## Kinematics Lab

## 1 Introduction

An object moving in one dimension and undergoing constant or uniform acceleration has a position given by:

$$
x(t)=x_{0}+v_{0} t+1 / 2 a t^{2}
$$

where $x_{0}$ is its initial position (its position at $t=0$ ), $v_{0}$ is its initial velocity, and $a$ is the value of the constant acceleration. The velocity of the object is:

$$
v(t)=v_{0}+a t
$$

You will attempt to measure the value for the acceleration due to gravity in two ways. First, you will measure positions at various times, and try to fit the best line you can to the data. Second, you will measure velocities at various times and fit the data to determine the acceleration.

You will also be required to do uncertainty or error analysis. You will also have random uncertainty that comes from random variations in wind currents, or how precisely you release the glider. You will obtain this uncertainty by making many measurements and comparing the data. You will also determine your overall uncertainty by determining how well your data agree with the assumptions of constant velocity and constant acceleration. Many of the values will be obtained by the computer. It is important to understand how the computer determines these values, so you will determine many of the values yourself to allow you to compare them to the computer.

## 2 Equipment

- airtrack with air pump,
- bumpers,
- glider,
- pegs for gliders,
- computer with measurement software,
- two Sonar Rangers


## 3 Procedures

Read through the procedures and familiarize yourself with the equipment. Make sure you understand the purpose of each measurement and devise a strategy for taking data. How can you best minimize the uncertainty of you measurements? Repeat each measurement several times for each set of initial conditions.

### 3.1 Learning to use the Sonar Ranger Software

1. Set up the air track. After the air supply has run 2 or 3 minutes at a setting of 2 or higher, the temperature of the track should stabilize. Carefully level the track so that a glider released from rest remains (nearly) at rest at several positions along the track.
2. Install a Sonar Ranger at each end of the air track.


- Select DataStudio from the desktop
- Select Create Experiment. You should see a Science Workshop Interface picture
- In the Experiment setup window, in the left scroll menu, scroll down and select Motion Sensor twice. After you select it twice, you should see two motion sensor symbols attached to the interface showing the correct connections of the yellow and black wires. Make sure you connect the wires correctly.
- In the Displays window, select Graph. You now have the option of plotting position, velocity or acceleration as a function of time. Note that the Sonar Ranger only measures positions as a function of time and the velocity and acceleration are calculated values.
- Select Position Ch $1 \& 2$ and also Position Ch $3 \& 4$. Two empty graphs should appear with axes labels Position and time.
- Make sure a blocker is placed in the glider and gently give it a push.
- Click the start button near the top and you should begin to see the glider's position as a function of time appearing on both graphs. Depending on whether the glider is approaching or receding from the Sonar Ranger you should see a positively or negatively sloped line. Note that if the slope is positive on one graph it should be negative on the other graph. You may see points that deviate far from the line. This occurs when the Sonar Ranger does not obtain a good signal. In these instances you will need to ignore these spurious data points. Look to see where you see a nice linear relationship between position and time. Note where you see deviations from linearity. This often happens when the glider is very near the Sonar Ranger. This will be important as you decide which data you will be using to determine your velocity.
- After taking some data select stop and compare the two graphs. You can change the scale with the mouse by moving the cursor near an axis until you see a symbol of a squiggly line with two arrows at each end. By holding down the left mouse button you can change the scale. You can also move the entire graph by moving the cursor near an axis until you see a hand symbol. Again by holding down the left mouse button, you can move the entire graph.
- Find a region on your graph where you have at least 7 data points that are good and using the left mouse button highlight them. They should turn yellow after they are selected. Click on the Fit button and within the Fit pull down menu select Linear Fit. A linear fit should appear on your graph showing the fit parameters. The slope on a position vs time graph is the velocity. Fit two regions for each of the Sonar Rangers. Are they the same, how much to they differ? Chose a different set of data points near where you fit. Does the fit change? This will give you some indication on the uncertainties involved and possible miscalibrations that exist with the Sonar Rangers.
- If you see large differences between the Sonar Rangers, you may need to calibrate them. Click on the motion sensor symbol that is attached to the interface. If you select calibration, you can place an object at 1 meter and hit calibrate. Here you can also change the frequency of the
measurements. Select a frequency that is proper for your measurement. Typically a frequency of 10 Hz should be fine for most experiments.
- Now select the velocity graph. You should see that the data points scatter around a lot. This shows that if the velocity is constant, the Sonar Ranger does not work well in reporting the velocity. For this reason, we will use the slope of the position vs time graph to determine the velocity.
- To remove old runs, select Experiment and scroll down to Delete ALL Data Runs, or Delete Last Data Run.


