

Assignment #8

Chapter 8 Problems:

8.28

a) The forces acting on the hog during its descent down the slope and across the horizontal portion of its path are:

1. its weight \vec{w} , a conservative force whose work is accounted for by the change in the gravitational potential energy due to the hog's change in height; and
2. the normal force \vec{N} which does no work since it is always perpendicular to the path of the hog.

When the hog encounters the spring, the spring also exerts a force \vec{F}_{spring} on the hog. This force also is conservative, so its work on the hog is accounted for by changes in the potential energy associated with the spring.

Use the CWE theorem with the initial position where the hog begins its descent and the final position where the hog has compressed the spring to its maximum.

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}).$$

There are no nonconservative forces, so $W_{\text{nonconservative}} = 0 \text{ J}$. Hence

$$0 \text{ J} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i).$$

In both its initial and final position, the hog is at rest, so $\text{KE}_i = \text{KE}_f = 0 \text{ J}$. Thus the last equation reduces to

$$\text{PE}_f = \text{PE}_i.$$

Let y be the height of the hog above the level of the spring, and let x be the amount that the spring is compressed from its equilibrium position. Choose the zero of the gravitational potential energy where $y = 0 \text{ m}$ (as usual), and the zero of the potential energy associated with the spring to be where $x = 0 \text{ m}$. Then the only initial nonzero potential energy is the gravitational potential, so

$$\text{PE}_i = mgy_i.$$

When the hog again comes to rest, the gravitational potential energy is zero, but the spring is compressed by x_f , and therefore the final potential energy is

$$\text{PE}_f = \frac{1}{2}kx_f^2.$$

Therefore,

$$\text{PE}_f = \text{PE}_i \implies \frac{1}{2}kx_f^2 = mgy_i \implies k = \frac{2mgy_i}{x_f^2} = \frac{2(100 \text{ kg})(9.81 \text{ m/s}^2)(5.00 \text{ m})}{(-1.50 \text{ m})^2} = 4.36 \times 10^3 \text{ N/m}.$$

b) Since the total mechanical energy, $\text{KE} + \text{PE}$, of the hog is conserved the hog will return to its initial height of 5.00 m.

c) In this case, there is work done by a nonconservative force, the kinetic force of friction, as the hog slides over the rough ground on the way to the spring. The force of kinetic friction is a constant force over the rough ground, so its work is $\vec{f}_k \cdot \Delta\vec{r}$. The force of kinetic friction is directed opposite to $\Delta\vec{r}$ while the hog is on rough ground. Hence the work done by the force of kinetic friction is

$$\vec{f}_k \cdot \Delta\vec{r} = f_k \Delta r \cos 180^\circ = -f_k \Delta r = -\mu_k N \Delta r.$$

The magnitude N of the normal force on the hog while it is on the horizontal portion of its path equals the magnitude mg of the weight of the hog. Δr is given as 2.00 m. Hence

$$\vec{f}_k \bullet \Delta \vec{r} = -\mu_k N \Delta r = -(0.30)(100 \text{ kg})(9.81 \text{ m/s}^2)(2.00 \text{ m}) = -5.9 \times 10^2 \text{ J}.$$

Apply the CWE theorem to the hog, with the initial position where the hog begins its slide, and the final position where the hog has compressed the spring to its greatest extent. We now have $W_{\text{nonconservative}} = -5.9 \times 10^2 \text{ J}$. We still have $\text{KE}_i = \text{KE}_f = 0 \text{ J}$, $\text{PE}_i = mgy_i$, and $\text{PE}_f = \frac{1}{2}kx_f^2$, so

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) \implies -5.9 \times 10^2 \text{ J} = \frac{1}{2}kx_f^2 - mgy_i \implies$$

$$x_f = \pm \sqrt{\frac{2(-5.9 \times 10^2 \text{ J}) + 2mgy_i}{k}} = \pm \sqrt{\frac{2(-5.9 \times 10^2 \text{ J}) + 2(100 \text{ kg})(9.81 \text{ m/s}^2)(5.00 \text{ m})}{4.36 \times 10^3 \text{ N/m}}} = -1.4 \text{ m}.$$

(We choose the negative root since the spring is compressed.) Therefore the spring is now compressed by only 140 cm, rather than the 150 cm in the original problem.

