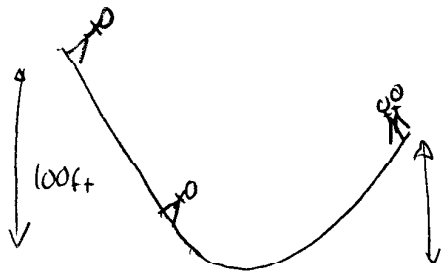
Problem A

Context-Rich Problems: Solutions Outline

FOCUS the PROBLEM

Draw a picture of the situation including ALL the information given in the problem.



$m_G = \frac{2}{3} m_B$

Question(s): What is the problem asking you to find?

How high will Bond + Gigi go

Approach: Outline the approach you will use.

Use conservation of mechanical energy to determine Bond's speed before collision

Use conservation of momentum during collision

Use conservation of energy to determine final height

DESCRIBE the PHYSICS

Draw physics diagram(s) and define ALL quantities uniquely.

Energy Diagram

Before	Transfer	After
$y_1 = 100ft$, $V_1 = 0$	None	$y_2 = 0$, $V_2 = ?$

Momentum Diagram

Before	Transfer	After
$V_{2B} = V_2$, $m_B = ?$, $m_G = \frac{2}{3} m_B$, $V_{2G} = 0$	None	$V_3 = V_{3B} = V_{3G}$, $m = m_B + m_G$

Energy Diagram

Before	Transfer	After
$y_3 = 0$, $V_3 = V_3$, $m = m_B + m_G$		$y_4 = ?$, $V_4 = 0$

Which of your defined quantities is your Target variable(s)? $\boxed{y_4}$

Quantitative Relationships: Write equations you will use to solve this problem.

$W_{ext} = \Delta U + \Delta K + \Delta E_{int}$

$U = mgh$

$K = \frac{1}{2} m v^2$

$\vec{I} = \Delta \vec{p}$

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

$\cancel{W_{ext}}^{0} = \Delta U + \Delta K + \cancel{\Delta E_{int}}^{0}$

$K_4 + U_4 = K_3 + U_3$

$mgy_4 = \frac{1}{2} m v_3^2$ $\boxed{y_4, v_3}$

$gy_4 = \frac{v_3^2}{2}$ (1)

Solve for v_3

$\vec{I} = \Delta \vec{P}$

$0 = \vec{P}_f - \vec{P}_i$

$m_B v_2 + 0 = (m_B + m_G) v_3$ (2) $\boxed{v_2}$

Solve for v_2

$E_1 = E_2$

$m_B g y_1 + 0 = \frac{1}{2} m_B v_2^2 + 0$

$v_2 = \sqrt{2gy_1}$ (3)

Plug (3) in (2)

$v_3 = \frac{m_B \sqrt{2gy_1}}{(m_B + m_G)}$ (4)

Plug (4) into (1)

$y_4 = \frac{m_B^2 y_1}{(m_B + m_G)^2} = \frac{m_B^2 y_1}{(m_B + \frac{2}{3} m_B)^2} = \frac{9}{25} y_1$

Check Units

(L)

EXECUTE the PLAN

Calculate Target Quantity(ies)

$y_4 = \frac{9}{25}(100\,ft) = 36\,ft$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes

Is Answer Unreasonable?

No, Less than 100, more than zero

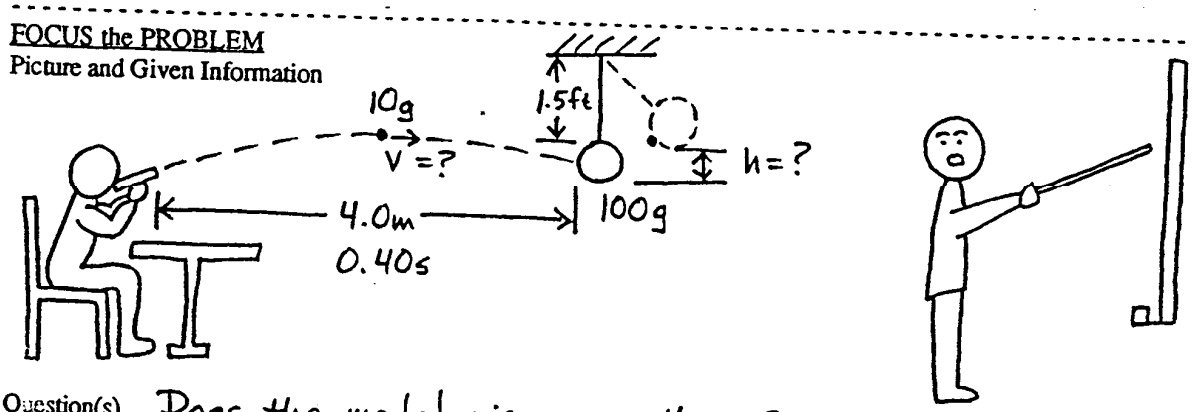
Is Answer Complete?