### 3.2 Constant Velocity Experiments

If the airtrack is level there should be no acceleration of the glider due to gravity. You should verify this by experiment. Start the glider with a push, and make the following measurements. Make sure you have installed the bumpers!

1. Start the glider with a push and push Start on the computer. After a few bounces end the run. Find regions where you have at least seven data points that are linear and select and fit the data to a straight line. Do this at an early time and a time near where you ended running. Change the fitting region and refit again. Record the fitted slopes and errors for both Sonar Rangers. When you change the number of data points to fit, does the slope change more than the uncertainty on the fitted line? Within the Displays window select Table. You will see data points of time and Position for your run. Record these numbers. Note that there is no error shown for the Position. You will use statistical analysis to try and determine the errors in the Position later. Repeat this for several runs, trying to start the glider with the same speed each time.

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
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| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
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| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |


| Time | Position | Time | Position | Time | Position | Time | Position |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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How constant is the speed of the glider on a given run?

How accurately could you reproduce the initial velocity?

How do the two readings from the Sonar Ranger agree? Give quantitative arguments using the ideas of uncertainties to determine if two quantities are consistent.
2. Within 1 run, determine the slope of your lines in at least 5 different time intervals. Determine the mean and the standard deviation of the velocity from this information. Note that the uncertainties for each measurement may be different so you will need to average these numbers correctly taking into account the uncertainties. If your velocity is changing as a function of time, does it make sense to average the 5 time intervals? Use these numbers to estimate how constant the velocity is on the air track.

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Time Interval $\qquad$

|  | Slope | Uncertainty |
| :--- | :--- | :--- |
| Sonar Ranger 1 |  |  |
| Sonar Ranger 2 |  |  |

Mean velocity: $\qquad$
Standard Deviation: $\qquad$

### 3.3 Acceleration due to Gravity

Now use a block or any other handy object (not your lab partner) to tilt the air track. Gravity will cause the glider to accelerate down the air track. If the air track is tilted an angle $\theta$ from the horizontal then the component of acceleration along the track is

$$
a=g \sin \theta .
$$



In order to proceed you must measure both the angle and acceleration as accurately as possible for several runs. You measure the angle of the tilt by again measuring the height of the air track above the table at two locations. You should then repeat the experiment at one or more angles.

1. Hold the glider with a pencil eraser at a known location at the high end of the track. Select velocity vs time graphs and position vs time graphs for both of the Sonar Rangers.

A single run will consist of releasing the glider from rest, and measuring the velocity and position as a function of time. Make at least 5 runs, replacing the glider each time at the start. Repeat the experiment several times, always starting the glider at rest from the same position. Find a region on the velocity vs time graph where the data are linear and fit the data to determine the slope for
each Sonar Ranger. Again change the number of data points and refit to get an idea of the variation in the slopes. Record the numerical values of velocity and position you find for each run. Since the acceleration due to gravity should be a constant, it is reasonable to average the slope for the 5 runs together. Determine the mean, standard deviation and error on the mean from the five runs. Estimate the uncertainties of your measurements. You should estimate both random uncertainties and systematic uncertainties.

| Run | Fitted Slope 1 | Uncertainty 1 | Fitted Slope 2 | Uncertainty 2 |
| :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

