Chapter 13

Temperature, Heat Transfer, and the First Law of Thermodynamics
Thermodynamics deals with
1. Temperature.
2. The transfer and transformation of energy.
3. The relationship between macroscopic properties and microscopic dynamics.
What is Temperature?

A property, or “state variable” of a system.

Other state variables are volume and pressure.

But this is not really a very good or complete definition.

We will ultimately discuss the characteristics of a substance that actually determine its temperature.
### Common Temperature Scales

<table>
<thead>
<tr>
<th>Units</th>
<th>Kelvin</th>
<th>Celsius</th>
<th>Fahrenheit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Our Sun</td>
<td>6000</td>
<td>5727</td>
<td>10445</td>
</tr>
<tr>
<td>Carbon Boils</td>
<td>5100</td>
<td>4927</td>
<td>8808</td>
</tr>
<tr>
<td>Carbon Melts</td>
<td>3825</td>
<td>3552</td>
<td>6490</td>
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<tr>
<td>Iron Boils</td>
<td>3023</td>
<td>2750</td>
<td>5032</td>
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<tr>
<td>A Cool Red Star</td>
<td>3000</td>
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<td>4990</td>
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<td>Iron Melts</td>
<td>1808</td>
<td>1535</td>
<td>2823</td>
</tr>
<tr>
<td>Water Boils</td>
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<td>100</td>
<td>212</td>
</tr>
<tr>
<td>Water Freezes</td>
<td>273</td>
<td>0</td>
<td>32</td>
</tr>
<tr>
<td>Oxygen Boils</td>
<td>90</td>
<td>-183</td>
<td></td>
</tr>
<tr>
<td>Oxygen Melts</td>
<td>55</td>
<td>-218</td>
<td></td>
</tr>
<tr>
<td>Absolute Zero</td>
<td>0</td>
<td>-273</td>
<td>-459</td>
</tr>
</tbody>
</table>

\[
t_C = \left(\frac{5}{9}\right)(t_F - 32^\circ) \\
T = t_C + 273.15 \text{ K}
\]
Thermal Equilibrium is the term given to two objects at the same temperature.
The Zeroth Law of Thermodynamics:
If two objects are in thermal equilibrium with a third, then they are in thermal equilibrium with each other.
How can you measure temperature?

Certain properties of substances, like volume or length, can change as temperature varies. These are called thermometric properties.

**Constant-volume gas thermometer**

At the triple point of water the temperature is defined as 273.16 K.
Thermal Expansion

Linear Expansion: Empirical measurements show that a change in temperature will usually cause an object to expand linearly with temperature.

\[ \Delta L = \alpha L_0 \Delta T \]

\(\alpha\) is the coefficient of linear expansion, \(\alpha = (1/L_0) \, dL/dT\)

Volume and area Expansion: Empirical measurements show that a change in temperature will usually cause an object to increase its volume and area with temperature.

\[ \Delta V = \beta V_0 \Delta T \]

\(\beta\) is the coefficient of volume expansion, \(\beta = (1/V_0) \, dV/dT\)

\(\alpha\) and \(\beta\) vary slightly with temperature, but we will ignore that small variation.
Consider a rectangular box with sides of length $L_1$, $L_2$, and $L_3$.

$$V = L_1 L_2 L_3$$

$$\left(\frac{1}{V_0}\right) \frac{dV}{dT} = \left(\frac{L_2 L_3}{V_0}\right) \frac{dL_1}{dT} + \left(\frac{L_1 L_3}{V_0}\right) \frac{dL_2}{dT} + \left(\frac{L_1 L_2}{V_0}\right) \frac{dL_3}{dT}$$

$$\beta = \left(\frac{1}{L_1}\right) \frac{dL_1}{dT} + \left(\frac{1}{L_2}\right) \frac{dL_2}{dT} + \left(\frac{1}{L_3}\right) \frac{dL_3}{dT}$$

$$\beta = 3 \alpha$$

In a similar manner:

$$\Delta A = (2\alpha)A_0 \Delta T$$
Interactive Question

A circular disk with a coefficient of linear expansion of $\alpha$ is heated and undergoes a temperature change of $\Delta T$. How does the diameter ($d$) of the disk change?