The total mechanical energy was reduced by $5.9 \times 10^2 \text{ J}$ due to the work of friction. When the hog travels back over the rough ground, it will lose another $5.9 \times 10^2 \text{ J}$ of mechanical energy. Therefore when it again comes to rest towards the top of the incline, its total mechanical energy will have been reduced by $2(5.9 \times 10^2 \text{ J})$. Since at that time its kinetic energy is zero, its new gravitational potential energy will be

$$\text{PE}_f = \text{PE}_i - 2(5.9 \times 10^2 \text{ J}) \implies mgy_f = mgy_i - 2(5.9 \times 10^2 \text{ J}) \implies$$

$$y_f = y_i - \frac{2(5.9 \times 10^2 \text{ J})}{mg} = 5.00 \text{ m} - \frac{2(5.9 \times 10^2 \text{ J})}{(100 \text{ kg})(9.81 \text{ m/s}^2)} = 3.8 \text{ m}.$$

8.29 Apply the CWE theorem to the Dean, with the initial position taken as the point of release and the final position as the instant just before impact on the surface of the Earth. The gravitational force of the Earth is the only force acting on the Dean during the descent and its work is accounted for in the CWE theorem by the change in the appropriate gravitational potential energy. Since the position of the Dean changes over distances comparable to the radius of the Earth, you cannot use mgy as the gravitational potential energy function. Instead you must use the more general form

$$\text{PE} = -\frac{GMm}{r}.$$

Let R be the radius of the Earth, M the mass of the Earth, m the mass of the Dean, and v the speed of the Dean just before impact. There is zero work done by nonconservative forces (since there are none), so the CWE theorem becomes

$$0 \text{ J} = W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE} + \text{PE})_f - (\text{KE} + \text{PE})_i$$

$$= \left[\frac{mv^2}{2} + \left(-\frac{GMm}{R} \right) \right] - \left[0 \text{ J} + \left(-\frac{GMm}{2R} \right) \right].$$

So $\frac{mv^2}{2} = \frac{GMm}{2R}$. Hence, solving for v ,

$$v = \sqrt{\frac{GM}{R}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{6.37 \times 10^6 \text{ m}}} = 7.91 \times 10^3 \text{ m/s} = 7.91 \text{ km/s}.$$

Converted to km/h, this is about 28 500 km/h!

8.40 Use the CWE theorem. Take the initial position to be the location of the rock as it begins its vertical descent and the final position where the rock passes the rangers. The work done by the nonconservative force of kinetic friction is equal to the change in the total mechanical energy of the rock.

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i)$$

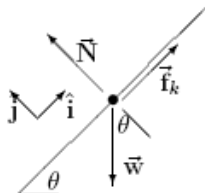
Let $\hat{\mathbf{j}}$ point straight up and choose the origin at the height where the rock passes the rangers. The appropriate form for the potential energy function is mgy , since the entire motion occurs close to the Earth's surface. Hence,

$$\begin{aligned} W_{\text{nonconservative}} &= \left(\frac{1}{2}mv_f^2 + mg(0 \text{ m}) \right) - \left(\frac{1}{2}m(0 \text{ m/s}) + mgy_i \right) \\ &= \frac{1}{2}(100 \text{ kg})(2.00 \text{ m/s})^2 - (100 \text{ kg})(9.81 \text{ m/s}^2)(50.0 \text{ m}) = -4.89 \times 10^4 \text{ J}. \end{aligned}$$

As the rock slides down the talus slope, the forces on the rock are:

1. its weight \vec{w} , of magnitude mg directed down;
2. the normal force \vec{N} of the surface on the rock; directed perpendicularly out of the surface; and
3. the force of kinetic friction \vec{f}_k , directed opposite to the motion of the rock.

Here's a second law force diagram and a convenient coordinate system. (The rock is the little black dot in the middle of the picture.)



There is no acceleration in the $\hat{\mathbf{j}}$ direction, so the total force in that direction is zero.

$$F_{y \text{ total}} = 0 \text{ N} \implies N - mg \cos \theta = 0 \text{ N} \implies N = mg \cos \theta.$$

Therefore the magnitude of the kinetic force of friction on the rock is

$$f_k = \mu_k N = \mu_k mg \cos \theta.$$

The kinetic force of friction is a constant force along the talus slope. The work done by the force is $W = \vec{f}_k \cdot \Delta\vec{r}$. The force \vec{f}_k is directed opposite to the velocity of the rock and so is opposite to $\Delta\vec{r}$. Therefore

$$W = f_k \Delta r \cos 180^\circ = -f_k \Delta r = (-\mu_k mg \cos \theta) \Delta r \implies \mu_k = -\frac{W}{(mg \cos \theta) \Delta r}.$$

The magnitude Δr is the length of the of the talus slope. From the geometry, we have

$$\Delta r = \frac{20 \text{ m}}{\sin 45^\circ} = 28 \text{ m}.$$

Make the substitutions into the expression for μ_k , recognizing that W is the work done by the nonconservative force that we found from the CWE theorem.