Mean: $\qquad$
Standard Deviation: $\qquad$
Error on Mean: $\qquad$
2. Repeat the above measurement but give your glider an unknown initial velocity vo. Record this data for at least 5 runs.

| Run | Fitted Slope 1 | Uncertainty 1 | Fitted Slope 2 | Uncertainty 2 |
| :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |

Mean: $\qquad$
Standard Deviation: $\qquad$
Error on Mean: $\qquad$

### 3.4 Analysis

1. Analysis of constant velocity data:
(a) Take your values of position as a function of time, graph them and draw (interpolate) a smooth line through the data and determine the slope. How well does it agree with the fit you obtained earlier? Calculate by hand using the equations given in the handout the exact values of the best fit slope and intercept using your data. How well do they agree with the fit from the computer.
(b) The uncertainties on the positions are not given so one can use statistical analysis to estimate them. Take ten values of position vs time and calculate the velocity by determining $\Delta \mathrm{v} / \Delta \mathrm{t}$. Since the velocity should be constant, one expects each velocity value to be a constant. Based on your calculated velocity determine the average velocity and standard deviation.

| position | time | velocity |
| :--- | :--- | :--- |
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Average velocity $\qquad$
Standard deviation $\qquad$
The standard deviation provides the expected uncertainty on the velocity measurement. This can be related to the uncertainty on the position. From propagation of errors we know that

$$
\begin{gathered}
v=D / t \\
\Rightarrow(\Delta v / v)^{2}=(\Delta D / D)^{2}+(\Delta t / t)^{2}
\end{gathered}
$$

If we assume that the uncertainty on the time measurement is small, then we can neglect the uncertainty in time. With this assumption, we then find that the fractional uncertainty in the
velocity is the same as the fractional uncertainty in the position.

Take your values of position as a function of time, graph them and draw (interpolate) a smooth line through the data and determine the slope. Use as the uncertainty on each point, the standard deviation you calculate previously. How well does it agree with the average velocity value you calculated previously.
2. Analysis of acceleration due to gravity.
(a) Take your values of velocity as a function of time, graph them and draw (interpolate) a smooth line through the data and determine the slope. How well does it agree with the fit you obtained earlier? Calculate by hand the exact values of the slope and intercept using your data. How well do they agree with the fit from the computer. The uncertainty on the velocity should be the standard deviation you determined previously. Calculate by hand the uncertainty on the slope and intercept. Does this agree with the computer? Based on the uncertainty that the computer provides for the slope and intercept, what uncertainty is the computer using for the position? Which do you believe is a better representation of the true uncertainty on the points, your determination or the computers?
(b) Take your data of position as a function of time. Plot position x versus $t^{2}$ for the data with zero initial velocity. Do you get a straight line? Use this graph to determine the acceleration of the glider and thus g . Which technique gives you a smaller uncertainty, using the position data or the velocity data?
(c) Determine g using the data where the glider has an initial velocity. Compare your results to those obtained in part (a). Does it make any difference to your measurement if you give the glider an initial velocity?

## 4 Your Report:

Your report should consist of well organized presentation of the above measurements, showing graphs of what happened for the level track and what happened for the tilted track. You should provide an interpretation of these graphs. You should also determine a value of g , the acceleration due to gravity, from measuring the velocity as a function of time, and from measuring the velocity at different locations. Your values should be accompanied by their uncertainty. Finally, evaluate the techniques you used, whether you think they were accurate, and probable sources of error.

If you wish to make additional plots of your data, you can print out the numbers and use one of the plotting routines in the computer lab.

In addition, please answer the following questions.

1. List three possible sources of systematic error, and three sources of random error. Which is worse for determining g in this experiment? What are the biggest sources of error in this experiment?
2. If the friction of the air track is not negligible, and if the force of friction is not a constant (for example if it depends on velocity) then your acceleration may not be constant. How would this show up in your data? Do you see any effects that might be due to non-constant acceleration?
3. Which method was better for determining $g$ ? Why?



