

# Rotational Motion

## 1 Introduction

In this lab you will investigate different aspects of rotational motion, including moment of inertia and the conservation of energy using the “smart pulley” and the rotation table. You will look at the acceleration of a rotating disk under the action of a constant torque.

## 2 Equipment

- 3-hole smart pulley with phototimer,
- string or thread,
- 2 platters, one with inner metal guide for string,
- heavy metal collar
- weights and hanging weight-holder
- computer with measurement software

## 3 Procedures

Read through the procedures and familiarize yourself with the equipment. This lab write up will provide space for you to record and analyze your data. You will have to do some analytic calculations in the lab itself in order to process the data.

### 3.1 Initializing the Software

1. Select the “Smart Pulley” from the initialization menu.
2. When the smart pulley software is running, it will give you a menu. Select “O” for “Other Options”.
3. Specify the type of pulley you have: either a three spoke or a ten-spoke pulley. Return to the main menu.

### 3.2 Single Platter Experiment

1. One of your platters will have a metal hub to which a string can be tied. Mount this platter on the spindle.
2. Tie your string to the screw on the center hub on the platter. Wrap the string several times around the hub.

3. The central hub has a series of “steps” each with a different radius. You need to know the radius of each of these steps. You may determine this by pulling the string so that the platter rotates once (or any integer number of times) and then measuring the length of string that has unwound. Measure each of the three possible radii.

Ring #	$N_{\text{turns}}$	$\Delta L$	Radius
1			
2			
3			

4. Attach the mass holder to the end of the string, and loop it over the smart pulley, as shown in the figure below. Add a small mass to the holder (you do not want to have too large a force applied).

5. Select the “Motion Timer” from the software menu. Release the weight allowing it to cause the platter to spin. Hit “return” to stop data acquisition before the weight hits the floor.
6. Use the software to graph and analyze the data. You should use the “smart pulley linear string” option. Have the program plot the velocity versus time.<sup>1</sup> When choosing plot options, select “regression” and “statistical analysis”. This will give you the slope, and intercept of the line, as well as the correlation coefficient,  $r$ . The correlation coefficient should be close to one for a good fit. The slope of the line is the acceleration of the falling weight in meters/sec<sup>2</sup>.

Record the radius of hub around which the string was wound, the total mass hanging off the string (including the holder), and the measured acceleration for several radii and masses. Leave the column marked I blank for the moment.

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<sup>1</sup>The software does not do any smoothing when it calculates derivatives. This amplifies the noise in the data. The acceleration calculated by the software has too much noise to be useful. (Plot it and see.)

Run #	Radius	$M_{\text{tot}}$	a	I
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

### 3.3 Analytic Calculation

There are two objects that are being accelerated: the platter (which has angular acceleration) and the hanging masses (which have linear acceleration). You need to write down the 1D form of Newton's 2nd law for each one, and then solve the system of equations.

*Question:* Draw a small, abstract sketch to represent the hanging masses. What are all the forces on them? Draw the forces on the sketch, and label them. Write down Newton's second law for the hanging masses.

*Question:* There is a torque applied to the platter. From the figure below, write down an expression for the torque in terms of  $T$  and radius of the hub. Note: The tension is not equal to  $mg$ . Write down the rotational analog of Newton's 2nd law for the platter.

We have two equations. We have measured the acceleration of the hanging masses, so we know  $a$ . We have three unknowns: The tension in the rope, the rotational acceleration of the platter, and the moment of inertia of the platter,  $I$ . We would like to solve for  $I$ . We need one more equation, namely the relationship between  $a$  and  $\alpha$ .

*Question:* What is the relationship between  $a$  of the falling weights and  $\alpha$  of the rotating platter?

*Question:* Using the above information, solve for  $I$  in terms of  $M_{\text{tot}}$ ,  $r$ ,  $a$  and  $g$ , the acceleration due to gravity.

Using the above formula, go back and calculate the moment of inertia of the platter for each run. Average the results and calculate the standard deviation of the mean.

Measured Moment of Inertia:  $I \pm \Delta I =$  \_\_\_\_\_

*Question:* Weigh the platter, and measure its outer radius. From this information, calculate its approximate moment of inertia.

Calculated Moment of Inertia:  $I_{\text{calc}} =$  \_\_\_\_\_

### 3.4 Addition of Moments

Place the platter back on the spindle with the metal hub underneath. Place either the second platter, the metal bar, or the metal ring on top of the platter. Repeat the above experiment and determine the moment of inertia of the total system.

Run #	Radius	$M_{\text{tot}}$	a	I
1				
2				
3				
4				
5				
6				

Total Measured Moment of Inertia:  $I_{\text{tot}} =$  \_\_\_\_\_

Moment of added object:  $I_{\text{tot}} - I_{\text{platter}} =$  \_\_\_\_\_

Look up the formula for the theoretical value of the moment of inertia for added objects. Weigh the object, and measure its dimensions. Use this information to calculate its moment of inertia.

Calculated Moment of Inertia:  $I_{\text{calc}} =$  \_\_\_\_\_

### 3.5 Conservation of Energy: Rotational, Translational and Gravitational

1. Shorten the string so that when it is entirely unwound, the weights do not quite touch the floor. Measure this minimum height,  $h_{\min}$
2. Repeat the first experiment, but measure the initial height of the hanging masses,  $h_{\max}$ . Use the graphing software to determine the maximum velocity of the falling mass which will be when it hits  $h_{\min}$ .

*Question:* Write down an expression for the total energy of the system. Include the kinetic energy of the platter, the masses, and the gravitational potential energy.

3. Calculate the total energy at the start,  $E_i$  (when all objects are at rest), and  $E_f$  the total energy when the hanging mass reaches  $h_{\min}$ . You will need to determine the angular velocity of the platter from the velocity of the falling weights.

Run #	Radius	$M_{\text{tot}}$	$v_{\text{max}}$	$h_{\text{max}}-h_{\text{min}}$	$E_i$	$E_f$
1						
2						
3						
4						

### 4) Questions

Please answer the following questions on a separate sheet of paper. Use complete sentences and diagrams when appropriate.

1. Did your observed and calculated values for  $I$  agree within your experimental uncertainty? List any possible sources of random and systematic error, and discuss if they are significant.
2. Show from the definition of the moment of inertia that it is additive. (That is, show that if the moments of inertia of two objects about a given axis are  $I_1$  and  $I_2$ , then the moment of inertia of the two together is just  $I_1 + I_2$ )
3. Did you observe that energy was conserved? Devise an experiment to measure the relative importance of rotational friction for the system.
4. Why is the tension in the string **not** equal to the weight,  $mg$ ? In what limiting cases would it be nearly equal to  $mg$ ? In what cases would it be nearly equal to zero?