Assignment #6

Chapter 6 Questions:

6.26 Let the planet at distance r be labeled #1 and that at 2r be labeled #2.

Periods (using Kepler's third law) $(T_1/T_2)^2 = (r_1/r_2)^3$
$T_1/T_2 = (r_1/r_2)^{3/2} = (1/8)^{1/2}$

Speeds (use Newton's 2nd law) $GMm/r^2 = v^2/r$
$v = (GMm/r)^{1/2}$
$v_1/v_2 = \sqrt{2}$

Accelerations (use Newton's 2nd law with centripetal acceleration) $GMm/r^2 = ma$
$a = GM/r^2$
$a_1/a_2 = 4$

6.31 Each orbital plane must contain the center of the earth at on focus of the orbital ellipse. The centers of the Artic and Antarctic circles and the circles of the Tropics of Cancer and Capricorn are not the center of the earth.

Chapter 7 Questions:

7.14
a) The frequency is independent of the amplitude, A, so the frequency is unaffected
b) The maximum speed is $A\omega$, so increasing A by a factor of 3 triples the maximum speed.
c) The magnitude of the maximum acceleration is $\omega^2 A$, so increasing A by a factor of 3 triples the magnitude of the maximum acceleration

7.21
a) Nothing happens
b) Nothing happens
c) The effective value of the acceleration magnitude g (in Equation 7.22) increases, so the period decreases.
d) The effective value of the acceleration magnitude g (in Equation 7.22) decreases, so the period increases.

Chapter 6 Problems:

6.22

a) Use Kepler's third law of planetary motion.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

where M is the mass of the Earth and r is the radius of the orbit of the satellite. Solve for the radius.

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}}.$$

The period T of the satellite is one day $= 8.6400 \times 10^4$ s. Therefore

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.98 \times 10^{24}\ \mathrm{kg}))(8.6400 \times 10^4\ \mathrm{s})^2}{4\pi^2}} = 4.23 \times 10^7\ \mathrm{m}.$$

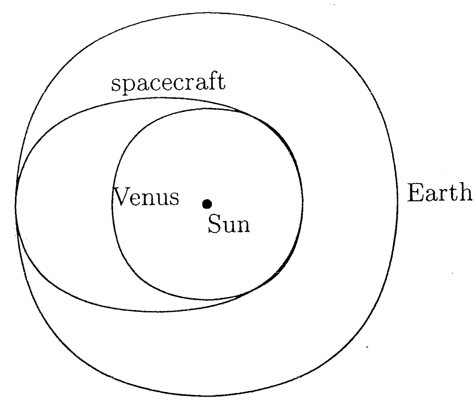
b) The speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(4.23 \times 10^7\ \mathrm{m})}{8.6400 \times 10^4\ \mathrm{m}} = 3.08 \times 10^3\ \mathrm{m/s}.$$

c) The acceleration of the satellite is

$$a_{\mathrm{centripetal}} = \frac{v^2}{r} = \frac{(3.08 \times 10^3\ \mathrm{m/s})^2}{4.23 \times 10^7\ \mathrm{m}} = 0.224\ \mathrm{m/s^2}.$$

6.58 Here's the picture.



a) From the diagram, the major axis of the spacecraft orbit is the sum of the semimajor axes of the orbits of Venus and the Earth. Using the data on the inside cover, we have

$$2a = 1.00\ \mathrm{AU} + 0.723\ \mathrm{AU} \implies a = 0.862\ \mathrm{AU}.$$

Now use Kepler's third law with customized units to find the spacecraft orbital period.

$$T^2 = 1\frac{\mathrm{y}^2}{\mathrm{AU}^3}a^3 = 1\frac{\mathrm{y}^2}{\mathrm{AU}^3}(0.862\ \mathrm{AU})^3 \implies T = 0.800\ \mathrm{y}.$$

The spacecraft only traverses half of its orbit, so the flight time is half the period. Thus

$$t_{\mathrm{flight}} = \frac{T}{2} = 0.400\ \mathrm{y}.$$

b) The aphelion distance of the spacecraft is 1.00 AU. The aphelion distance is

$$r_{\mathrm{aphelion}} = a(1+\epsilon) \implies 1.00\ \mathrm{AU} = (0.862\ \mathrm{AU})(1+\epsilon) \implies \epsilon = 0.160.$$

