# The Pendulum

## Introduction:

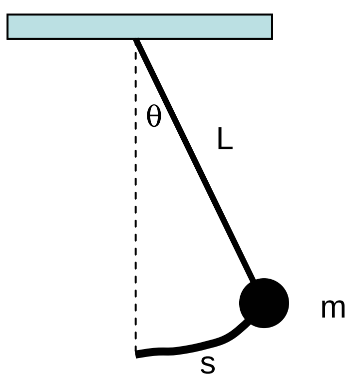
The purpose of this tab is to predict the motion of various pendulums and compare these predictions with experimental observations.

## Equipment:

- Simple pendulum made from string and a bob
- Stop watch
- Protractor
- Physical pendulum made from a metal bar
- Pasco Rotation sensor
- Various Sails

## THEORY:

The simple pendulum is a physical system consisting of a single mass suspended at the end of a string. The force of gravity determines the motion of the pendulum. The size of the mass should be small compared to the length of the string, and the mass of the string is neglected.




Let L be the length of the string and θ be the angle that the string makes with the vertical axis, as shown. θ will be positive when the object swings to the right and negative when it swings to the left. Let S be the arc length measured along the circular path from its intersection with the vertical axis. From the definition of radians, we know that $S=L\theta$ when θ is measured in radians.

To calculate the motion of the object we need to determine the forces on it. These are the tension, T, in the string and the weight, mg, of the object. We can resolve these forces into radial and tangential components.

Draw a force diagram for the simple pendulum

Use Newton's second law to show that:

$$d^2\theta/dt^2 = - g/L \sin\theta \qquad (1)$$

Newton's second law thus predicts that the angle $\theta$ will vary in time as determined by the above differential equation. Note that the time dependence of $\theta$ does not depend on the mass. The function that satisfies this equation is hard to find so we will make a small angle approximation.

We will assume that the angle $\theta$ is small enough that

$$\text{Sin}\,\theta \approx \theta \qquad (2)$$

This simplifies equation (1) so that:

$$d^2\theta/dt^2 = -g/L\,\theta \qquad (3)$$

A function that satisfies this equation is

$$\theta(t)= \theta_o \cos([g/L]^{1/2} t + \delta) \qquad (4)$$

Show that this function satisfies equation (3) using the chain rule for differentiation.

At what angle $\theta$ does $\theta = \sin\theta$ to within 1%

Plot the function and show that it is <u>periodic</u>, repeating itself every time t changes by $2\pi\ [g/L]^{1/2}$

This is called the <u>period</u>, T, of the motion. The <u>angular frequency</u> is then $\omega = 2\pi / T = [g/L]^{1/2}$

# Procedure:

You will first measure the period of the simple pendulum using a stop watch.

Set up a simple pendulum and measure the length L and mass m

1. For fixed L, measure the period of oscillation for several different masses. Do this for small angles. Make sure that the initial angle you choose is small enough that the small angle approximation is valid.

1a. Measure the period by determining the time it takes for the pendulum to make one oscillation. Repeat this five times and using the five measurements determine the average period of the pendulum with its correct error. Make sure you use propagation of errors to determine the uncertainty on your expected period.

L=_______________
m=_______________

| Measurement | Period |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |

Average Period__________________
Expected Period__________________

1b. Measure the period by determining how long it takes the pendulum to make five oscillations. Do this once and determine the pendulum period and error.

Average Period:___________________
Expected Period___________________

Question: Do you expect the two measurements to give you the same answer and the same uncertainty. Explain.

Which technique should you use to determine the average Period?

2. Keeping the same length of the pendulum, change the mass of the pendulum and measure its period.

L=________________
m=_________________

Average Period=______________
Expected Period=______________

Does the period change?   Is your answer expected?  Does your measured period agree with the expected period.  Make sure you use statistical ideas to determine if two numbers agree.

3. Keeping the mass the same, change the length of the pendulum.

L=_________________
m=________________

Average period=________________
Expected Period=_______________

3a.  Repeat with a different length pendulum

L=___________________
m=__________________

Average period=_______________
Expected Period=_______________

Is the period changing as expected?  Explain

4. For fixed L and m, measure the period for several different initial angles, including both large angles ($\theta > 20^o$) and small angles ($\theta < 5^o$).

L=__________________
m=_________________
θ=_________________ (small angle)

Average Period=________________
Expected Period using small angle approximation=_____________

L=__________________
m=_________________
θ=_________________ (large angle)

Average Period=________________
Expected Period using small angle approximation=_____________

L=__________________
m=_________________
θ=_________________ (large angle)

Average Period=________________
Expected Period using small angle approximation=_____________

Question: For large angles, does the period you determine agree with the period you calculated using the small angle approximation. How many standard deviations are they apart? Based on the number of standard deviations that they are apart, can you conclude that the small angle approximation is incorrect for your large angle measurements?