A) $\Delta d/d_0 = \alpha \Delta T$
B) $\Delta d/d_0 = 2\alpha \Delta T$
C) $\Delta d/d_0 = 3\alpha \Delta T$
D) $\Delta d/d_0 = \sqrt{\alpha \Delta T}$
E) None of the above
Problem: When the temperature of a copper penny is increased by 100°C, its diameter increases by 0.18%. (a) What is the coefficient of linear expansion. (b) What is the percent increase in the thickness, (c) the area of the face, (b) the volume, and (e) the mass of the penny?
Interactive Question

A bimetallic strip is made by putting two metals together. As the temperature changes the strip will bend. What can you say about the coefficient of linear expansion of the two metals.

\[
\begin{array}{|c|c|}
\hline
\text{Metal 1} & \alpha_1 \\
\hline
\text{Metal 2} & \alpha_2 \\
\hline
\end{array}
\]

\[ T = T_0 \]

\[ T > T_0 \]

A) \( \alpha_1 = \alpha_2 \)
B) \( \alpha_1 > \alpha_2 \)
C) \( \alpha_1 < \alpha_2 \)
D) Not enough information to say anything
The figure shows a rectangular brass plate. If the temperature is raised.

A) $x$ will increase and $y$ will decrease.
B) Both $x$ and $y$ will decrease.
C) $x$ will decrease and $y$ will increase.
D) both $x$ and $y$ will increase.
E) the changes in $x$ and $y$ depend on the dimension $z$. 
Problem: Mercury fills 90% of the volume of a sealed glass container at 20ºC. What temperature does the glass/Hg have to be heated to so that the mercury completely fills the container? \( \alpha_{\text{Hg}} = 6 \times 10^{-5} \, ^\circ\text{C}^{-1}, \alpha_{\text{glass}} = 0.4 \times 10^{-5} \, ^\circ\text{C}^{-1} \).
Ideal Gas Law

This has been experimentally determined to be true

\[ PV \propto T \]

\( P \): Pressure, \( V \): Volume, \( T \): Temperature

\[ PV = CT \]

\( C \) is a constant

\[ PV = NkT \]

\( N \): The number of particles (molecules) of gas

\( k = 1.381 \times 10^{-23} \text{ J/K} \) (Boltzmann’s Constant)

\[ PV = nRT \]

\( n = N/N_A \): Number of moles

1 mole is \( N_A = 6.022 \times 10^{23} \) particles (Number of \(^{12}\text{C} \) atoms in 12 grams of \(^{12}\text{C} \)).

\( R = 8.314 \text{ J/mol} \cdot \text{K} \) (the universal gas constant)
Interactive Question

Have you already seen the ideal gas law and done problems using it in another class?

A) Yes, and I still remember the concepts
B) Yes, but I don’t remember much about it
C) No
D) What is the ideal gas law anyway?
Interactive Question

Which statement is true?

A) $R = Nk$
B) $R = N_A k$
C) $R = k/N$
D) $R = k/N_A$
E) $R = kN/N_A$
What is the slope of this graph as measured for an ideal gas?

A) $k$
B) $R$
C) $nR$
D) $kT$
E) More than one of the above
Which curve represents a constant temperature for a certain amount of an ideal gas?
Does it matter which temperature scale you use in the ideal gas law? Why or why not?

\[ PV = nRT \]

What about units for Pressure or Volume?
Problem: A 2 liter bottle is filled with nitrogen (N₂) at STP and closed tight. (STP is “Standard Temperature and Pressure” of 273 K and 1 atm.)

(a) What is the mass of the N₂?

(b) If the temperature is raised to 100° C, what will be the new pressure.

1 atm = 1.013×10⁵ Pa = 1.013×10⁵ N/m²

1 L = 1000 cm³ (1 m/1000 cm)³ = 1×10⁻³ m³
Interactive Question

In part (b) above, could we have used lb/in² as the pressure measurement or Celsius as the temperature measurement?

<table>
<thead>
<tr>
<th>lb/in²</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>B) Yes</td>
<td>No</td>
</tr>
<tr>
<td>C) No</td>
<td>Yes</td>
</tr>
<tr>
<td>D) No</td>
<td>No</td>
</tr>
</tbody>
</table>
Interactive Question

Two identical rooms in a house are connected by an open doorway. The temperatures in the two rooms are maintained at different values. Which room contains more air?

