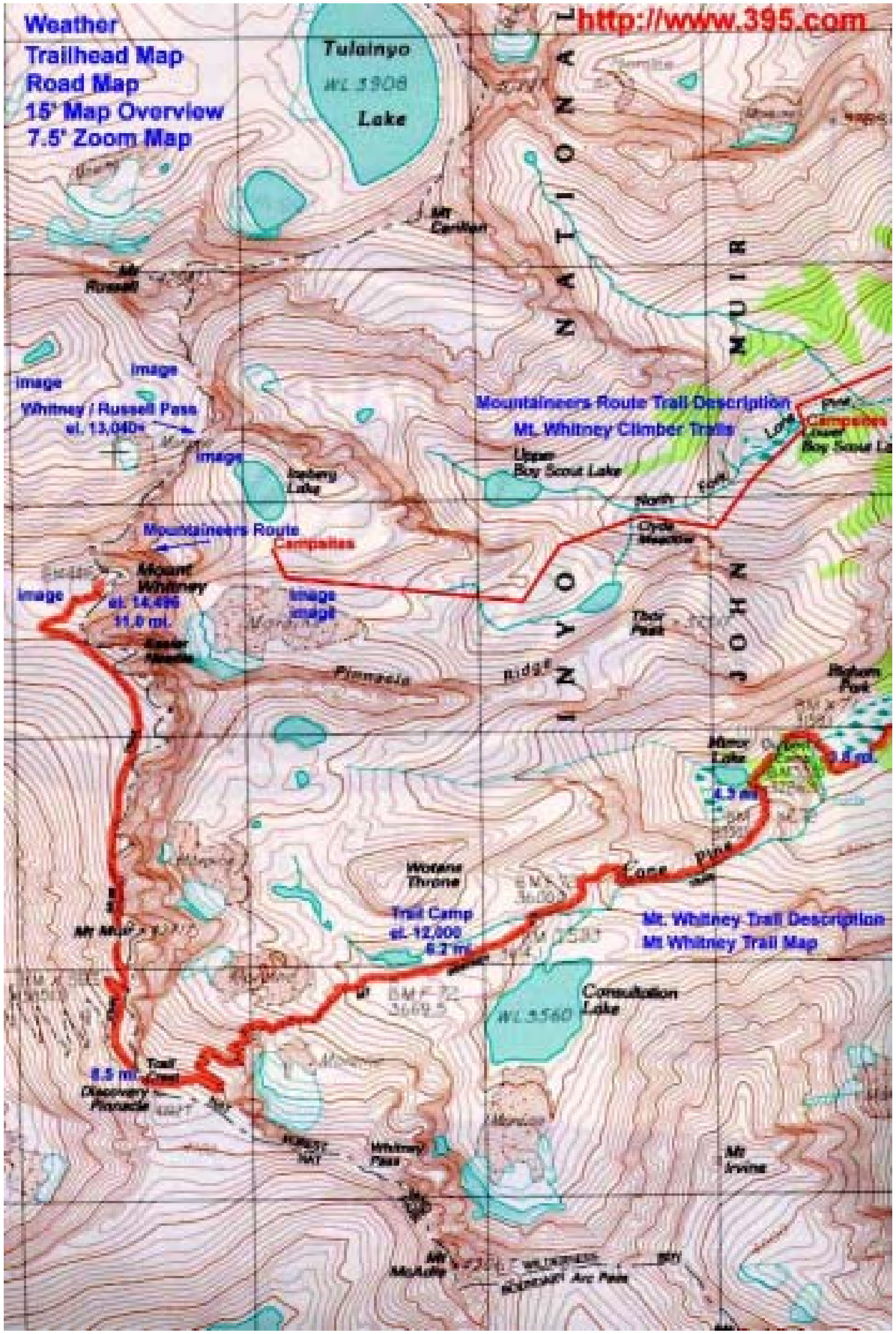
- Equipotential surfaces are imaginary surfaces that are at the same potential. For gravity, equipotential surfaces are lines of constant elevation.
- Equipotential surfaces are always perpendicular to field lines ($\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$).
- No work is required to move a charge along an electrostatic equipotential.



