Assignment #7

Chapter 8 Questions:

8.3 The work done is not force times distance. In Example 8.4 the force is perpendicular to the differential change in the position vector at every point along the path so zero work is done.

8.5 Yes, the force is always anti-parallel to the velocity.

8.6 Each has chosen a different location for the origin of their y coordinate. The one who says the potential energy is zero has the origin at the cola can. The one who says the potential energy is positive has the origin below the can, and the one who says it is negative has the origin above the can.

8.26 The impact speed is the same for all cases. Using the CWE theorem with no work done by non-conservative forces, $0 = \Delta K + \Delta U$. The change in the potential energy is the same for all paths since the initial and final elevations are the same. Since each has the same initial kinetic energy (i.e. speed and mass), then the final kinetic energy is the same in each of the cases. So the final speed is the same.

Chapter 8 Problems:

8.6 The force is a constant force so the work can be found using $W = \vec{\mathbf{F}} \bullet \Delta\vec{\mathrm{r}}$. The change in the position vector is

$$\Delta\vec{\mathrm{r}} = \vec{\mathrm{r}}_{\mathrm{f}} - \vec{\mathrm{r}}_{\mathrm{i}} = (-4.00\ \mathrm{m})\hat{\mathrm{i}} + (1.00\ \mathrm{m})\hat{\mathrm{j}}) - \left((8.00\ \mathrm{m})\hat{\mathrm{i}} + (2.00\ \mathrm{m})\hat{\mathrm{j}}\right) = -(12.00\ \mathrm{m})\hat{\mathrm{i}} - (1.00\ \mathrm{m})\hat{\mathrm{j}}.$$

The work done is

$$W = \left((60.0\ \mathrm{N})\hat{\mathrm{i}} - (40.0\ \mathrm{N})\hat{\mathrm{j}} + (25.0\ \mathrm{N})\hat{\mathrm{k}}\right) \bullet \left(-(12.00\ \mathrm{m})\hat{\mathrm{i}} - (1.00\ \mathrm{m})\hat{\mathrm{j}}\right) = -680\ \mathrm{J}\,.$$

8.18 Model the Earth as a spherical mass M of radius R. Then the gravitational potential energy of a mass m located a distance $r \geq R$ from its center is

$$\mathrm{PE} = -\frac{GMm}{r}$$

The potential energy of m on the surface of the Earth is

$$\mathrm{PE}_{\mathrm{surface}} = -\frac{GMm}{R}.$$

We want to find r so that

$$\mathrm{PE} = 0.0100\,\mathrm{PE}_{\mathrm{surface}} \implies -\frac{GMm}{r} = 0.0100\left(-\frac{GMm}{R}\right) \implies r = 100R.$$

This is the distance of m from the *center* of the Earth. The distance of m above the *surface* of the Earth is therefore

$$99R = 99(6.37 \times 10^{6}\ \mathrm{m}) = 6.31 \times 10^{8}\ \mathrm{m}.$$

8.19 The forces on the mass are:

1. its weight $\vec{w}$, of magnitude mg, directed down;
2. the normal force $\vec{N}$ of the surface, directed up;
3. the force of kinetic friction $\vec{f}_k$, of magnitude $\mu_k N$, directed opposite to the velocity of the mass; and
4. The force $\vec{F}$ pushing the mass horizontally across the surface.

Let $\hat{j}$ point up. Then $\vec{w} = -mg\hat{j}$ and $\vec{N} = N\hat{j}$. These are the only forces on m with nonzero $\hat{j}$ components. Since the mass is not accelerating in the $\hat{j}$ direction these forces sum to 0 N. Hence

$$-mg\hat{j} + N\hat{j} = 0\text{ N} \implies N = mg.$$

Therefore

$$f_k = \mu_k N = \mu_k mg.$$

Along Path 1, the force of kinetic friction is constant, so we can use $W = \vec{F} \bullet \Delta\vec{r}$ to calculate its work. Take the origin at point A and let $\hat{i}$ point in the direction from A to B. Then

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i = (0.500\text{ m})\hat{i} - 0\text{ m} = (0.500\text{ m})\hat{i}.$$

The force of kinetic friction along this path is

$$f_k = -\mu_k g\hat{i} = -(0.25)(3.00\text{ kg})(9.81\text{ m/s}^2)\hat{i} = -(7.4\text{ N})\hat{i}.$$

The work done by friction along Path 1 is therefore

$$W_{\text{Path }1} = \vec{f}_k \bullet \Delta\vec{r} = (-7.4\text{ N})\hat{i} \bullet (0.500\text{ m})\hat{i} = -3.7\text{ J}.$$

Along Path 2, the force of kinetic friction is not a constant force, since it changes direction at the apex of the equilateral triangle. However, along each side of the triangle, the force of kinetic friction is constant. Indeed, since the triangle is equilateral, the sides have equal length so the work done by the force of kinetic friction along each side of the triangle will be the same as calculated for Path 1. Hence the work done by this force along Path 2 is

$$W_{\text{Path }2} = 2(-3.7\text{ J}) = -7.4\text{ J}.$$

Since the work done by kinetic friction is different for these two different paths from A to B, kinetic friction is *not* conservative.

