Data and Error Analysis

1 Introduction

In this lab you will learn a bit about taking data and error analysis. The physics of the experiment itself is not the essential point. (Indeed, we have not completed our study of constant acceleration in a gravitational field yet in class.) Instead we will focus on ideas such as random vs. systematic error, and precision vs. accuracy in measurements. The equipment is decidedly low-tech; the point of this lab is to focus on understanding the ideas of uncertainties in experiments.

Be sure to read the handout on error analysis before the lab, and ask your instructor if you have any questions. These lab instructions presume you have already read the handout. In this document I will denote the standard deviation of the variable x by σ_x, and the standard deviation of the mean of x by $\bar{\sigma}_x$.

You do not have to write up a formal lab report for this lab (with statement of purpose, description of method etc.). You only need to complete this handout, and answer the questions at the end.

2 Equipment

- Rulers/meter stick
- Stop watch
- Inclined metal track
- Ball bearing

3 Procedures

Read through the procedures and familiarize yourself with the equipment. This lab write up will provide space for you to record and analyze your data. You will have to do some analytic calculations in the lab itself in order to process the data. Try to work in groups of two or three.

3.1 Human Reaction Times

1. One lab partner (A) should hold a ruler such that one end hangs between the finger and thumb of a second lab partner (B). Person A should drop the ruler, and B should try to catch it by pinching their finger and thumb together. Try to avoid anticipation; person A should vary the time they wait before releasing the ruler. Record the distance the ruler falls before person B catches it. (Think about how best to do this.) Repeat this for each person in the group, 15 times for each person in the group.

   Calculate the average reaction "distance" for each person, $\bar{d}$, and the standard deviation, σ_d (Often your calculator can do this for you. Otherwise, use the formula in the handout.)

Trial #	Distance for A	Distance for B	Distance for C
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
$\bar{d}$			
σ_d			

Question: On a standard ruler it is easy to measure to about half a millimeter (0.0005m). This is the uncertainty in each of your measurements. Which is more meaningful in describing your data, this uncertainty, or σ_d? Why?

2. Histogram your own reaction data on the graph below. You should think how to choose your bins. If they are too big, all the data will fall in one bin. If they are too small, each bin will have only one data point. Mark the average of the distribution, $\bar{d}$, and the points $\bar{d} \pm \sigma_d$ (You have calculated these above, just mark those results here.) Make sure you label your axes!

3. The "reaction distance" can be converted to a reaction time via the formula:

$$t = \sqrt{2d/g}$$

   where $g = 9.80 \pm .01\ \mathrm{m/s^2}$. Using error propagation write down the equation which relates the uncertainty on g and the uncertainty on d to the uncertainty on t. Use the two formulas to calculate your reaction time, and its error.

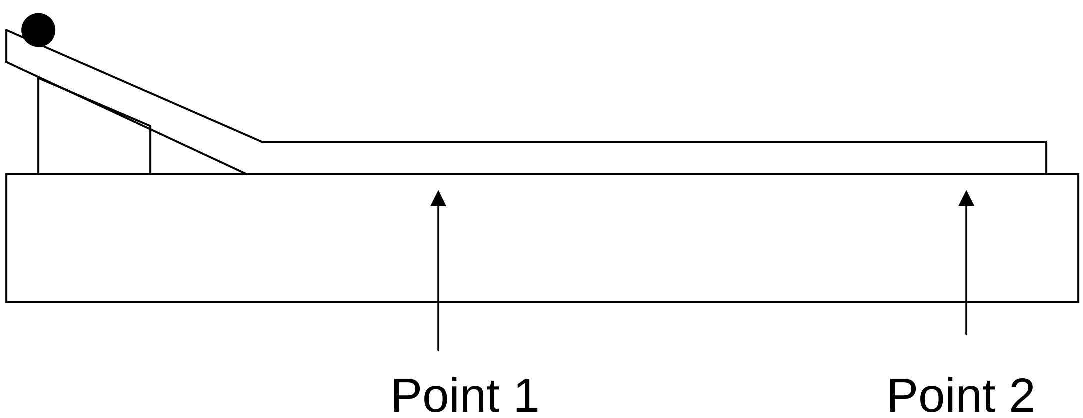
4. Using the above graph, what fraction of your data fall within ± 1 σ? If your data was a true Gaussian distribution, what fraction of the data should fall within ± 1 σ?