Positive Charge

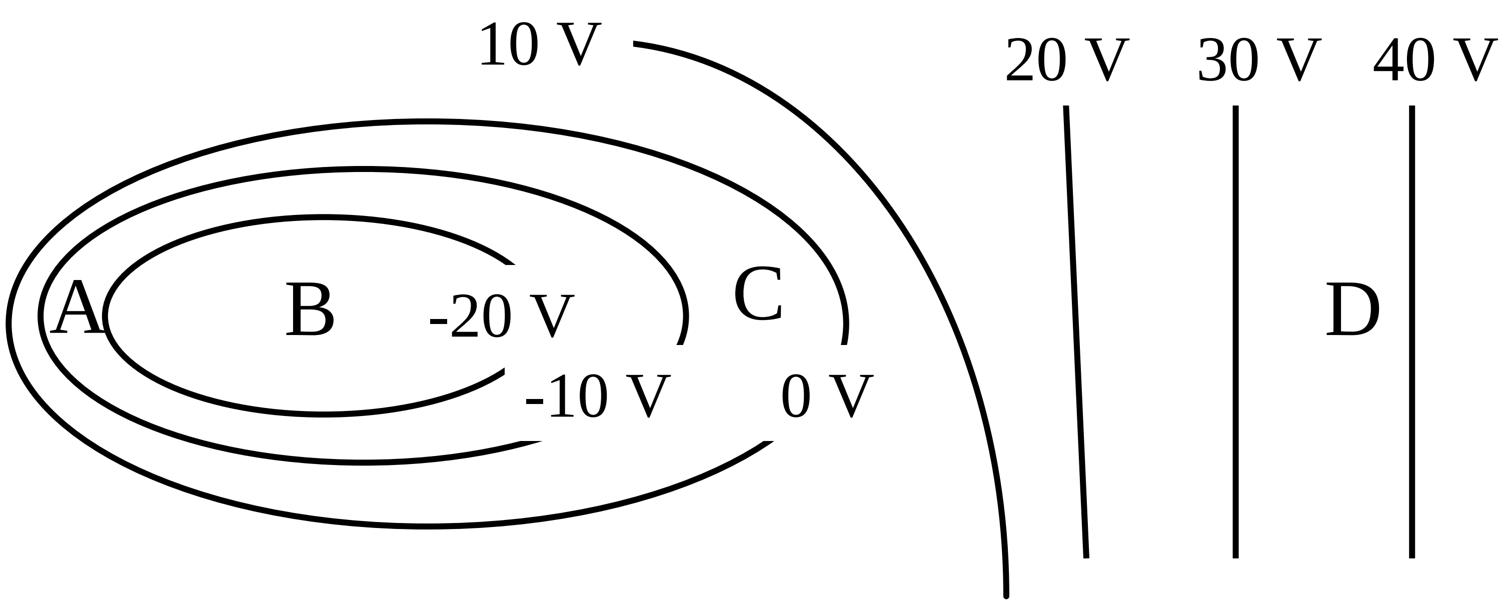


Uniform Field


Weather
Trailhead Map
Road Map
15' Map Overview
7.5' Zoom Map
http://www.395.com
Tulainyo
Lake
Mt Carillon
Mt Russell
image
image
Whitney / Russell Pass
image
Iceberg Lake
Mountaineers Route Trail Description
Mt. Whitney Climber Trails
Upper Boy Scout Lake
Campsites
Boy Scout La
North Fork
Mountaineers Route
Campsites
Clyde Meadow
Mount Whitney
image
image
image
Thor Peak
Pinnacle
Ridge
Mirror Lake
Wotans Throne
Trail Camp
Mt. Whitney Trail Description
Mt Whitney Trail Map
Mt Muir
Consultation Lake
Trail Crest
Discovery Pinnacle
Whitney Pass
Mt Irvine
Mt McAdie
Arc Pass
INYO NATIONAL
JOHN MUIR