$$\mu_k = -\frac{-4.89 \times 10^4 \text{ J}}{(100 \text{ kg})(9.81 \text{ m/s}^2)(\cos 45^\circ)(28 \text{ m})} = 2.5.$$

8.43

a) The work done by the gravitational force on the book is, by definition, the negative of the change in the gravitational potential energy of the book.

$$W_{\text{gravity}} = -\Delta\text{PE}.$$

Take $\hat{\mathbf{j}}$ to point straight up, and choose the origin to be at the bottom of the incline. The appropriate gravitational potential energy to use is mgy . Hence

$$W_{\text{gravity}} = -(\text{PE}_f - \text{PE}_i) = -mg(y_f - y_i) = -(2.00 \text{ kg})(9.81 \text{ m/s}^2)(0 \text{ m} - 10.0 \text{ m}) = 196 \text{ J}.$$

b) The normal force of the surface on the book does zero work because it is perpendicular to $\Delta\vec{\mathbf{r}}$ at every point along the path.

c) The change in the kinetic energy of the book is

$$\Delta\text{KE} = \text{KE}_f - \text{KE}_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(2.00 \text{ kg})(0 \text{ m/s})^2 - \frac{1}{2}(2.00 \text{ kg})(2.00 \text{ m/s})^2 = -4.00 \text{ J}.$$

d) The change in the gravitational potential energy of the book is

$$\Delta\text{PE} = \text{PE}_f - \text{PE}_i = mgy_f - mgy_i = mg(y_f - y_i) = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(0 \text{ m} - 10.0 \text{ m}) = -196 \text{ J}.$$

e) We first use the CWE theorem to find the work done by the nonconservative force of kinetic friction on the book. Then we use the work done by friction to compute the force of friction. Finally, we compute the normal force of the inclined plane on the book and use this together with the force of friction to find the coefficient of kinetic friction.

The work done by friction is

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = \Delta\text{KE} + \Delta\text{PE} = -4.00 \text{ J} + (-196 \text{ J}) = -200 \text{ J}.$$

The work done by friction is also given by $W = \vec{\mathbf{f}}_k \bullet \Delta\vec{\mathbf{r}}$. Since $\vec{\mathbf{f}}_k$ opposes the motion, it is directed opposite to $\Delta\vec{\mathbf{r}}$. Hence

$$W = f_k \Delta r \cos 180^\circ = -f_k \Delta r.$$

From the geometry, $\frac{10.0 \text{ m}}{\Delta r} = \sin 15^\circ$, so $\Delta r = \frac{10.0 \text{ m}}{\sin 15^\circ} = 39 \text{ m}$. Thus

$$W = -f_k(39 \text{ m}) \implies -200 \text{ J} = -f_k(39 \text{ m}) \implies f_k = 5.1 \text{ N}.$$

We now need the normal force of the surface on the book. The forces on the book as it slides down the plane are:

1. The weight $\vec{\mathbf{w}}$ of the book of magnitude mg , directed straight down;
2. the normal force $\vec{\mathbf{N}}$ of the surface on the book, directed perpendicularly out of the surface; and
3. the force of friction $\vec{\mathbf{f}}_k$, directed to oppose the motion of the book.

Let $\hat{\mathbf{j}}$ point perpendicularly out of the surface — parallel to $\vec{\mathbf{N}}$. The book is not accelerating in this direction, so the total force on the book in this direction must be zero. Thus

$$N - mg \cos 15^\circ = 0 \text{ N} \implies N = mg \cos 15^\circ = (2.00 \text{ kg})(9.81 \text{ m/s}^2) \cos 15^\circ = 19 \text{ N}.$$

Finally, the magnitude of the force of friction is

$$f_k = \mu_k N \implies \mu_k = \frac{f_k}{N} = \frac{5.1 \text{ N}}{19 \text{ N}} = 0.27.$$

8.58 Note that since $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$,

$$v = \left| \frac{dy}{dt} \right| = |v_{0y} - gt|.$$

Hence the expression for E is

$$E = \text{KE} + \text{PE} = \frac{1}{2}m(v_{0y} - gt)^2 + mg \left(y_0 + v_{0y}t - g\frac{t^2}{2} \right)$$

When we expand the square and then simplify, this becomes

$$E = \frac{1}{2}mv_{0y}^2 + mgy_0 = \frac{1}{2}mv_{0y}^2 + mgy_0.$$

This is just the sum of the initial kinetic energy $\frac{1}{2}mv_{0y}^2$ and the initial potential energy mgy_0 , and is independent of time.