8.23

a) The change in the velocity is

$$\begin{aligned}\Delta\vec{v} &= \vec{v}_f - \vec{v}_i \\ &= \Big(-(4.00\text{ m/s})\hat{i} + (6.00\text{ m/s})\hat{j} - (8.00\text{ m/s})\hat{k}\Big) - \Big((3.00\text{ m/s})\hat{i} - (2.00\text{ m/s})\hat{j} + (5.00\text{ m/s})\hat{k}\Big) \\ &= -(7.00\text{ m/s})\hat{i} + (8.00\text{ m/s})\hat{j} - (13.00\text{ m/s})\hat{k}.\end{aligned}$$

b) To find the change in the kinetic energy, first find the initial and final speeds.

$$v_i = \sqrt{(3.00\text{ m/s})^2 + (-2.00\text{ m/s})^2 + (5.00\text{ m/s})^2} = 6.16\text{ m/s}, \quad \text{and}$$
$$v_f = \sqrt{(-4.00\text{ m/s})^2 + (6.00\text{ m/s})^2 + (-8.00\text{ m/s})^2} = 10.8\text{ m/s}.$$

The change in the kinetic energy is

$$\Delta\text{KE} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}m(v_f^2 - v_i^2) = \frac{1}{2}(3.00\text{ kg})[(10.8\text{ m/s})^2 - (6.16\text{ m/s})^2] = 118\text{ J}.$$

c) In general, $\Delta v^2 \neq (\Delta v)^2$, since

$$(\Delta v)^2 = (v_\mathrm{f} - v_\mathrm{i})^2 = v_\mathrm{f}^2 - 2v_\mathrm{f}v_\mathrm{i} + v_\mathrm{i}^2 = \Delta v^2 - 2v_\mathrm{f}v_\mathrm{i} + 2v_\mathrm{i}^2.$$

Therefore, in general,

$$\Delta\mathrm{KE} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_\mathrm{f}^2 = \frac{1}{2}m(v_\mathrm{f}^2 - v_\mathrm{i}^2) = \frac{1}{2}m\Delta v^2 \neq \frac{1}{2}m\,(\Delta v)^2\,.$$

d) According to the CWE theorem, the work done by the total force on the system is equal to the change in the kinetic energy of the system, which we found in part b) to be 118 J.

8.26

a) The forces acting on the tooth fairy system are:

1. her weight $\vec{\mathrm{w}}$, a conservative force, whose work is accounted for by the change in gravitational potential energy of the system at different heights; and
2. the normal force $\vec{\mathrm{N}}$ of the surface on the fairy, which does zero work since it is always perpendicular to the fairy's path.

The CWE theorem states that

$$W_\mathrm{nonconservative} = \Delta(\mathrm{KE} + \mathrm{PE}).$$

There are no nonconservative forces acting on the system, so the left-hand side of the equation is zero, which means the total mechanical energy of the system is conserved (ie, the same) throughout the motion. The total mechanical energy at Point 1 is purely gravitational potential energy, since the tooth fairy begins at rest. Thus, at Point 1, the total mechanical energy is

$$\mathrm{KE} + \mathrm{PE} = 0\ \mathrm{J} + mgy = (4.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(3.00\ \mathrm{m}) = 118\ \mathrm{J}\,.$$

The total mechanical energy at Points 2, 3, and 4 is also 118 J since the total mechanical energy is conserved.

b) The potential energy of the system at each location is $\mathrm{PE} = mgy$. Therefore

$$\begin{aligned}
&\text{At Point 1,} \quad \mathrm{PE} = (4.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(3.00\ \mathrm{m}) &&= 118\ \mathrm{J}\\
&\text{At Point 2,} \quad \mathrm{PE} = (4.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(0\ \mathrm{m}) &&= 0\ \mathrm{J}\\
&\text{At Point 3,} \quad \mathrm{PE} = (4.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(-4.00\ \mathrm{m}) &&= -157\ \mathrm{J}\\
&\text{At Point 4,} \quad \mathrm{PE} = (4.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(0\ \mathrm{m}) &&= 0\ \mathrm{J}
\end{aligned}$$

c) The kinetic energy at each location is KE = Total mechanical energy − PE = 118 J − PE. Hence