A) The room with the higher temperature
B) The room with the lower temperature
C) Neither because they both have the same volume
D) Neither because they both have the same pressure
Two identical containers are filled, one with H₂ gas and one with N₂ gas. Objects of identical mass are placed on top of moveable pistons and the two gasses fill identical volumes.

If both gasses have the same mass, which is at a higher temperature?

A) H₂  B) N₂
C) They are at the same temperature
D) There is not enough information to tell
A certain amount of gas is put into a container and the container is sealed. The volume of the container is decreased by a factor of 2 and the temperature is decreased by a factor of 3. If the original pressure was $P$, what is the final pressure of the gas?

A) $\frac{1}{6} \ P$
B) $\frac{2}{3} \ P$
C) $\frac{3}{2} \ P$
D) $6 \ P$
E) None of the above
What is heat?

Energy transferred from one system to another because of a temperature difference.

What happens when heat is added to a system?

1) A temperature increase
2) A phase change
3) An isothermal expansion
Temperature change due to heat:

\[ Q = C \Delta T = mc \Delta T = nc_{\text{molar}} \Delta T \]

\( \Delta T \) is always \( T_f - T_i \)

\( C \) is the heat capacity, \( c \) is the specific heat, \( c' \) is the molar specific heat.

\( C \) is energy change per temperature change
\( c \) is energy change per temperature change per mass
\( c_{\text{molar}} \) is energy change per temperature change per mole

(in chapter 14 and beyond, \( c \) is used for \( c_{\text{molar}} \). This can be confusing.)

\[ c_{\text{molar}} = C/n = mc/n = Mc \]

Where \( M \) is the molar mass (\( M=m/n: \) grams/mole)
1 cal is approximately the amount of energy needed to raise 1 g of water by 1° C

1 Btu is approximately the amount of energy needed to raise 1 pound of water by 1° F

1 cal = 4.186 J
1 kcal = 1 Calorie

1 cal/g·C° = 1 Btu/lb·F° = 4186 J/kg·K

The heat capacity depends on how the heat is transferred, particularly for gases

\[ c_V = \frac{1}{m}(dQ/dT)_V \]
\[ c_P = \frac{1}{m}(dQ/dT)_P \]
\[ c_{V \text{-molar}} = \frac{1}{n}(dQ/dT)_V \]
\[ c_{P \text{-molar}} = \frac{1}{n}(dQ/dT)_P \]
Interactive Question

Given that the same amount of heat will affect the following objects in the following ways. Which object has the greatest specific heat?

A) Raises the temperature of 3 g of substance 1 by 10 K
B) Raises the temperature of 4 g of substance 2 by 4 K
C) Raises the temperature of 6 g of substance 3 by 15 K
D) Raises the temperature of 8 g of substance 4 by 6 K
E) Raises the temperature of 10 g of substance 5 by 10 K
Interactive Question

Two objects of different temperatures are brought together. Eventually the objects reach thermal equilibrium. Which of the following statements is true once they have reached thermal equilibrium?

A) They are at the same temperature.
B) The absolute value of the temperature change of each object was the same.
C) Both objects have the same specific heat.
D) More than one of the above is true.
When two systems are placed in thermal contact and allowed to come to thermal equilibrium, heat will flow from the hotter system to the cooler system until they are at the same temperature. Since energy must be conserved, the amount of heat leaving one system is the same amount of heat absorbed by the other.

The book uses $Q_{\text{in}} = Q_{\text{out}}$, but I like the following better. Conservation of energy requires that the change in total energy is zero, so:

\[ Q_1 + Q_2 = 0 \]
\[ m_1 c_1 \Delta T_1 + m_2 c_2 \Delta T_2 = 0 \]

Note the signs of $\Delta T_1$ and $\Delta T_2$. 
Interactive Question

The heat capacity of object B is twice that of object A. Initially, object A is at 300 K and object B is at 450 K. They are placed in thermal contact and the combination is isolated. The final temperature of both objects is:

A) 300 K
B) 350 K
C) 375 K
D) 400 K
E) 450 K
Problem: A copper coin with a mass of 75 g is heated to a temperature of 312°C and then dropped into a beaker containing 220 g of water. The initial temperature of the water and beaker is 12°C and the heat capacity of the beaker is 45 cal/K. Find the final temperature of the system.
Energy Added

T

Boiling, or Vaporizing

Melting, or fusion

Energy Added

\[ Q_f = mL_f \]
\[ Q_V = mL_V \]

\( Q_f \): “latent heat of fusion”
\( L_f \): “latent heat of fusion”
\( L_V \): “latent heat of vaporization.”
Problem: A 1.00 kg block of copper at 20°C is dropped into a large vessel of liquid nitrogen at 77.3 K. How many kilograms of nitrogen boil away by the time the copper reaches 77.3 K?
Problem: A physics student drops 50 g of ice, initially at −20°C into a styrofoam cup containing 250g of water, initially at 25°C. Neglecting the heat capacity of the cup, what will be the final temperature of the drink? Will there still be ice in the cup?
Transfer of thermal energy

1) Convection: Mass carrying thermal energy is transported. “Hot air rises”

2) Conduction: Mass interacts to transport energy. Molecules may collide with other molecules. “Don’t grab a metal spoon that is partially submerged in hot water.”

3) Radiation: No mass is transported. Thermal energy is transported by electromagnetic radiation. “Always wear sunscreen.”
Conduction

\[ \frac{dQ}{dt} = -kA \frac{dT}{dx} \]

**A:** Cross sectional area of the object(s) transferring heat.

**k:** Thermal conductivity (SI units of W/m·K)

1) The units are J/s or Watts

2) This looks a lot like a current with heat “flowing” just like charge “flows”. \( H = \frac{dQ}{dt} \)

3) The minus sign indicates that heat flows in the opposite direction from the temperature gradient.

4) After integration, this can be written as

\[ \frac{\Delta Q}{\Delta t} = kA \frac{(T_H - T_L)}{d} \]

where \( d \) is the distance the heat is transferred \((x_f - x_i)\).
We can write this in a form that looks like Ohm’s law
\[ \Delta T = HR' \] with \( R' = L/kA \)

However, the term “thermal resistance” \( (R) \) usually means
\[ R = R' / A = L/k \]

This is the \( R \) value quoted for building insulation. The units in the U.S. are \( \text{ft}^2\cdot\text{hr}\cdot\text{°F}/\text{Btu} \).

These thermal resistances can be added in series or in parallel like resistors in an electric circuit.

With this thermal resistance, we can write:
\[ H = dQ/dt = A(T_H - T_L)/R \]
Let’s put two objects next to each other and calculate the total thermal resistance for these objects in “series”. The heat flowing through each object must be the same.

\[
\frac{\Delta Q}{\Delta t} = k_1 A \frac{(T_M - T_L)}{d_1} = k_2 A \frac{(T_H - T_M)}{d_2}
\]

\[
k_1 \frac{T_M}{d_1} - k_1 \frac{T_L}{d_1} = k_2 \frac{T_H}{d_2} - k_2 \frac{T_M}{d_2}
\]

\[
T_M = \frac{(k_2 \frac{T_H}{d_2} + k_1 \frac{T_L}{d_1})}{(k_2/d_2 + k_1/d_1)}
\]

\[
T_M = \frac{(k_2 T_H d_1 + k_1 T_L d_2)}{(k_2 d_1 + k_1 d_2)}
\]

Put this into the first equation:

\[
\frac{\Delta Q}{\Delta t} = k_1 A \frac{(T_M - T_L)}{d_1} = k_1 A \frac{((k_2 T_H d_1 + k_1 T_L d_2))}{(k_2 d_1 + k_1 d_2)} - T_L \] /d_1
\]

\[
= k_1 A (k_2 T_H d_1 + k_1 T_L d_2 - k_2 T_L d_1 - k_1 T_L d_2)
\]

\[
\div \{d_1 (k_2 d_1 + k_1 d_2)\}
\]

\[
= k_1 A (k_2 T_H d_1 - T_L k_2 d_1) / d_1 (k_2 d_1 + k_1 d_2)
\]
Continuing from the previous slide

\[
\frac{\Delta Q}{\Delta t} = d_1 k_1 k_2 A (T_H - T_L) / d_1 k_2 k_1 (d_1/k_1 + d_2/k_2)
\]

\[
\frac{\Delta Q}{\Delta t} = A (T_H - T_L) / (d_1/k_1 + d_2/k_2)
\]