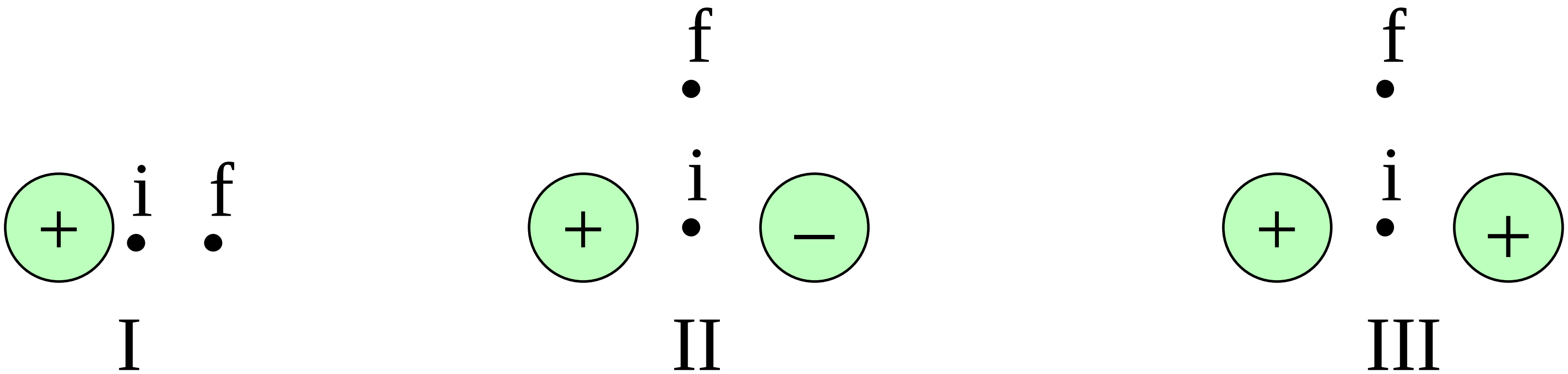
Interactive Question

The picture shows equipotential lines. Where is the electric field the greatest?



Interactive Question

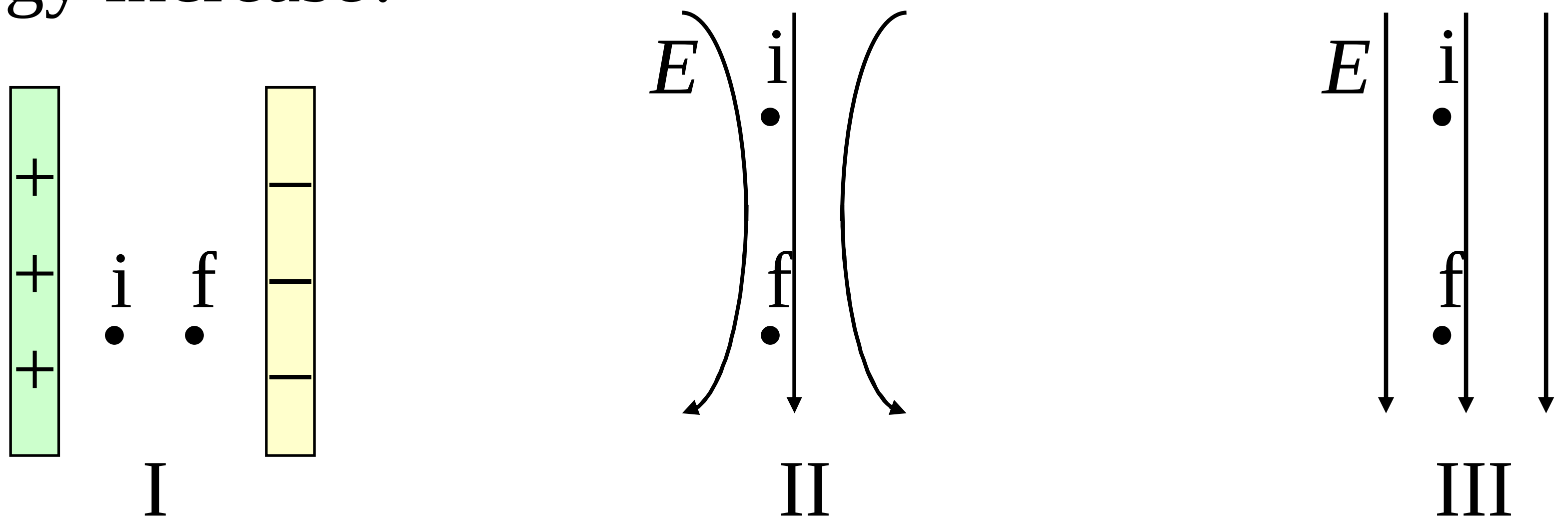
A proton is moved from the point labeled “i” to the point labeled “f”. In which case(s) does the potential energy increase?