8.67 The total mechanical energy E of the oscillator is

$$E = \frac{1}{2}kA^2.$$

Since $E = \text{KE} + \text{PE}$, this implies

$$\frac{1}{2}kA^2 = \text{KE} + \text{PE} = \text{KE} + \frac{1}{2}kx^2.$$

Thus, at the position $x = \frac{A}{2}$,

$$\frac{1}{2}kA^2 = \text{KE} + \frac{1}{2}k \left(\frac{A}{2} \right)^2 \implies \text{KE} = \frac{3}{8}kA^2.$$

Therefore,

$$\frac{\text{KE}}{E} = \frac{\frac{3}{8}kA^2}{\frac{1}{2}kA^2} = \frac{3}{4}.$$

8.77 Convert the final speed of the car from km/h to m/s.

$$v = 150 \text{ km/h} = (150 \text{ km/h}) \left(\frac{10^3 \text{ m}}{\text{km}} \right) \left(\frac{\text{h}}{3600 \text{ s}} \right) = 41.7 \text{ m/s}.$$

According to the CWE theorem, the work done by the total force is equal to the change in the kinetic energy of the car, so

$$W_{\text{total}} = \Delta \text{KE} = \frac{mv_{\text{f}}^2}{2} - \frac{mv_{\text{i}}^2}{2} = \frac{m(v_{\text{f}}^2 - v_{\text{i}}^2)}{2} = \frac{(1.00 \times 10^3 \text{ kg}) [(41.7 \text{ m/s})^2 - (0 \text{ m/s})^2]}{2} = 8.69 \times 10^5 \text{ J}.$$

The average power of the total force is

$$P_{\text{ave}} = \frac{W_{\text{total}}}{\Delta t} = \frac{8.69 \times 10^5 \text{ J}}{8.00 \text{ s}} = 1.09 \times 10^5 \text{ W} = 109 \text{ kW}.$$

Chapter 9 Questions:

9.9 Since $K = p^2/2m$, if the kinetic energy is equal, the larger mass will have the larger momentum.

9.12 Airbags are designed to bring you to rest over a longer time interval than otherwise would occur, thus decreasing the force acting on you for a given change in momentum. To change your momentum, an impulse is needed and the same impulse can be provided by either a large force acting over a short time interval or a smaller force acting over a longer time interval. Airbags are used so the latter scenario is the case.

Chapter 9 Problems:

9.6 Convert the speed from km/h to m/s.

$$100 \text{ km/h} = (100 \text{ km/h}) \left(\frac{10^3 \text{ m}}{\text{km}} \right) \left(\frac{\text{h}}{3600 \text{ s}} \right) = 27.8 \text{ m/s}.$$

a) The velocity of the car is

$$\vec{v} = (27.8 \text{ m/s})(\cos 135^\circ)\hat{i} + (27.8 \text{ m/s})(\sin 135^\circ)\hat{j} = (-19.7 \text{ m/s})\hat{i} + (19.7 \text{ m/s})\hat{j},$$

so, the momentum is

$$\begin{aligned} \vec{p} = m\vec{v} &= (1.20 \times 10^3 \text{ kg}) \left((-19.7 \text{ m/s})\hat{i} + (19.7 \text{ m/s})\hat{j} \right) \\ &= -(2.36 \times 10^4 \text{ kg}\cdot\text{m/s})\hat{i} + (2.36 \times 10^4 \text{ kg}\cdot\text{m/s})\hat{j}. \end{aligned}$$

b) The magnitude of the momentum is the magnitude of this vector — the square root of the sum of the squares of its components:

$$p = \sqrt{(2.36 \times 10^4 \text{ kg}\cdot\text{m/s})^2 + (2.36 \times 10^4 \text{ kg}\cdot\text{m/s})^2} = 3.34 \times 10^4 \text{ kg}\cdot\text{m/s}.$$

We can also find the magnitude of the momentum more directly by taking the product of the mass with the speed:

$$p = |\vec{p}| = |m\vec{v}| = m|\vec{v}| = mv = (1.20 \times 10^3 \text{ kg})(27.8 \text{ m/s}) = 3.34 \times 10^4 \text{ kg}\cdot\text{m/s}.$$

Note that this is valid only because m is a *scalar* — not a vector.

9.9 Use the CWE theorem to determine the speed of the cockleshell the instant before impact. Choose a coordinate system with \hat{j} pointing up, \hat{i} pointing horizontally in the direction of the gull's flight, and origin at ground level directly below the release point.

