

## Assignment #9

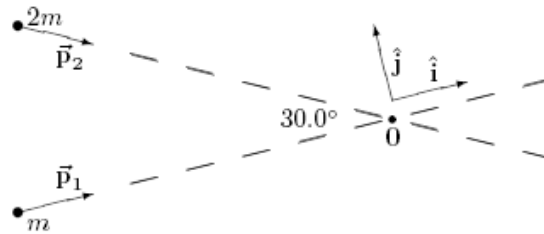
### Chapter 9 Questions:

- 9.20 Momentum is conserved in such decays. In cases (a) and (c), the two momenta are in opposite directions, so it is quite possible for the two momenta to sum (using vector addition) to zero, since the total momentum of the particle initially at rest before decay is zero. In case (b), the two momentum vectors can not sum to zero, so there must exist a third particle, at least, in the decay so that the three momenta can vector sum to zero. The scenario in (b) was observed in the  $\beta$  decay of radioactive nuclei and led to the prediction in the early 1930's of a new particle called the neutrino, which was eventually detected directly in the 1950's.
- 9.21 The momentum of the gum is not conserved because there is an external force acting on it (the force of the wall.) The momentum of the gum is transferred to the composite gum-wall system. Since the mass of the gum-wall system is effectively infinite compared with the mass of the gum, the gum-wall system has zero speed.

### Chapter 9 Problems:

#### 9.31

- a) Since the particles stick together after the collision, the collision is completely inelastic.  
 b) Use the coordinate system shown below.



- c) The total momentum before the collision is the vector sum of the momenta of the particles before the collision.

$$\begin{aligned}\vec{p}_{\text{total before}} &= \vec{p}_1 + \vec{p}_2 = m(1.00 \text{ m/s})\hat{i} + 2m \left( (1.00 \text{ m/s}) \cos 30.0^\circ \hat{i} - (1.00 \text{ m/s}) \sin 30.0^\circ \hat{j} \right) \\ &= m(2.73 \text{ m/s})\hat{i} - m(1.00 \text{ m/s})\hat{j}.\end{aligned}$$

- d) Let  $\vec{v}'$  be the velocity of the composite particle after the collision. The total momentum before and after the collision is conserved, so

$$\vec{p}_{\text{total before}} = \vec{p}_{\text{total after}} \implies m(2.73 \text{ m/s})\hat{i} - m(1.00 \text{ m/s})\hat{j} = 3m\vec{v}' \implies \vec{v}' = (0.910 \text{ m/s})\hat{i} - (0.333 \text{ m/s})\hat{j}.$$

- e) The speed is

$$v' = |\vec{v}'| = \sqrt{(0.910 \text{ m/s})^2 + (-0.333 \text{ m/s})^2} = 0.969 \text{ m/s}.$$

- f) Let  $\phi$  be the angle between  $\hat{i}$  and  $\vec{v}'$ . Then

$$\hat{i} \cdot \vec{v}' = |\hat{i}| |\vec{v}'| \cos \phi \implies 0.910 \text{ m/s} = 1(0.969 \text{ m/s}) \cos \phi \implies \cos \phi = 0.939 \implies \phi = 20.1^\circ.$$

g) The change in the kinetic energy of the system is

$$\Delta KE = KE_f - KE_i = \frac{1}{2}(3m)(0.969 \text{ m/s})^2 - \left( \frac{1}{2}m(1.00 \text{ m/s})^2 + \frac{1}{2}2m(1.00 \text{ m/s})^2 \right) = (-0.09 \text{ J/kg})m.$$

9.36 Let the system be the cart together with all the rain that falls into it.

a) Since all the external forces (gravity, for example) involved are in the vertical direction, the total momentum of the system in the horizontal direction is conserved.