$$\begin{aligned}
&\text{At Point 1,} \quad \mathrm{KE} = 118\ \mathrm{J} - 118\ \mathrm{J} &&= 0\ \mathrm{J}\\
&\text{At Point 2,} \quad \mathrm{KE} = 118\ \mathrm{J} - 0\ \mathrm{J} &&= 118\ \mathrm{J}\\
&\text{At Point 3,} \quad \mathrm{KE} = 118\ \mathrm{J} - (-157\ \mathrm{J}) &&= 275\ \mathrm{J}\\
&\text{At Point 4,} \quad \mathrm{KE} = 118\ \mathrm{J} - 0\ \mathrm{J} &&= 118\ \mathrm{J}
\end{aligned}$$

d) The speed at each location may be found from the kinetic energy at that location,

$$\mathrm{KE} = \frac{1}{2}mv^2 \implies v = \sqrt{\frac{2(\mathrm{KE})}{m}} = \sqrt{\frac{2(\mathrm{KE})}{4.00\ \mathrm{kg}}}.$$

So,

$$\text{At Point 1,} \quad v = \sqrt{\frac{2(0\ \mathrm{J})}{4.00\ \mathrm{kg}}} = 0\ \mathrm{m/s}$$

$$\text{At Point 2,} \quad v = \sqrt{\frac{2(118\ \mathrm{J})}{4.00\ \mathrm{kg}}} = 7.68\ \mathrm{m/s}$$

$$\text{At Point 3,} \quad v = \sqrt{\frac{2(275\ \mathrm{J})}{4.00\ \mathrm{kg}}} = 11.7\ \mathrm{m/s}$$

$$\text{At Point 4,} \quad v = \sqrt{\frac{2(118\ \mathrm{J}}{4.00\ \mathrm{kg}}} = 7.68\ \mathrm{m/s}$$

Here's a summary.

Point	$E = \mathrm{KE} + \mathrm{PE}$	KE	PE	v
1	118 J	0 J	118 J	0 m/s
2	118 J	118 J	0 J	7.68 m/s
3	118 J	275 J	−157 J	11.7 m/s
4	118 J	118 J	0 J	7.68 m/s

8.35 Use the CWE theorem. The only force on the ball after it's thrown is the gravitational force of the Earth. Its work is accounted for in the CWE theorem by the change in the gravitational potential energy. There is no work done by nonconservative forces since there are no such forces acting on the ball. We cannot use mgy for the gravitational potential energy since the path is not confined to distances close to the surface of the Earth. Instead we must use the more general gravitational potential energy function

$$\text{PE} = -\frac{GMm}{r}.$$

Take the initial position of the ball to be on the surface of the Earth just as it leaves Superman's hand. Take the final position to be the maximum height position at a distance r from the center of the Earth where its speed and kinetic energy are zero. The CWE then is

$$\begin{aligned} 0\ \text{J} &= W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE}_\text{f} + \text{PE}_\text{f}) - (\text{KE}_\text{i} + \text{PE}_\text{i}) \\ &= \left[0\ \text{J} + \left(\frac{-GMm}{r}\right)\right] - \left[\frac{1}{2}mv^2 + \left(\frac{-GMm}{R}\right)\right] = \frac{-GMm}{r} - \frac{1}{2}mv^2 + \frac{GMm}{R}. \end{aligned}$$

The initial speed v is given as half the escape speed.

$$v = \frac{1}{2}\sqrt{\frac{2GM}{R}}.$$

Substitute this expression for v in the CWE theorem,

$$0\ \text{J} = \frac{-GMm}{r} - \frac{1}{2}m\left(\frac{1}{2}\sqrt{\frac{2GM}{R}}\right)^2 + \frac{GMm}{R} = \frac{-GMm}{r} + \frac{3}{4}\frac{GMm}{R} \implies r = \frac{4}{3}R.$$

The distance r is measured from the center of the Earth, so the maximum height of the ball above the surface of the Earth is

$$\frac{4}{3}R - R = \frac{1}{3}R.$$

8.45

a) Use the CWE theorem. During its descent, the forces acting on the vehicle are its weight $\vec{w}$ and the normal force $\vec{N}$. The normal force does zero work and the work done by the weight is accounted for by the potential energy term in the CWE theorem. The appropriate potential energy is mgy. Take $\hat{j}$ to point upward and choose the origin at point A. There are no nonconservative forces, so they do zero work. Take the initial position to be where the vehicle begins its descent at zero speed, and take point A as the final position. The CWE theorem becomes

$$\begin{aligned} 0\ \text{J} &= W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = (\text{KE} + \text{PE})_\text{f} - (\text{KE} + \text{PE})_\text{i} = \left(\frac{mv_A^2}{2} + 0\ \text{J}\right) - (0\ \text{J} + mgy_\text{i}) \\ &\implies v_A = \sqrt{2gy_\text{i}} = \sqrt{2gh}. \end{aligned}$$