A) I only
B) II only
C) III only
D) More than one of the above
E) None of the above

Interactive Question

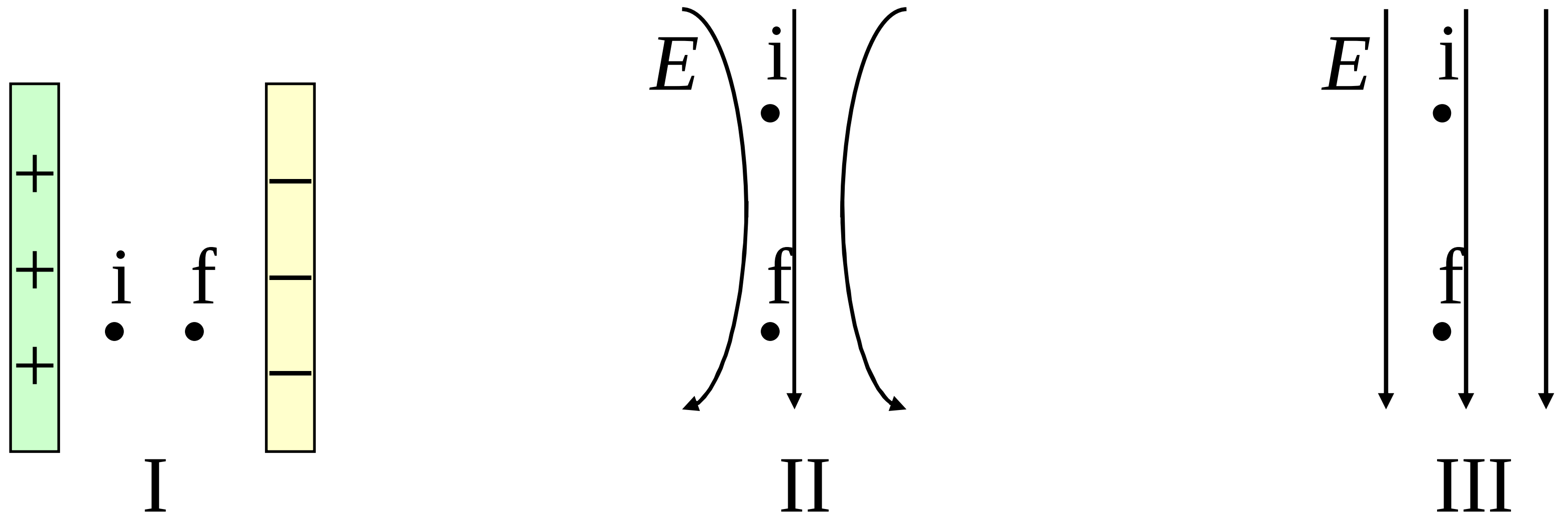
An electron is moved from the point labeled "i" to the point labeled "f". In which case(s) does the potential energy increase?



A) I only
B) II only
C) III only
D) All of the above
E) None of the above

Interactive Question

An electron is moved from the point labeled "i" to the point labeled "f". In which case(s) does the *potential* increase?