b) Let  $\hat{i}$  be in the direction the cart is moving, and let  $m$  be the mass of the rain that falls into the cart. Before the rain falls into the cart, the velocity of the rain in the  $\hat{i}$  direction is zero. Afterwards, it is the same as the cart's hence

$$\begin{aligned} \vec{P}_{\text{horizontal before}} &= \vec{P}_{\text{horizontal after}} \\ \implies (400 \text{ kg})(10.0 \text{ m/s})\hat{i} + m(0 \text{ m/s})\hat{i} &= (400 \text{ kg})(8.00 \text{ m/s})\hat{i} + m(8.00 \text{ m/s})\hat{i} \\ &\implies m = 100 \text{ kg}. \end{aligned}$$

9.38

a) The table does not move since there is no frictional force between the block and the table surface.

b) When the block and bullet system falls off the table it becomes a projectile. We'll first use the range and height of the flight, along with the equations for motion with a constant acceleration, to find the horizontal speed  $v_{\text{system}}$  at which the *system* left the table. We'll then use this together with conservation of momentum to find the speed  $v_{\text{bullet}}$  of the bullet just before impact.

Let  $\hat{j}$  point straight up, let  $\hat{i}$  point in the direction of the horizontal component of velocity, and choose the origin directly below the point at which the block and bullet leave the table.

$$\begin{array}{ll} \begin{array}{l} x \text{ direction} \\ v_x(t) = v_{x0} + a_x t = v_{\text{system}}. \\ x(t) = x_0 + v_{x0}t + a_x \frac{t^2}{2} = v_{\text{system}}t. \end{array} & \begin{array}{l} y \text{ direction} \\ v_y(t) = v_{y0} + a_y t = -gt. \\ y(t) = y_0 + v_{y0}t + a_y \frac{t^2}{2} = 0.750 \text{ m} - g \frac{t^2}{2}. \end{array} \end{array}$$

On impact with the floor, the  $y$  coordinate is zero, so we may use this in the  $y(t)$  equation to find the time  $t_{\text{impact}}$  of impact.

$$0 \text{ m} = 0.750 \text{ m} - g \frac{t_{\text{impact}}^2}{2} \implies t_{\text{impact}} = 0.391 \text{ s}.$$

The  $x$  coordinate on impact is 2.50 m, so from the  $x(t)$  equation,

$$2.50 \text{ m} = v_{\text{system}}0.391 \text{ s} \implies v_{\text{system}} = 6.39 \text{ m/s}.$$

So, the completely inelastic collision of the bullet with the block of wood results in a composite particle traveling at speed  $v_{\text{system}} = 6.39 \text{ m/s}$ .

Now let  $m_{\text{bullet}}$  and  $m_{\text{block}}$  be the masses of the bullet and block. The momentum of the bullet-block system is conserved before and after the collision, so

$$\begin{aligned} \vec{P}_{\text{total before}} &= \vec{P}_{\text{total after}} \implies m_{\text{bullet}}v_{\text{bullet}}\hat{i} = (m_{\text{bullet}} + m_{\text{block}})v_{\text{system}}\hat{i} \\ \implies v_{\text{bullet}} &= \left( \frac{m_{\text{bullet}} + m_{\text{block}}}{m_{\text{bullet}}} \right) v_{\text{system}} = \left( \frac{0.0100 \text{ kg} + 3.00 \text{ kg}}{0.0100 \text{ kg}} \right) 6.39 \text{ m/s} = 1.92 \times 10^3 \text{ m/s}. \end{aligned}$$

9.40

- a) Choose a coordinate system with  $\hat{i}$  pointed in the direction of the sports car's initial velocity vector, and  $\hat{j}$  in the direction of the truck's. Let the origin be at the point of impact.
- b) and
- c) Let  $v$  be the speed of the smashed car-truck system immediately after the collision, and let  $v_{\text{car}}$  be the initial speed of the car. The total momentum of the system is conserved immediately before and after the collision, so

$$\begin{aligned}\vec{P}_{\text{total before}} &= \vec{P}_{\text{total after}} \\ \implies (1.50 \times 10^3 \text{ kg})v_{\text{car}}\hat{i} + (3.00 \times 10^3 \text{ kg})(25.0 \text{ m/s})\hat{j} &= (4.50 \times 10^3 \text{ kg})(v \cos 70.0^\circ \hat{i} + v \sin 70.0^\circ \hat{j})\end{aligned}$$