b) At point B, if the normal force of the track on the vehicle is zero, the only force on it is its weight, and this must provide the centripetal acceleration. Applying Newton's second law to the vehicle at point B, using the magnitudes of the vectors:

$$F_{\text{total}} = ma \implies mg = m\frac{v_B^2}{R} \implies v_B = \sqrt{gR}.$$

c) Use the CWE theorem. Taking the initial position to be where the vehicle begins its journey, and the final position to be point B, we have

$$0\text{ J} = W_{\text{nonconservative}} = \Delta(\text{KE}+\text{PE}) = (\text{KE}+\text{PE})_{\text{f}} - (\text{KE}+\text{PE})_{\text{i}} = \left(\frac{mv_B^2}{2} + mg(2R)\right) - (0\text{ J} + mgh)$$
$$= \frac{mv_B^2}{2} + mg(2R) - mgh.$$

Substitute the expression for v_B from part b):

$$0\text{ J} = \frac{mRg}{2} + mg(2R) - mgh \implies \frac{h}{R} = \frac{5}{2}.$$

d) Since the total mechanical energy of the vehicle is conserved, the vehicle has the same speed when it exits the loop as it does when it enters the loop. Hence, from part a) the exit speed is $v_A = \sqrt{2gh}$.

e) The work done by the gravitational force is the negative of the change in the gravitational potential energy of the vehicle,

$$W_{\text{gravity}} = -\Delta\text{PE} = -(\text{PE}_{\text{f}} - \text{PE}_{\text{i}}) = -(0\text{ J} - mgh) = mgh.$$

f) Only the answer to part e) depends upon the mass of the vehicle.

8.48

a) The kinetic energy of the mass is

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}(0.070\text{ kg})(7.0\text{ m/s})^2 = 1.7\text{ J}.$$

b) Use the CWE theorem. There are no nonconservative forces, so they do zero work. Take the initial position to be where the mass is resting against the compressed spring and the final position to be where the mass leaves the spring (at the equilibrium unstretched position of the spring).

$$0\text{ J} = W_{\text{nonconservative}} = \Delta(\text{KE}+\text{PE}) = (\text{KE}_{\text{f}}+\text{PE}_{\text{f}}) - (\text{KE}_{\text{i}}+\text{PE}_{\text{i}})$$
$$= (1.7\text{ J} + 0\text{ J}) - (0\text{ J} + \text{PE}_{\text{i}}) \implies \text{PE}_{\text{i}} = 1.7\text{ J}.$$

c) The initial potential energy is associated with the spring, so letting $x = -0.050\text{ m}$ be the amount that it is compressed, and letting k be the spring constant,

$$\text{PE}_{\text{i}} = \frac{1}{2}kx^2 \implies k = \frac{2\text{PE}_{\text{i}}}{x^2} = \frac{2(1.7\text{ J})}{(-0.050\text{ m})^2} = 1.4\times 10^3\text{ N/m}.$$

8.56

a) The period of the satellite measured in seconds is

$$T = 9.84\text{ h} = (9.84\text{ h})\left(\frac{3600\text{ s}}{\text{h}}\right) = 3.54 \times 10^4\text{ s}.$$

The gravitational force of Jupiter is the only force on the satellite. The orbit is circular, so the acceleration is centripetal. Hence

$$F = ma \implies \frac{GMm}{r^2} = m\frac{v^2}{r}$$

The speed v of the satellite is the circumference of its orbit divided by the period T.

$$v = \frac{2\pi r}{T}.$$

Substitute this expression for v into the previous equation and solve for r.

$$\frac{GMm}{r^2} = m\frac{\left(\frac{2\pi r}{T}\right)^2}{r} \implies$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90 \times 10^{27}\text{ kg})(3.54 \times 10^4\text{ s})^2}{4\pi^2}} = 1.59 \times 10^8\text{ m}.$$

b) The speed is

$$v = \frac{2\pi r}{T} = \frac{2\pi(1.59 \times 10^8\text{ m})}{3.54 \times 10^4\text{ s}} = 2.82 \times 10^4\text{ m/s}.$$

c) The total mechanical energy of the satellite is

$$\begin{aligned} E &= \text{KE} + \text{PE} \\ &= \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) \\ &= \frac{1}{2}(1200\text{ kg})(2.82 \times 10^4\text{ m/s})^2 - \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90 \times 10^{27}\text{ kg})(1200\text{ kg})}{1.59 \times 10^8\text{ m}} \\ &= -4.79 \times 10^{11}\text{ J}. \end{aligned}$$

d) The escape speed from this distance from Jupiter is

$$v_{\text{escape}} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.90 \times 10^{27}\text{ kg})}{1.59 \times 10^8\text{ m}}} = 3.99 \times 10^4\text{ m/s}.$$