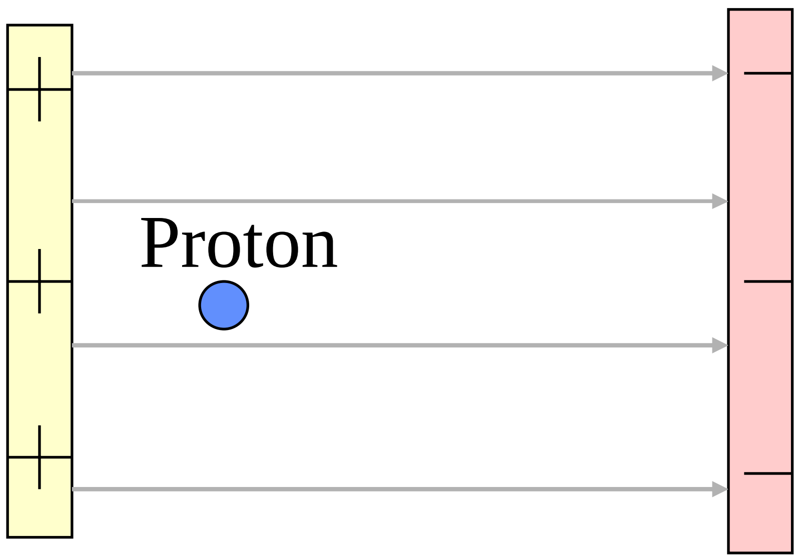
A) I only
B) II only
C) III only
D) All of the above
E) None of the above

Problem: A proton is released from rest in a uniform electric field of magnitude 8×10^4 V/m. After the proton has moved 0.5 meters,

(a) What is the change in electric potential?

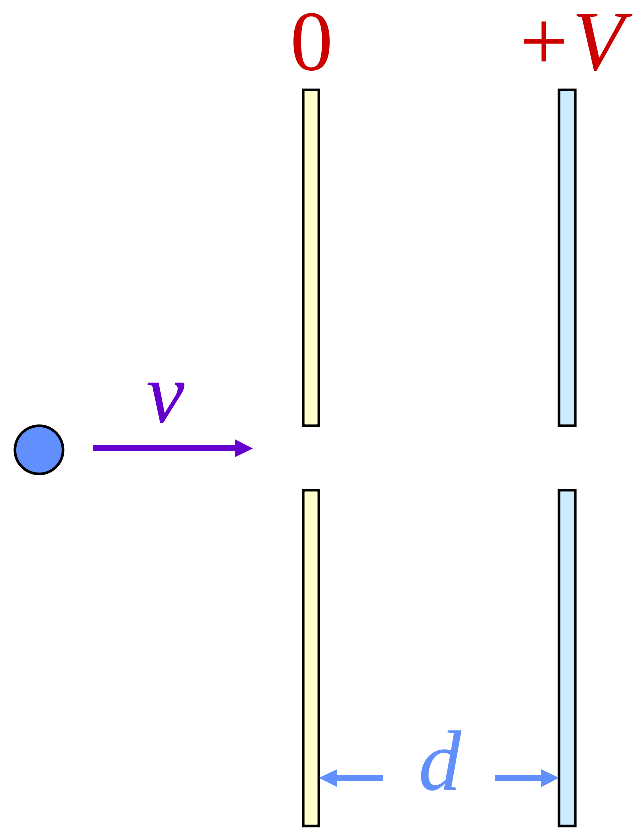
(b) What is the change in potential energy?

(c) What is the speed of the proton?



Interactive Question

An object with mass m and charge $-q$ is projected with speed v into the region between two plates. The potential difference between the plates is V and their separation is d. The change in kinetic energy as the particle traverses this region is:



A) $-qV/d$

B) $2qV/mv$

C) qV

D) $-qV$

E) $mv^2/2$

Interactive Question

The electric potential at a certain point in space is 12V. What is the electric potential energy of a -3μC charge placed at that point?