Two vectors are equal to each other if and only if their respective components are the same, so

$$(1.50 \times 10^3 \text{ kg})v_{\text{car}} = (4.50 \times 10^3 \text{ kg})v \cos 70.0^\circ \quad \text{and} \quad (3.00 \times 10^3 \text{ kg})(25.0 \text{ m/s}) = (4.50 \times 10^3 \text{ kg})v \sin 70.0^\circ.$$

Solve the second equation for  $v = \frac{(3.00 \times 10^3 \text{ kg})(25.0 \text{ m/s})}{(4.50 \times 10^3 \text{ kg}) \sin 70.0^\circ} = 17.7 \text{ m/s}$ . Now use this in the first equation to find

$$v_{\text{car}} = \frac{(4.50 \times 10^3 \text{ kg})v \cos 70.0^\circ}{1.50 \times 10^3 \text{ kg}} = \frac{(4.50 \times 10^3 \text{ kg})(17.7 \text{ m/s}) \cos 70.0^\circ}{1.50 \times 10^3 \text{ kg}} = 18.2 \text{ m/s}.$$

9.47

- a) Use the conservation of momentum of the system of two particles before and after the collision. The 3.50 kg particle is at rest after the collision. Let  $\vec{v}$  be the velocity of the 5.00 kg particle after the collision. Then

$$\begin{aligned}\vec{P}_{\text{total before}} &= \vec{P}_{\text{total after}} \\ \implies (3.50 \text{ kg})(4.00 \text{ m/s})\hat{i} + (5.00 \text{ kg})(1.50 \text{ m/s})\hat{j} &= (3.50 \text{ kg})0 \text{ m/s} + (5.00 \text{ kg})\vec{v} \\ &\implies \vec{v} = (2.80 \text{ m/s})\hat{i} + (1.50 \text{ m/s})\hat{j}.\end{aligned}$$

- b) To see if the collision is elastic or inelastic, compare the kinetic energy of the system of two particles before and after the collision. Before the collision

$$\text{KE}_{\text{before}} = \frac{1}{2}(3.50 \text{ kg})(4.00 \text{ m/s})^2 + \frac{1}{2}(5.00 \text{ kg})(1.50 \text{ m/s})^2 = 33.6 \text{ J}.$$

After the collision the speed of the 3.50 kg mass is zero, and the speed of the 5.00 kg mass is

$$v = \sqrt{(2.80 \text{ m/s})^2 + (1.50 \text{ m/s})^2} = 3.18 \text{ m/s}.$$

So, after the collision

$$\text{KE}_{\text{after}} = \frac{1}{2}(3.50 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}(5.00 \text{ kg})(3.18 \text{ m/s})^2 = 25.3 \text{ J}.$$

The kinetic energy of the system of colliding particles is conserved in elastic collisions. Since  $\text{KE}_{\text{after}} < \text{KE}_{\text{before}}$ , the collision is inelastic.

9.49 Use the CWE theorem to find the speed  $v$  of  $m$  the instant before it collides with  $M$ . Take the initial position to be where  $m$  is released and the final position to be where it is about to collide with  $M$ . Choose a coordinate system with origin at the bottom of the inclined plane,  $\hat{j}$  pointing up, and  $\hat{i}$  pointing horizontally to the right.