6.66

a) Refer to Figure P.66 on page 277 of the text. The magnitude of the gravitational field of the Earth at the point P is

$$g_{\text{Earth}} = \frac{Gm_{\text{Earth}}}{r^2_{\text{Earth}}} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{(4.16 \times 10^8\ \text{m})^2} = 2.30 \times 10^{-3}\ \text{m/s}^2 .$$

The magnitude of the gravitational field of the Moon at P is

$$g_{\text{Moon}} = \frac{Gm_{\text{Moon}}}{r^2_{\text{Moon}}} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.36 \times 10^{22}\ \text{kg})}{(1.60 \times 10^8\ \text{m})^2} = 1.92 \times 10^{-4}\ \text{m/s}^2 .$$

These fields point in different directions, toward the Earth and Moon, respectively. The total gravitational field is the vector sum of the two.

So, introduce a coordinate system to form the vector sum. Let $\hat{\mathbf{i}}$ point from P to the Moon. Let $\hat{\mathbf{j}}$ point down, in Figure P.66 — so $\hat{\mathbf{j}}$ is parallel to a vector from Moon to Earth. Let θ be the angle between the lines from P to Moon and P to Earth. Then

$$\vec{\mathbf{g}}_{\text{Earth}} = (2.30 \times 10^{-3}\ \text{m/s}^2)(\cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}) = (0.886 \times 10^{-3}\ \text{m/s}^2)\hat{\mathbf{i}} + (2.12 \times 10^{-3}\ \text{m/s}^2)\hat{\mathbf{j}}.$$

and

$$\vec{\mathbf{g}}_{\text{Moon}} = (1.92 \times 10^{-4}\ \text{m/s}^2)\hat{\mathbf{i}}.$$

Therefore

$$\vec{\mathbf{g}}_{\text{total}} = \vec{\mathbf{g}}_{\text{Earth}} + \vec{\mathbf{g}}_{\text{Moon}} = (10.78 \times 10^{-4}\ \text{m/s}^2)\hat{\mathbf{i}} + (2.12 \times 10^{-3}\ \text{m/s}^2)\hat{\mathbf{j}}.$$

The magnitude of this vector is

$$g_{\text{total}} = \sqrt{(10.78 \times 10^{-4}\ \text{m/s}^2)^2 + (2.12 \times 10^{-3}\ \text{m/s}^2)^2} = 2.37 \times 10^{-3}\ \text{m/s}^2 .$$

b) The acceleration of the salt lick is equal to the gravitational field at that point, so its magnitude is

$$2.37 \times 10^{-3}\ \text{m/s}^2 .$$

c) The magnitude of the force on the 4.00 kg salt lick is

$$(4.00\ \text{kg})(2.37 \times 10^{-3}\ \text{m/s}^2) = 9.48 \times 10^{-3}\ \text{N} .$$

Chapter 7 Problems:

7.6 First convert the angular speed from revolutions per second to radians per second.

$$\omega = 25.0 \text{ rev/s} = (25.0 \text{ rev/s})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 157 \text{ rad/s}\,.$$

The force of the spring on the mass provides the centripetal acceleration. Therefore, using magnitudes, we have

$$F = ma \implies kx = mr\omega^2 \implies k = \frac{mr\omega^2}{x} = \frac{(0.150 \text{ kg})(0.060 \text{ m})(157 \text{ rad/s})^2}{0.040 \text{ m}} = 5.5 \times 10^3 \text{ N/m}\,.$$

7.8 Assume the mass is on a horizontal frictionless surface. Stretch and hold the mass at a distance x from its equilibrium position with a force $\vec{\mathbf{F}}_{\mathrm{we}}$. The total force in the horizontal direction is zero. Each spring is stretched a distance x and so each provides a force $-kx\hat{\imath}$. Hence

$$\vec{\mathbf{F}}_{\mathrm{we}} - kx\hat{\imath} - kx\hat{\imath} = 0 \text{ N} \implies \vec{\mathbf{F}}_{\mathrm{we}} - 2kx\hat{\imath} = 0 \text{ N}\,.$$

Imagine replacing the two springs with a single spring with an effective force constant k_{e}. When the single spring is stretched a distance x,

$$\vec{\mathbf{F}}_{\mathrm{we}} - k_{\mathrm{e}}x\hat{\imath} = 0 \text{ N}\,.$$

Compare this with the equation for the two springs to find

$$k_{\mathrm{e}} = 2k.$$

Note that if the two springs had *different* spring constants k_1 and k_2, then we would have found $k_{\mathrm{e}} = k_1 + k_2$. This result is sometimes stated as "spring constants in parallel add."