As the angle increases, does the difference between the calculated value and your measured period increase, decrease or remain constant as the angle is increased.

# Period for large angle oscillations:

For large angles the period can be shown to be

$$T = T_0\left[1 + \frac{1}{4}\sin^2\frac{\phi_0}{2} + \frac{1}{4}\left(\frac{3}{4}\right)^2\sin^4\frac{\phi_0}{2} + \ldots\right]$$

Where $T_0 = 2\pi\sqrt{\frac{L}{g}}$

Calculate the period for your previous measurements using this relationship. Compare to your experimental measurements.

L=___________________
m=__________________
θ=__________________ (large angle)

Average Period=_________________
Expected Period for general angle =______________

L=___________________
m=__________________
θ=__________________ (large angle)

Average Period=_________________
Expected Period for general angle =______________

Question: For large angles, does the period you determine agree with the period you calculated using the general angle equation.

# Physical Pendulum:

If we replace the string with a solid rod, we can longer neglect the mass of the rod and therefore we can no longer consider the pendulum a simple pendulum.

In the case of a physical pendulum the period can be shown to be equal to

$$T = 2\pi\sqrt{\frac{I}{mgD}}$$

where I is the moment of inertia about the rotation axis and D is the distance from the center of mass to the rotation axis.

Show that this equation reduces to T= $2\pi\sqrt{\frac{L}{g}}$ in the case of a simple pendulum

5) Measure the length and mass of the metal bar.

Bar Length: ___________________________
Bar Mass: ______________________________

Setup a physical pendulum using the metal bar. Insert the metal bar into the PASCO rotation sensor.

What is the moment of inertia of your physical pendulum______________________

What is the distance D for your physical pendulum _________________________

What is the expected period of the physical pendulum__________________________

Does your expected period depend on the mass of the physical pendulum? ________________

Setup the software to measure the angle of the rotation sensor as a function of time. Using an initial small angle, start the physical pendulum and hit start on the computer. Based on the PASCO sensor determine the period of the pendulum. You can do this many different ways. You can record the data as a function of time, interpolate between the points and determine the period. You can also fit the data to a sine wave to determine the period (note, the fitted frequency fits okay, but the amplitude does not seem to fit properly)

Measured Period ___________________

Does the measured period and predicted period agree?

Start the physical pendulum with a large angle and measure the period.

Measured period___________________

Does the period of the physical pendulum change if the initial angle is changed?

Start the physical pendulum with a different mass m

Measured period_____________________

Start the physical pendulum with a different length.

Measured period______________________

Does the period of the physical pendulum change as expected.

# 5. Underdamped oscillations.

The oscillation of a pendulum will eventually stop because mechanical energy is dissipated by frictional forces. Such motion is called damped. In the case that the damping is small enough such that the system oscillates with an amplitude that slowly decreases with time, the motion is called underdamped. See Figure 1

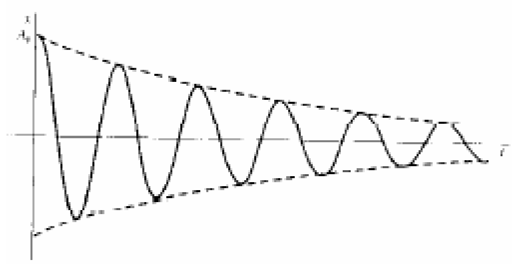


Figure 1.  Damped oscillation curve

In this case the amplitude A decreases as a function of time $A=A_0e^{(-t/\tau)}$ where $A_0$ is the amplitude at time t=0 and $\tau$ is the characteristic time.

Attach a large sail onto the physical pendulum to increase the frictional forces on the pendulum. Start the pendulum with a small angle and measure the amplitude as a function of time. Determine the characteristic time for your pendulum.  Note that at time $t=\tau$ the amplitude will have changed by $e^{-1}$.

$\tau$=______________________

Change the size of the sail and repeat your measurement

$\tau$=______________________

Does the characteristic time change as you change the size of the sail?  Is the change expected? Explain.

# DISCUSSION:

1. Describe the kinetic energy and potential energy of the pendulum during its motion. How do they change with time? Derive an expression for kinetic and potential energy as a function of $\theta$. Show that the mechanical energy is constant as a function of time if we ignore friction.

2. Prove that the following are also solutions to equation (4).

$$\theta(t) = \theta o \sin(\omega t + \delta)$$
$$\theta(t) = A \cos(\omega t + \delta) + B \sin(\omega t + \delta),$$

where $\omega = [g/L]^{1/2}$ . Which solution is appropriate if the clock is started at t = 0 when the pendulum is at its maximum amplitude? Which is appropriate when the pendulum is started when it is hanging straight down with some initial velocity? Which is appropriate if the pendulum is started in some intermediate position?

3. Compare the equation governing the simple pendulum (in the small angle approximation) to the equation governing the motion of a mass on a spring. Compare the solutions to both equations. *Why are the equations similar?*