Since the slide is frictionless, there is no work done by the force of kinetic friction while  $m$  slides down the plane. There is work done by the force of kinetic friction along the rough, horizontal ground. Along the straight section of horizontal rough ground of length 0.20 m, the normal force is equal in magnitude to that of the weight of  $m$ . The force of kinetic friction is constant here, so its work is

$$W_{\text{friction}} = \vec{f}_k \bullet \Delta\vec{r} = f_k \Delta r \cos 180^\circ = (\mu_k mg)(0.20 \text{ m})(-1) = (-0.20 \text{ m})\mu_k mg.$$

This is the  $W_{\text{nonconservative}}$  in the CWE theorem. The initial height of  $m$  is  $y_i = (2.0 \text{ m}) \sin 30^\circ$ . So, applying the CWE theorem,

$$\begin{aligned} (-0.20 \text{ m})\mu_k mg &= W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i) \\ &= \left( \frac{mv^2}{2} + 0 \text{ J} \right) - (0 \text{ J} + mg(2.0 \text{ m}) \sin 30^\circ) \\ &= \frac{mv^2}{2} - mg(2.0 \text{ m}) \sin 30^\circ = \frac{mv^2}{2} - \frac{mg(2.0 \text{ m})}{2}. \end{aligned}$$

Solve for  $v$ , and substitute the values for  $\mu_k$  and  $g$ .

$$v = \sqrt{g(2.0 \text{ m}) - 2(0.20 \text{ m})\mu_k g} = \sqrt{(9.81 \text{ m/s}^2)(2.0 \text{ m}) - 2(0.20 \text{ m})(0.25)(9.81 \text{ m/s}^2)} = 4.4 \text{ m/s}.$$

Now let the system be  $m$  and  $M$ . Conserve the total momentum of the system before and after the collision. The particles stick together after the collision since it is completely inelastic. Let  $v'$  be their common speed after the collision. Then

$$\vec{p}_{\text{total before}} = \vec{p}_{\text{total after}} \implies mv = (m + M)v' \implies \frac{M}{m} = \frac{v}{v'} - 1 = \frac{4.4 \text{ m/s}}{0.25 \text{ m/s}} - 1 = 17.$$

**9.54** Choose a coordinate system with origin on the ground and  $\hat{j}$  pointing up.

a) Apply the CWE theorem to the falling mass  $M$ . Take the initial position to be the point of release and the final position to be just before impact with the floor. There are no nonconservative acting on the falling ball, so the CWE theorem becomes

$$\begin{aligned} 0 \text{ J} = W_{\text{nonconservative}} &= \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i) = \left( \frac{1}{2} M v^2 + 0 \text{ J} \right) - (0 \text{ J} + Mgh) \\ &\implies v = \sqrt{2gh}. \end{aligned}$$

Since the ball rebounds from the floor with as much kinetic energy as it had just before impact, the initial upward speed of the ball is also

$$v = \sqrt{2gh}.$$

This result is independent of the mass  $M$ , so the speed of the small ball is also  $v = \sqrt{2gh}$  just before impact.

b) The velocity of the small ball with respect to the large ball is

$$\begin{aligned} \vec{v}_{\text{small large}} &= \vec{v}_{\text{small ground}} + \vec{v}_{\text{ground large}} \\ &= \vec{v}_{\text{small ground}} - \vec{v}_{\text{large ground}} \\ &= -v\hat{j} - v\hat{j} = -2v\hat{j}. \end{aligned}$$

Hence the speed of the small ball as seen by the large ball just before their collision is

$$2v = 2\sqrt{2gh}.$$

c) The elastic collision of the small ball with the large one is like a BB pellet colliding with a bowling ball: the small ball's rebound speed is the same as its incident speed —  $2v = 2\sqrt{2gh}$ .

d) Use the relative velocity addition equation again. The velocity of the small ball with respect to the ground is

$$\begin{aligned}\vec{v}_{\text{small ground}} &= \vec{v}_{\text{small large}} + \vec{v}_{\text{large ground}} \\ &= 2v\hat{j} + v\hat{j} = 3v\hat{j}.\end{aligned}$$