A) +4 μJ
B) –4 μJ
C) +36 μJ
D) –36 μJ
E) zero

Calculating the Potential

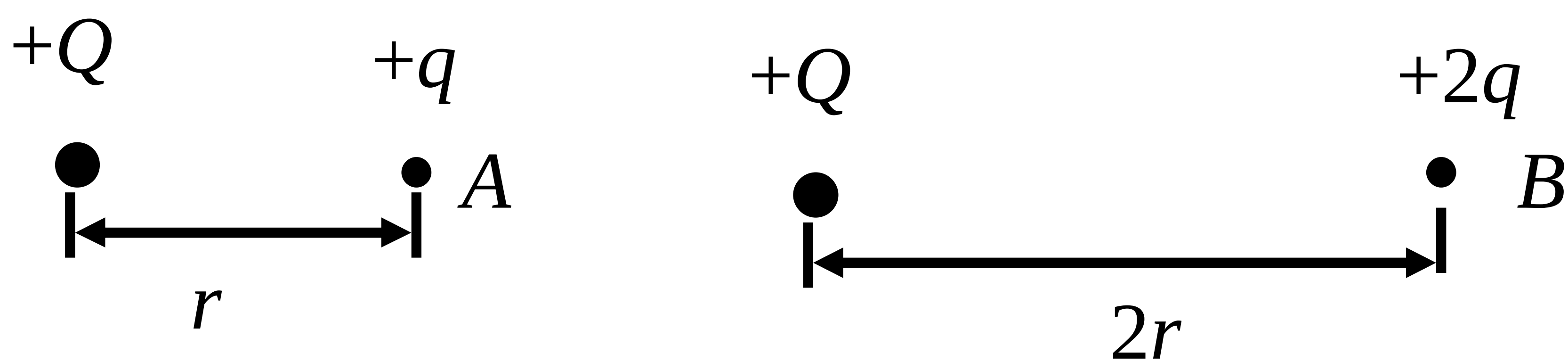
1) Series of point charges: $V = \sum_i (k_i q_i / r_i)$
2) Distribution of charge: $V = \int (k\, dq/r)$
3) From the electric field: $\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$

Calculating the Potential Energy

$$\Delta U_E = q\, \Delta V$$

Interactive Question

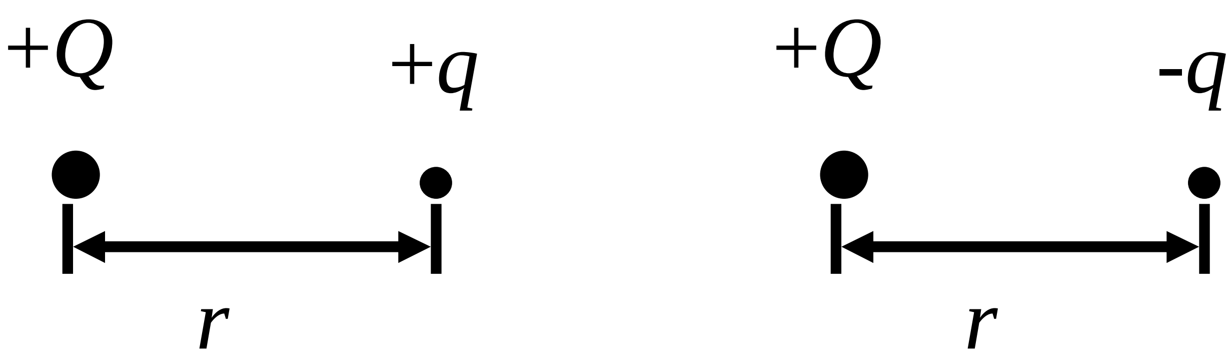
Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to a point A a distance r from $+Q$. Next, $+q$ is removed and a test charge of $+2q$ is brought to point B a distance $2r$ from $+Q$. Compared to the electrostatic *potential* of the charge at A, that of the charge at B is



A) greater B) smaller C) the same

Interactive Question

Two test charges are brought separately into the vicinity of a charge $+Q$. First, test charge $+q$ is brought to a point a distance r from $+Q$. Then this charge is removed and a test charge of $-q$ is brought to point to the same point. The electrostatic *potential energy* of which test charge is greater?