Yes

(extra space if needed)

Problem B

FOCUS the PROBLEM
Picture and Given Information



Question(s) Does the model rise more than 2 cm above its rest height?

Approach Use kinematics to find horizontal velocity of spitball initially.
Assume motion of spitball is essentially all horizontal -- const. velocity.
Use conservation of momentum to find recoil speed of model/spitball right after the collision. System (for momentum sub-problem): spitball and model
Use conservation of energy to determine how high model/spitball swings. System (for energy sub-problem): spitball, model, and Earth
Time: Initial time is the instant after spitball is fired.
Middle time #1 is the instant just before the collision.
Middle time #2 is the instant just after the collision.
Final time is the instant model/spitball reaches maximum height.
Just before collision mom. due to spitball. Just after, mom. due to model/s.b.
No net momentum transfer between momentum-system and environment.
Energy just after collision is kinetic.
Energy at maximum height is gravitational potential.
No energy transfer between energy-system and envir.

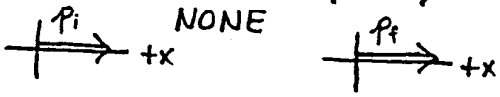
DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

motion diagram

$x_0 = 0$ $x_1 = 4.0m$
$t_0 = 0$ $t_1 = 0.40s$

momentum diagram

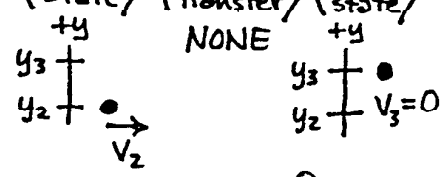
⟨initial state⟩ ⟨momentum transfer⟩ ⟨final state⟩



$V_0 = V_1 = ?$ $m_s = 10g$
$V_2 = ?$ $m_m = 100g$
$V_3 = 0$ $M = m_s + m_m = 110g$

energy diagram

⟨initial state⟩ ⟨energy transfer⟩ ⟨final state⟩



$y_2 = 0$
$y_3 = ?$

Target Quantity(ies) $\boxed{y_3}$

Quantitative Relationships

Constant velocity in x-direction so use

$= V_1 = \bar{V}_x = \frac{x_1 - \cancel{x_0}^{\,0}}{t_1 - \cancel{t_0}^{\,0}}$

$\Rightarrow V_0 = V_1 = x_1/t_1$

$p_{fx} - p_{ix} = J_x = 0$
$p_{fx} = MV_2$
$p_{ix} = m_s V_1$
$\therefore MV_2 - m_s V_1 = 0$

$E_f - E_i = W = 0$
$E_f = Mgy_3$
$E_i = \frac{1}{2}MV_2^2$
$\therefore Mgy_3 - \frac{1}{2}MV_2^2 = 0$

PLAN the SOLUTION
Construct Specific Equations

Unknowns

$\boxed{y_3}$

Find y_3

$$\cancel{M}gy_3 - \frac{1}{2}\cancel{M}v_2^2 = 0 \quad (1)$$

$\boxed{v_2}$

Find v_2

$$Mv_2 - m_s v_1 = 0 \quad (2)$$

$\boxed{v_1}$

Find v_1

$$v_1 = x_1/t_1 \quad (3)$$

$$Mv_2 - m_s x_1/t_1 = 0$$

$$Mv_2 = m_s x_1/t_1$$

$$v_2 = \frac{m_s x_1}{Mt_1}$$

$$gy_3 - \frac{1}{2}\left(\frac{m_s x_1}{Mt_1}\right)^2 = 0$$

$$gy_3 = \frac{1}{2}\left(\frac{m_s x_1}{Mt_1}\right)^2$$

$$\boxed{y_3 = \frac{1}{2g}\left(\frac{m_s x_1}{Mt_1}\right)^2}$$

Check Units

$$\frac{[g]^2[m]^2}{[m/s^2][g]^2[s]^2} = [m] \quad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$y_3 = \frac{(10g)^2(4.0m)^2}{2(9.8m/s^2)(110g)^2(0.40s)^2}$$

$$y_3 = 0.042m = 4.2cm$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected y_2 has units of length.

Is Answer Unreasonable?

No. The spitball has 1/10th the mass of the model so it moves it quite a bit (but not nearly 1.5ft ~ 0.5m, for example).

Is Answer Complete?

Not yet. $y_3 = 4.2cm > 2cm$ so the teacher could notice the vertical motion. This answers the question.

(extra space if needed)