The speed is the magnitude of the velocity, so

$$v_{\text{small ground}} = 3v = 3\sqrt{2gh}.$$

e) Apply the CWE theorem to the rising small ball. Take the initial position to be just after it collides with the large ball and the final position where it ceases to rise.

$$\begin{aligned}0 \text{ J} = W_{\text{nonconservative}} &= \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i) = (0 \text{ J} + mgy_f) - \left(\frac{1}{2}m(3v)^2 + 0 \text{ J}\right) \\ &\implies y_f = \frac{(3v)^2}{2g} = \frac{(3\sqrt{2gh})^2}{2g} = 9h.\end{aligned}$$

9.57 The position vector of the center of mass is

$$\begin{aligned}\vec{r}_{\text{CM}} &= \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3}{m_1 + m_2 + m_3} \\ &= \frac{(2.50 \text{ kg})(0 \text{ m}) + (3.00 \text{ kg})(0.400 \text{ m})\hat{j} + (1.50 \text{ kg})[(-0.600 \text{ m})\hat{i} + (0.400 \text{ m})\hat{j}]}{2.50 \text{ kg} + 3.00 \text{ kg} + 1.50 \text{ kg}} \\ &= \frac{(-0.900 \text{ kg}\cdot\text{m})\hat{i} + (1.80 \text{ kg}\cdot\text{m})\hat{j}}{7.00 \text{ kg}} = (-0.129 \text{ m})\hat{i} + (0.257 \text{ m})\hat{j}.\end{aligned}$$

9.60 Choose a coordinate system with origin at the center of the large circle,  $\hat{i}$  pointing to the right, and  $\hat{j}$  pointing up in Figure P.60.

The surface mass density  $\sigma$  of the plate is constant and is equal to the mass of the plate divided by its area.

$$\sigma = \frac{m}{\pi R^2 - \pi \left(\frac{R}{2}\right)^2} = \frac{4m}{3\pi R^2}.$$

Use the method of “judicious subtraction.” View the given plate as a complete circular plate of radius  $R$ , which we'll call plate 1, that has had a second plate, plate 2 of radius  $\frac{R}{2}$ , cut away from it. Let  $m_1$  be the mass of plate 1 and  $m_2$  the mass of plate 2. Then

$$m_1 = \sigma\pi R^2 = \frac{4m}{3}, \quad \text{and} \quad m_2 = \sigma\pi \left(\frac{R}{2}\right)^2 = \frac{m}{3}.$$

The center of mass of plate 1 is

$$\vec{r}_1 = 0 \text{ m}.$$

The center of mass of plate 2 is

$$\vec{r}_2 = \frac{R}{2}\hat{i}.$$

Therefore the center of mass of the given plate is

$$\vec{r}_{\text{CM}} = \frac{m_1 \vec{r}_1 - m_2 \vec{r}_2}{m_1 - m_2} = \frac{\frac{4m}{3} \mathbf{0} \text{ m} - \frac{m}{3} \frac{R}{2} \hat{\mathbf{i}}}{\frac{4m}{3} - \frac{m}{3}} = -\frac{R}{6} \hat{\mathbf{i}}.$$

#### 9.64

a) All forces in the horizontal direction on the student-canoe system are internal forces. The total force component in the horizontal direction is zero. Hence the total momentum of the system is conserved. The momentum is initially zero and so remains zero. Hence, the center of mass of the system remains stationary.

b) Choose a coordinate system with origin where the stern (back) of the canoe was before the student began to run, and  $\hat{\mathbf{i}}$  in the direction that the student runs. Keep the coordinate system stationary with respect to the water.