5. Do you expect your data to follow a Gaussian distribution, why or why not?

3.2 Inclined Plane Experiment

The above experiment introduced you to the idea of standard deviations. However, the reaction time of a human being isn't truly a constant. It varies with time of day, etc. Even if there were no errors in the measurement, no uncertainty in the data, your reaction time would still vary from trial to trial. Error analysis (or more properly called uncertainty analysis) comes to the fore when there is an actual, precise physical constant to be measured, a constant that would be reproduced again and again in an "ideal" experiment. In the real world measurements are subject to a variety of errors. So long as these errors are random in sign, we can use error analysis to close in on the true value they obscure.

1. Arrange the ramp so that it is relatively flat. Choose two points on the flat section over which you will measure the average velocity.



2. Place the ball bearing on the curved part of the track at a fixed point. (You may wish to mark the point with a pencil.) Release the ball and measure the time it takes to travel between the two fixed points. Repeat this measurement 5(20) times, recording your values below making sure you report the proper uncertainties in your measurement. Remember that you will need to use propagation of errors to relate your uncertainties in the measurements you make (distance and time) to the uncertainty in the velocity.

3. Repeat this process for 4 different heights, as time allows.

Run 1

Distance between marks: ___________________________

Height of released ball: _______________________________

Trial #	Time in Seconds	Velocity in cm/s
1		
2		
3		
4		
5		

Run 2

Distance between marks: _______________________________

Height of released ball: ___________________________________

Trial #	Time in Seconds	Velocity in cm/s
1		
2		
3		
4		
5		

Run 3

Distance between marks: ___________________________

Height of released ball: ____________________________

Trial #	Time in Seconds	Velocity in cm/s
1		
2		
3		
4		
5		

Run 4

Distance between marks: ______________________________

Height of released ball: ______________________________

Trial #	Time in seconds	Velocity in cm/s
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		

4. For each of the above runs, calculate the standard deviation of the mean, the average velocity and its standard deviation. Make sure you correctly calculate the uncertainty for each value

Run #	Height	v_{avg}	σ_v	$\overline{\sigma_v}$
1				
2				
3				
4				

5. Do you expect your four runs to have approximately the same σ_v? Why or why not

6. Do you expect your four runs to have approximately the same $\overline{\sigma}_v$? Why or why not

7. Plot your data on the log-log graph paper provided, using $\overline{\sigma}_v$ as error bars. Note that you will not need the whole page!

8. Log-log graph paper is nice for turning power law relationships into straight lines (see handout). But sometimes it is nice to actually calculate the exponent itself. Let us assume that there is a relationship

$$v=\beta h^z$$

Then if we want to determine z we take the log of both sides to obtain

$$\ln v = z \ln h + \ln \beta$$

In order to extract z from your data, draw the best fit line through your error bars, and extend it across the graph. This best-fit line will happen to pass through points where a horizontal and a vertical graph line intersect. Pick two such, widely separated points (h_1,v_1), and (h_2, v_2) through which your line passes cleanly. These points are *not* data points - they are the intersection of your best fit line with two other lines. The point-slope formula then gives the slope of the best-fit line:

$$z = \frac{\ln v_2 - \ln v_1}{\ln h_2 - \ln h_1}$$

We have to take logs because we are fitting to a log-log plot. Determine the error in z by drawing the steepest and shallowest lines and recalculating, as discussed in the error analysis handout.

z = ________________________

9. The value of β can be determined by solving for the intercept. Determine your value for β and its error.

   β = ______________________

4 Questions

Please answer the following questions on a separate sheet of paper. Use complete sentences, and diagrams when appropriate. You may need to consult the handout on error analysis.

1. Under what circumstances will taking many trials allow you to reduce your error? When does it fail? Give explicit examples of both cases.
2. Assume your experiment had a systematic error that shifted your calculated velocity by a constant, v_0. How would this affect your results? When would the effect be largest? When would it be smallest? Can you think of a simple way to determine v_0?
3. Discuss the sources of error in determining the relationship between v and h in your experiment. (There are many!) Which are most significant? Do they affect the *accuracy* or the *precision* of your result? (See handout.) How would you reduce them?
4. Derive the formula for z and β from the knowledge that $v_1 = \beta h_1^z$ and $v_2 = \beta h_2^z$.