7.15 Choose the small mass m as the system. The forces on m are:

1. its weight $\vec{\mathrm{w}}$, of magnitude mg and directed downward;
2. the normal force $\vec{\mathrm{N}}$ of the surface of M on m, directed upward; and
3. the static force of friction $\vec{\mathrm{f}}_s$, directed horizontally in the direction of acceleration so as to oppose slippage.

Choose a coordinate system with $\hat{\imath}$ pointed horizontally to the right in Figure P.15, and with $\hat{\jmath}$ pointing up (parallel to $\vec{\mathrm{N}}$). Then

x direction.	y direction.
$F_{x\ \mathrm{total}} = ma_x \implies f_s = ma_x.$	$F_{y\ \mathrm{total}} = ma_y \implies N - mg = m(0 \text{ m/s}^2) \implies N = mg.$

The force of static force of friction on m reaches its maximum magnitude when the acceleration component in the x direction reaches its maximum magnitude. Since the oscillation is described by $x(t) = A\cos(\omega t + \theta)$, the acceleration component is

$$a_x(t) = \frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \theta).$$

The maximum magnitude of the acceleration is therefore $a_{\max} = A\omega^2$. Hence, if m is not slipping but is on the verge of slipping

$$f_{s\,\max} = ma_{max} \implies \mu_s N = mA\omega^2 \implies \mu_s mg = mA\omega^2 \implies \mu_s g = A\omega^2.$$

The total mass on the end of the spring is $M + m$. The angular frequency of the oscillation is

$$\omega = \sqrt{\frac{k}{M+m}}.$$

Hence

$$\mu_s g = A\omega^2 = A\frac{k}{M+m} \implies k = \frac{\mu_s(M+m)g}{A}$$

If m is increased and k is held constant, the angular frequency of the oscillation will decrease, so the maximum magnitude of acceleration will also decrease, and the force of static friction needed to prevent slippage will be less than $f_{s\,\max}$. Hence, increasing m will mean that m is no longer on the verge of slipping.

7.17

a) The angular frequency is found from

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\ \mathrm{N/m}}{1.50\ \mathrm{kg}}} = 11.5\ \mathrm{rad/s}.$$

When $t = 0\ \mathrm{s}$, the mass is released at $x = 0.100\ \mathrm{m}$, so the general equation for x becomes

(1) $$0.100\ \mathrm{m} = A\cos[\omega(0\ \mathrm{s}) + \phi] = A\cos\phi.$$

Likewise, when $t = 0\ \mathrm{s}$, the velocity component is $v_x = 2.00\ \mathrm{m/s}$. The velocity at any instant is

$$v_x(t) = \frac{d}{dt}x(t) = -A\omega\sin(\omega t + \phi).$$

So

(2) $$2.00\ \mathrm{m/s} = -A\omega\sin[\omega(0\ \mathrm{s}) + \phi] = -A(11.5\ \mathrm{rad/s})\sin\phi.$$

Divide equation (2) by equation (1).

$$\begin{aligned}\frac{2.00\ \mathrm{m/s}}{0.100\ \mathrm{m}} = \frac{-A(11.5\ \mathrm{rad/s})\sin\phi}{A\cos\phi} &\implies 20.0\ \mathrm{s}^{-1} = -11.5\ \mathrm{rad/s}\tan\phi \\ &\implies \tan\phi = -1.74 \\ &\implies \phi = -1.05\ \mathrm{rad}.\end{aligned}$$

Now use this value for ϕ in equation (1) to find A:

$$0.100\ \mathrm{m} = A\cos\phi = A\cos(-1.05\ \mathrm{rad}) \implies A = 0.201\ \mathrm{m}.$$

Hence

$$x(t) = (0.201\ \mathrm{m})\cos[(11.5\ \mathrm{rad/s})t - 1.05\ \mathrm{rad}].$$

b) The period T of the oscillation is the inverse of the frequency ν:

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega} = \frac{2\pi}{11.5\ \mathrm{rad/s}} = 0.546\ \mathrm{s}.$$

c) The velocity component at any time is

$$v_x(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi).$$

Since the maximum magnitude of the sine is 1, the maximum speed is

$$v_{\max} = \omega A = (11.5\ \mathrm{rad/s})(0.201\ \mathrm{m}) = 2.31\ \mathrm{m/s}.$$