This can be generalized to more than two objects with the same heat flowing through them:

\[
\frac{\Delta Q}{\Delta t} = A (T_H - T_L) / \sum (d_i/k_i) = A (T_H - T_L) / R_{eq}
\]

This is the same as an equivalent \( R \) value of:

\[
R_{eq} = \sum R_i
\]

For thermal resistors in “parallel” it can be shown that

\[
(1/R_{eq}) = \sum (1/R_i)
\]
Problem: (a) What is the rate of heat loss (in W/m²) through a glass window 3.0 mm thick if the outside temperature is -10°C and the inside air is 20°C? (b) What would be the rate of heat loss for the same total thickness of glass with a 7.5 cm air gap between two panes of glass. Assume conduction is the only heat loss mechanism. 

\[ k_{\text{glass}} = 0.84 \text{ W/m} \cdot \text{K}, \quad k_{\text{air}} = 0.023 \text{ W/m} \cdot \text{K}, \]
Interactive Question

Two identical rectangular rods of metal are welded together as shown and 10 J of heat is conducted through the rods in 2.0 min.

How long would it take 10 J to be conducted through the rods if they were welded together like this?

A) 0.5 min  B) 1.0 min  C) 2.0 min  D) 4.0 min  B) 16 min
Radiation

The transfer of thermal energy by electromagnetic (em) radiation without any exchange of matter. This can happen in the absence of matter, like through outer space.

The rate at which an object emits or absorbs EM radiation:

\[ P = \frac{\Delta Q}{\Delta t} = \sigma \varepsilon A T^4 \]

- \( A \): surface area
- \( \varepsilon \): emissivity (a number between 0 and 1). An object that absorbs radiation easily also emits radiation easily with \( \varepsilon \) near 1.
- \( \sigma \): Stefan-Boltzmann constant (5.6703×10^-8 W/m^2·K^4)
- \( T \): Temperature of the object gives the power radiated.
  Temperature of the surroundings gives power absorbed by the object.
Interactive Question

Compare a white coat and a black coat illuminated by sunlight. The white coat gives off more light than the dark black coat. Which likely has the greater emissivity?

A) The white one
B) The black one
C) They are both the same
Problem: While camping out one night, you want to make some ice. Unfortunately, the air temperature only gets down to 6°C at night. However, a clear moonless night does act like a blackbody radiator with a temperature of –23°C. You pour a thin layer of water with a mass of 4.5 g into a container and insulate it from the warm ground with a layer of styrofoam. The water has a surface area of 9.0 cm², a depth of 5.0 mm, and an emissivity of 0.90 at an initial temperature of 6.0 °C. Will the water freeze?
Problem: Suppose you want to lose weight. The fastest you can burn calories while doing high aerobic activity is about 700 kcal (Calories)/hour. (250 while walking, 60 while sleeping/watching TV). Compare this rate with sitting naked in a cold room (16°C, 60°F). The human body has a surface area of about 1.7 m². Assume your skin is a blackbody at a temperature of 35°C. (98.6 °F = 37°C)
Problem: Suppose you want to lose weight. You sit in a bathtub with water temperature of 16°C with your skin at a temperature of about 35°C. The water next to your skin is warm, but cools to 16°C over a distance of about 1 cm. How many Calories do you burn in an hour?
When you get out of bed on a cold winter morning, a tile floor will feel colder than a carpeted floor. This is because:

A) A given mass of tile contains more heat than the same mass of carpet.
B) Tile conducts heat better than carpet.
C) Heat tends to flow from tile to carpet.
D) The temperature of the tile is lower than the temperature of the carpet.
E) The human body, being organic, resembles carpet more closely than it resembles tile.
Interactive Question

A thermos bottle works well because

A) Its glass walls are thin
B) Silvering reduces convection
C) Vacuum reduces heat radiation
D) Silver coating is a poor heat conductor
E) None of the above
Terminology for Different Processes

Isothermal: Temperature does not change
Isobaric: Pressure does not change
Isochoric: Volume does not change
Adiabatic: No heat flow ($Q=0$)
Free Expansion: A gas expands adiabatically, without doing any work, so $Q=0$, $W=0$.
Cyclic: The system returns to the same physical state that it started from.
Reversible and Irreversible Processes

Reversible processes are those that can proceed through very small steps so that the system is always at equilibrium.