A) $+q$

B) $-q$

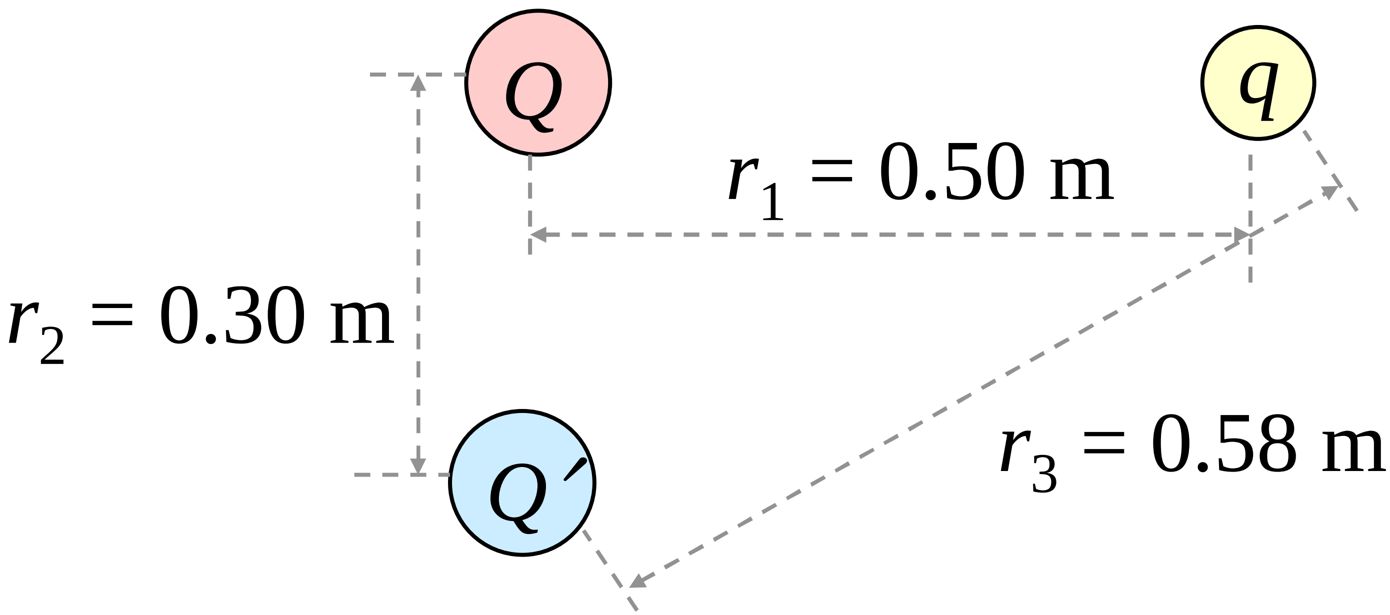
C) it is the same for both.

Calculating the Potential

Problem: Objects with charges of 2.0 μC are placed on opposite corners of a square, 0.025 m on each side. On another corner an object with a charge of –4.0 μC is placed. What is the potential at the fourth corner?

Calculating the Potential Energy

Problem: How much more work does it take to set up the following situation if $q = -3.0\ \mu C$, $Q = 20.0\ \mu C$ and $Q' = 5.0\ \mu C$ to the point shown?



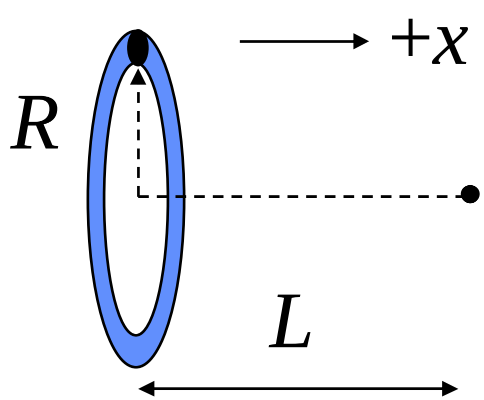
Interactive Question

Two positive point charges Q and q are separated by a distance R. If the distance between the charges is reduced to $R/2$, what happens to the total electric potential energy of the system?

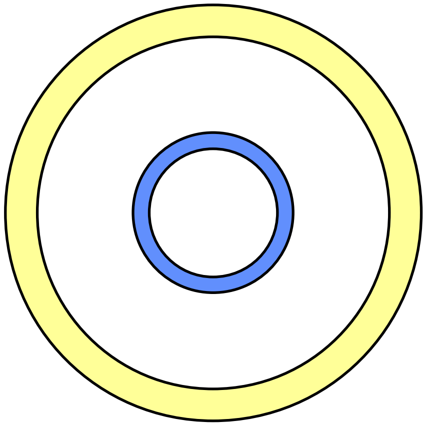
A) It is doubled
B) It remains the same
C) It increases by a factor of 4
D) It is reduced to one-half of its original value
E) It is reduced to one-fourth of its original value

Calculating the Potential for Charge Distributions

Problem: What is the electrostatic potential from a ring a ring of charge with a total charge Q and a uniform charge density and radius R?