Let  $\vec{r}_{\text{canoe}} = x_{\text{canoe}} \hat{\mathbf{i}}$  be the position vector of the center of mass of the canoe, and let  $\vec{r}_{\text{student}} = x_{\text{student}} \hat{\mathbf{i}}$  be that of the student. Let  $m_{\text{canoe}} = 20.0 \text{ kg}$ , and  $m_{\text{student}} = 80.0 \text{ kg}$  be their masses. Then the position vector for the center of mass of the student-canoe system is

$$\vec{r}_{\text{CM}} = \frac{m_1 \vec{r}_{\text{canoe}} + m_2 \vec{r}_{\text{student}}}{m_{\text{canoe}} + m_{\text{student}}} = \frac{(20.0 \text{ kg})x_{\text{canoe}} \hat{\mathbf{i}} + (80.0 \text{ kg})x_{\text{student}} \hat{\mathbf{i}}}{20.0 \text{ kg} + 80.0 \text{ kg}} = (0.200x_{\text{canoe}} + 0.800x_{\text{student}}) \hat{\mathbf{i}}.$$

Differentiate this last expression with respect to  $t$ .

$$(1) \quad \frac{d}{dt} \vec{r}_{\text{CM}} = \left( 0.200 \frac{d}{dt} x_{\text{canoe}} + 0.800 \frac{d}{dt} x_{\text{student}} \right) \hat{\mathbf{i}}.$$

Now  $\frac{d}{dt} x_{\text{canoe}} = v_{x \text{ canoe}}$ , the velocity component of the canoe, and  $\frac{d}{dt} x_{\text{student}} = v_{x \text{ student}}$ , the velocity component of the student. Also, the center of mass of the system is stationary, so  $\frac{d}{dt} \vec{r}_{\text{CM}} = \mathbf{0} \text{ m/s}$ . Therefore (1) becomes

$$\mathbf{0} \text{ m/s} = (0.200v_{x \text{ canoe}} + 0.800v_{x \text{ student}}) \hat{\mathbf{i}}.$$

Because our reference frame is stationary with respect to the water, both of these velocity components are with respect to the water. So the last equation implies

$$(2) \quad v_{x \text{ canoe water}} = -4.00v_{x \text{ student water}}.$$

Now use this equation in the relative velocity addition equation. In component form, we have

$$\begin{aligned} v_{x \text{ student water}} &= v_{x \text{ student canoe}} + v_{x \text{ canoe water}} = 3.0 \text{ m/s} - 4.00v_{x \text{ student water}} \\ &\implies 5.00v_{x \text{ student water}} = 3.0 \text{ m/s} \implies v_{x \text{ student water}} = 0.60 \text{ m/s}. \end{aligned}$$

So,  $v_{x \text{ student water}} = 0.60 \text{ m/s}$ . Use this result in equation (2) to find

$$v_{x \text{ canoe water}} = -4.00(0.60 \text{ m/s}) = -2.4 \text{ m/s}.$$

#### 9.69

a) Let  $\hat{\mathbf{i}}$  be in the direction of motion. The one-dimensional collision is completely inelastic. Conserve the total momentum before and after the collision (noting in this case that the momentum only has an  $x$ -component):

$$\begin{aligned} p_x \text{ total before} &= p_x \text{ total after} \\ &\implies (120 \times 10^3 \text{ kg})(2.0 \text{ m/s}) = (120 \times 10^3 \text{ kg} + 10 \times 10^3 \text{ kg}) v_x \implies v_x = 1.8 \text{ m/s}. \end{aligned}$$

The final speed is 1.8 m/s.

b) The change in the kinetic energy is

$$\begin{aligned}\Delta KE &= KE_f - KE_i = \frac{(130 \times 10^3 \text{ kg})(1.8 \text{ m/s})^2}{2} - \frac{(120 \times 10^3 \text{ kg})(2.0 \text{ m/s})^2}{2} \\ &= 2.1 \times 10^5 \text{ J} - 2.4 \times 10^5 \text{ J} = -0.3 \times 10^5 \text{ J} = -3 \times 10^4 \text{ J}.\end{aligned}$$

c) The velocity of the center of mass is unchanged in the collision. Since the cars couple together, the velocity (and speed) of the center of mass is the same as that of the coupled system, which is 1.8 m/s.