No law of physics that we have studied prevents the eggs from coming back together, but they won’t.
Which of the following is a reversible process?

A) A glass vase falls on the floor and shatters.
B) Ice is taken from the freezer and melts.
C) Air flows into a chamber that initially contained a vacuum.
D) None of the above.
What is Required for Reversible Processes?

• A real reversible process must always be at equilibrium.
  • No real processes of change are reversible
• A quasi-static process is approximately reversible

• An process with no heat transferred (adiabatic) can be reversible.
• If heat is transferred with no temperature difference, or in the limit, is transferred infinitesimally slowly in an isothermal process, then the process can be reversible
• Adiabatic and isothermal processes without any friction or other energy loss are the only reversible processes.
Interactive Question

Match the curves on the $PV$ diagram with the correct process for an ideal gas.

<table>
<thead>
<tr>
<th></th>
<th>Isothermal</th>
<th>Isobaric</th>
<th>Isochoric</th>
<th>Adiabatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>B)</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>C)</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>2</td>
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<tr>
<td>D)</td>
<td>2</td>
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<td>1</td>
<td>3</td>
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<tr>
<td>E)</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
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Conservation of Energy, and the First Law of Thermodynamics

\[ W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta E_{\text{mech}} + \Delta E_{\text{int}} \]

\[ W_{\text{ext}} = \Delta K + \Delta PE + \Delta E_{\text{therm}} + \Delta E_{\text{chem}} + \Delta E_{\text{other}} \]

We can add energy to the system by adding heat. Let’s neglect chemical and other energy, and consider the thermal energy to be the internal energy of the system \((U)\).

\[ Q + W_{\text{ext}} = \Delta K + \Delta PE + \Delta U \]

In Physics I, we didn’t deal with heat or internal energy, so this became: \( W_{\text{ext}} = \Delta K + \Delta PE \)

In Thermodynamics, we usually don’t deal with changes in potential energy or kinetic energy, and we usually deal with the work done by the system \((W)\), so this becomes:
\[ Q = \Delta U + W \]

where \( W = -W_{\text{ext}} \)

This is the first law of thermodynamics, which is a special case of the general principle of conservation of energy:

\[ Q - W = \Delta E_{\text{mech}} + \Delta U + \Delta E_{\text{chem}} + \Delta E_{\text{other}} \]
Work done by a gas:

Consider a piston with a cross sectional area of $A$ that can move only along the $x$ axis.

$$W = \int F \, dx = \int PA \, dx = \int P \, dV$$

It is true, in general, that $W = \int P \, dV$
Problem: How much work is done when an ideal gas is expanded isothermally from $V_1$ to $V_2$?
Pressure vs. Volume graphs for a certain gas undergoing five different processes are shown. Rank these from the greatest work done to the least. Positive work should be ranked higher than negative work.

Interactive Question

A) IB, L, IT, A, IV
B) IT, A, L, IB, IV
C) IB, IV, L, A, IT
D) IV, IB, L, IT, A
E) None of the Above
Pressure vs. Volume graphs for a certain gas undergoing five different processes are shown. During which cycle does the gas do the greatest positive work?

(A) 

(B) 

(C) 

(D) 

(E)
Interactive Question

Of the following, which might not vanish over one cycle of a cyclic process?

A) $\Delta U$
B) $\Delta P$
C) $W$
D) $\Delta V$
E) $\Delta T$
Work done by Various Processes

**Isothermal**

\[ W = \int P \, dV \]

For an ideal gas: \[ W = nRT \ln \frac{V_2}{V_1} \]

**Isobaric**

\[ W = \int P \, dV = P \int dV = P \, \Delta V \]

For an ideal gas: \[ W = nR \, \Delta T = nRT_2(1 - \frac{V_1}{V_2}) \]

**Isochoric**

\[ W = \int P \, dV = 0 \]