The conservative gravitational force of the Earth is the only force acting on the shell during its fall. There are no nonconservative forces, so their work is zero. The CWE theorem becomes

$$0 \text{ J} = W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_f + \text{PE}_f) - (\text{KE}_i + \text{PE}_i) = \left(\frac{mv_f^2}{2} + mg(0 \text{ m}) \right) - \left(\frac{mv_i^2}{2} + mgy_i \right)$$

$$\implies v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(15.0 \text{ m/s})^2 + 2(9.81 \text{ m/s}^2)(20.0 \text{ m})} = 24.8 \text{ m/s}.$$

The magnitude of the momentum of the shell the instant before impact is

$$p = mv_f = (0.200 \text{ kg})(24.8 \text{ m/s}) = 4.96 \text{ kg}\cdot\text{m/s}.$$

9.12

a) Choose a coordinate system with \hat{i} pointing horizontally in the same direction as the ball's initial horizontal velocity component, \hat{j} pointing straight up, and origin at the ball's position before it is whacked.

b) The impulse is equal to the change in the momentum of the ball.

$$\vec{I} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i.$$

The initial velocity of the ball resting on the tee before the impact of the club is $\vec{v}_i = 0 \text{ m/s}$. The final velocity of the ball after leaving the club head is

$$\vec{v}_f = (60.0 \text{ m/s}) \left(\cos 45.0^\circ \hat{i} + \sin 45.0^\circ \hat{j} \right) = (42.4 \text{ m/s})\hat{i} + (42.4 \text{ m/s})\hat{j}.$$

Therefore, the impulse is

$$\vec{I} = m\vec{v}_f - m\vec{v}_i = (0.045 \text{ kg}) \left((42.4 \text{ m/s})\hat{i} + (42.4 \text{ m/s})\hat{j} \right) - (0.045 \text{ kg})(0 \text{ m/s}) = (1.9 \text{ N}\cdot\text{s})\hat{i} + (1.9 \text{ N}\cdot\text{s})\hat{j}.$$

c) From part b) the magnitude of the impulse \vec{I} is

$$I = \sqrt{(1.9 \text{ N}\cdot\text{s})^2 + (1.9 \text{ N}\cdot\text{s})^2} = 2.7 \text{ N}\cdot\text{s}.$$

The magnitude of the average force times the time interval it acts on the ball is equal to the magnitude of the impulse given to the ball, so

$$F_{\text{ave}}\Delta t = I \implies F_{\text{ave}} = \frac{I}{\Delta t} = \frac{2.7 \text{ N}\cdot\text{s}}{1.00 \times 10^{-3}} = 2.7 \times 10^3 \text{ N}.$$

9.22

a) Since the collision of each marble with the wall is elastic, the kinetic energy of each marble is conserved immediately before and after the collision. Hence, each marble rebounds from the wall with the same speed as the incident speed. The momentum change of each marble is thus

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i) = m \left((-v\hat{i}) - v\hat{i} \right) = -2mv\hat{i}.$$

b) During the impact with the wall, the force of the wall on a marble is much greater than the weight of the marble; hence, neglect the weight. It is the impulse of the wall on each marble that causes its change in momentum. According to the impulse-momentum theorem, the total impulse on each marble is equal to the change in its momentum. Hence, the impulse of the wall on each marble is

$$\vec{I}_{\text{wall on marble}} = -2mv\hat{i}.$$

c) From Newton's third law, the impulse of the marble on the wall has the same magnitude as the impulse of the wall on the marble, but is in the opposite direction. Hence the impulse of the marble on the wall is

$$\vec{\mathbf{I}}_{\text{marble on wall}} = 2mv\hat{\mathbf{i}}.$$

d) With n marbles per second incident upon the wall, the total impulse provided to the wall during one second is $n2mv\hat{\mathbf{i}}$. This equals the average force on the wall times one second.

$$n2mv\hat{\mathbf{i}} = \vec{\mathbf{F}}_{\text{ave}}(1.00 \text{ s}) \implies \vec{\mathbf{F}}_{\text{ave}} = (2.00 \text{ s}^{-1})nmv\hat{\mathbf{i}}.$$

It is, of course, important to retain the units of s^{-1} . Otherwise the two sides of the equation would not have the same dimensions. If, for example, n marbles were thrown per *hour*, then $\vec{\mathbf{F}}_{\text{ave}} = (2.00 \text{ h}^{-1})nmv\hat{\mathbf{i}}$. The magnitude of this force is only $1/3600$ times the force when n marbles are thrown every second.

e) The force per unit area is the force on the entire wall divided by its area.

$$\frac{(2.00 \text{ s}^{-1})nmv\hat{\mathbf{i}}}{A}$$