## 2D Collisions Lab

## 1 Introduction

In this lab you will perform measurements on colliding pucks in two dimensions. The spark table will allow you to determine the velocity and trajectory of the pucks. You will collide pucks of equal masses and unequal masses, both elastically and inelastically.

## 2 Equipment

- spark table with carbon paper
- air pump,
- spark generator,
- pucks of various masses,
- connecting hoses and wires,
- paper,
- rulers, protractors and pens


## 3 Procedures

Read through the procedures and familiarize yourself with the equipment. Do not turn on the spark generator until you know what you are doing! Make sure you understand the purpose of each measurement and devise a strategy for taking data. How can you best minimize the uncertainty of you measurements?

### 3.1 Getting Acquainted with Mr. Spark Table:

1. Place a large sheet of paper (newsprint) on the carbon papered surface of the spark table. Put the pucks on top of the table. The pucks should be connected to the pump through the hoses. Turn on the pump and watch the pucks drift about. You should level the spark table in much the same way you leveled the airtrack: put the pucks at rest and watch for any drift to one side or the other.
2. Turn the spark generator to a setting of 10 Hz . which means that it fires ten sparks in one second. Tap the pucks with a plastic or wooden ruler. Do not use your hands. When the pucks are gliding have someone depress the pedal switch for a few seconds. You should hear a clicking noise. You can use the ruler to guide the pucks about. Disappointingly, nothing will show up on your paper.
3. Turn off the pump and spark generator. Remove the paper and look at its underside. Aha! There are the marks left by the spark generator. You should see a set of dots that mark the trails of your pucks. When the puck is moving rapidly the dots will be further apart.

In each of the following experiments you will be expected to place a clean sheet of paper on the spark table when you take data. Practice the collisions before you press the switch and actually take data. Make sure that everyone in the group has their hands away from the table before pressing the pedal switch. Use an insulating object (plastic or wood) to move the pucks when the generator is on.

### 3.2 Elastic Collisions:

1. Practice arranging the pucks for an elastic collision. They should both have an initial velocity before the collision, and both should be moving afterwards.
2. Take data on a clean sheet of paper. Before removing the paper from the table, mark clearly the approximate of the incoming and outgoing paths. (Remember that directions are reversed on the back side of the paper.)

3. Remove the paper. Draw a set of x and y axes on it, centering at the point of collision. (You may wish to place the x -axis along the direction of one of the incoming pucks for simplicity.) Determine the velocity of the pucks by measuring the distance the pucks travel in a set number of intervals. For example, five intervals (a total of six dots, counting the endpoints) takes a total of half a second at the 10 Hz setting. By measuring the distance the puck moves in that period you can calculate the velocity. Take your interval somewhere in the middle of the trajectory, away from the collision or the end of the interval.
4. Measure the angles of the incoming and outgoing trajectories with respect to the x -axis. This will allow you to break the velocity up into components.
5. Weigh the pucks.
6. On the datasheet provided reproduce the momentum vectors on the graph provided. Reproduce the angles as best as possible. Below is an example of such a graph.

7. Determine the total momentum before the collision in the x and y directions, by taking components of $\vec{p}_{1}$ and $\vec{p}_{2}$ and then adding them.
8. Determine the total initial kinetic energy.
9. Repeat the above, calculating the total momentum and kinetic energy after the collision.
10. On your data set identify which points correspond to the position of the puck at the same instant in time. Connect these points in sets of straight lines (see figure). You should have several such lines before the collision, and several after the collision. The center of mass of the two-puck system will lie on that line. If the total distance between the two pucks is L then the center of mass will be on the line at a distance

$$
L_{\mathrm{cm}}=\frac{m_{2}}{m_{1}+m_{2}} L
$$

from puck 1. (The center of mass will always be closer to the more massive puck.) If the two pucks have the same mass it will be at the center of the line connecting them at equal times. Determine the velocity of the center of mass before and after the collision by locating it at several points, and then measuring how far it travels in a known interval of time.

Repeat all of the above with two pucks of unequal mass. (There should be pucks of different mass. You may also attach masses to the pucks themselves.

### 3.3 Inelastic Collisions:

There are velcro strips which can be attached to the pucks. These will cause the pucks to stick to one another when they collide. This will allow you to perform perfectly inelastic collisions. Make sure that the velcro does not scrape along the paper when the pucks travel.

Repeat the above experiments for the inelastic case, taking the same data. Don't forget that the mass of the puck will be different.

Practice your collisions before taking data. You want to create a collision where the pucks travel off together without rotating. If they are rotating it will be very hard to determine their final velocity, since the points will travel in loops.

## 4 Questions:

In addition, please answer the following questions.

1. Did you observe conservation of momentum and energy in your elastic collisions? When answering this question, you need to determine the dominant uncertainties in your measurement, and use statistical analysis to decide if your final and initial energies and momenta agree. To within how many standard deviations do you observe conservation of momentum and energy?
2. Did you observe conservation of momentum and energy in your inelastic collisions?
3. Describe the motion of the center of mass before and after the collisions. How does its velocity change before and after the collision?
4. List any possible sources of systematic and random errors that you did not include in your error calculations. State whether or not these errors would affect your ability to observe conservation of energy, conservation of momentum or both.
5. Assume that the spark generator did not fire at 10 Hz but at twice that rate. How would this affect your ability to observe conservation of energy and conservation of momentum?

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|  | Before | After |
| :--- | :--- | :--- |
| Speed of puck 1 |  |  |
| $\theta_{1}$ |  |  |
| Speed of puck 2 |  |  |
| $\theta_{2}$ |  |  |
| $p_{x}$ of puck 1 |  |  |
| $p_{\mathrm{x}}$ of puck 2 |  |  |
| Total $p_{\mathrm{x}}$ |  |  |
| $p_{\mathrm{y}}$ of puck 1 |  |  |
| $p_{\mathrm{y}}$ of puck 2 |  |  |
| Total $\mathrm{p}_{\mathrm{y}}$ |  |  |
| $\mathrm{E}_{\mathrm{k}}$ of puck 1 |  |  |
| $\mathrm{E}_{\mathrm{k}}$ of puck 2 |  |  |
| $p_{\mathrm{x}}$ of CoM |  |  |
| $p_{y}$ of CoM |  |  |


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