Problem: A thin conducting sphere with a radius of R_1 is placed inside the center of a larger conducting sphere with radius R_2. The total chare on the smaller sphere is Q_1 and on the larger sphere Q_2. If the electrostatic potential is zero an infinite distance away from the spheres, what is the potential at R_2, between R_1 and R_2, and inside R_1?



Interactive Question

Two conducting spheres, one having twice the diameter of the other are separated by some distance. The smaller sphere (1) has a charge of q and the larger sphere (2) is uncharged. If the spheres are connected by a thin wire:

A) 1 and 2 will have the same potential
B) 2 will have twice the potential of 1
C) 2 will have half the potential of 1
D) 1 and 2 will have the same charge
E) 2 will have a larger electric field at its surface

Problem: Two spherical objects, one with a smaller radius R_1 and one with a larger radius R_2 have charges on them of q_1' and q_2', respectively. The two spheres are connected by a wire and charge moves from one sphere to another until they reach equilibrium. What is the ratio of the surface charge density on one object compared to the other?

Calculating the Electric Field from the Potential

$\Delta V = -\int \mathbf{E} \cdot d\mathbf{s}$ (1)

In one dimension,

$\Delta V = -\int E_x \cdot dx$ (2)

Take the derivative

$dV/dx = -E_x$ (3)

Equations (2) and (3) are identical.

In three dimensions,

$\nabla V = -\mathbf{E}$ (4)

Equations (1) and (4) are identical, where,

$$\nabla = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$$

Problem: We previously saw that the potential for a ring of charge is given by $kQ/(x^2 + R^2)^{1/2}$. Find the electric field due to this charge distribution.

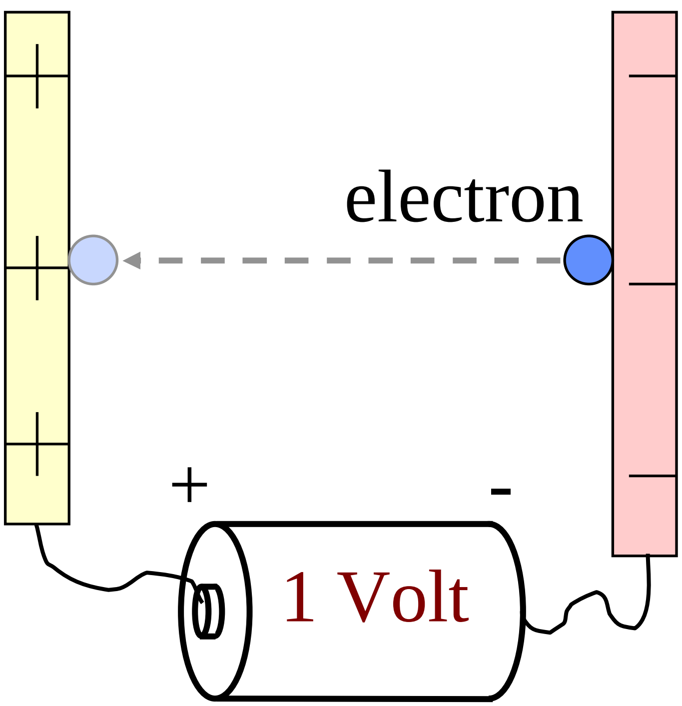
Problem: The electric potential due to a point charge Q at the origin may be written as

$$V = \frac{kQ}{r} = \frac{kQ}{\sqrt{x^2 + y^2 + z^2}}$$

Calculate E_x, E_y, and E_z.

The Electron Volt (A non-SI unit of energy)

Problem: An electron is released from rest and allowed to accelerate across a potential difference of one volt. How much kinetic energy does the electron have?



The answer is simple: It has one electron volt of energy.

Or in SI Units:

$$K_F + U_F = K_I + U_I$$

$$K_F = U_I - U_F = 0 - qV$$

$$= -(-1.6\times10^{-19}\text{ C})(1\text{ V})$$

$$= 1.6\times10^{-19}\text{ J}$$

So $1\text{ eV} = 1.6\times10^{-19}\text{ J}$