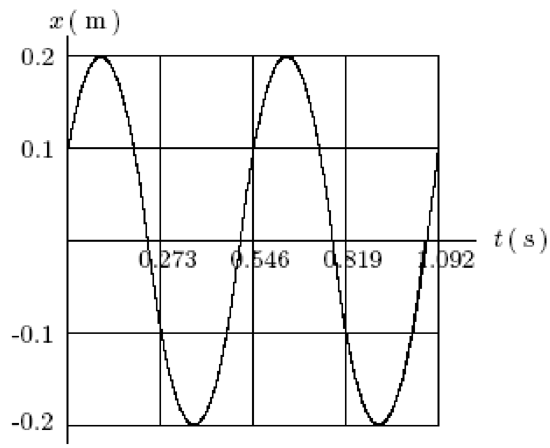
The acceleration component at any time is

$$a_x(t) = \frac{dv_x(t)}{dt} = -A\omega^2\cos(\omega t + \phi).$$

Since the maximum magnitude of the cosine is 1, the maximum magnitude of the acceleration is

$$a_{\max} = \omega^2 A = (11.5\ \mathrm{rad/s})^2(0.201\ \mathrm{m}) = 26.6\ \mathrm{m/s^2}.$$

d) A plot of $x(t)$ versus t for two periods is shown below.



7.30

a) Before the turkey is pulled down below its equilibrium position, the forces on it are:

1. its weight $\vec{w}$, of magnitude mg, directed downward; and

2. the Hooke's law force of the spring, of magnitude kx, where x is the amount by which the spring has stretched, directed upward.

Choose a coordinate system with $\hat{\imath}$ pointing down and origin at the point where the turkey would be if the spring were not extended.

In its equilibrium position, the turkey is not accelerating, so the total force on it is zero.

$$F_{x\ \mathrm{total}} = 0\ \mathrm{N} \implies mg - kx = 0\ \mathrm{N} \implies k = \frac{mg}{x} = \frac{(8.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2})}{0.10\ \mathrm{cm}} = 7.8\times10^2\ \mathrm{N/m}.$$

b) The amplitude of the oscillation is the additional distance the turkey is pulled down below its equilibrium position, $A = 0.040\ \mathrm{m}$. The angular frequency of the oscillation is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{7.8\times10^2\ \mathrm{N/m}}{8.00\ \mathrm{kg}}} = 9.9\ \mathrm{rad/s}.$$

When $t = 0\ \mathrm{s}$, the turkey is at $x = 0.040\ \mathrm{m}$, hence

$$x(t) = A\cos[\omega t + \phi] \implies 4.0\ \mathrm{m} = (0.040\ \mathrm{m})\cos[\omega(0\ \mathrm{s}) + \phi] \implies 1 = \cos\phi \implies \phi = 0\ \mathrm{rad}.$$

Therefore

$$x(t) = (0.040\text{ m})\cos[(9.9\text{ rad/s})t].$$

c) The velocity component is

$$v_x(t) = \frac{dx(t)}{dt} = -A\omega\sin(\omega t + \phi).$$

Since the maximum magnitude of the sine is 1, the maximum speed is

$$v_{\text{max}} = A\omega = (0.040\text{ m})(9.9\text{ rad/s}) = 0.40\text{ m/s}.$$

d) The acceleration component is

$$a_x(t) = \frac{dv(t)}{dt} = -A\omega^2\cos(\omega t + \phi).$$

Since the maximum magnitude of the cosine is 1, the maximum magnitude of the acceleration is

$$a_{\text{max}} = A\omega^2 = (0.040\text{ m})(9.9\text{ rad/s})^2 = 3.9\text{ m/s}^2.$$

7.46

a) Spring 1 exerts a force on the hero equal to $(-k_1 x)\hat{\mathbf{i}}$. (Directed toward the equilibrium position).

b) Spring 2 also exerts a force on the hero toward the equilibrium position. This force is $(-k_2 x)\hat{\mathbf{i}}$.

c) The total force on the hero is the vector sum of the two forces.

$$\vec{\mathbf{F}}_{\text{total}} = (-k_1 x)\hat{\mathbf{i}} + (-k_2 x)\hat{\mathbf{i}} = -(k_1 + k_2)x\hat{\mathbf{i}}.$$

d) Apply Newton's second law to the hero in the horizontal direction.

$$F_{x\text{ total}} = ma_x \implies -(k_1 + k_2)x = m\frac{d^2x}{dt^2} \implies \frac{d^2x}{dt^2} + \left(\frac{k_1 + k_2}{m}\right)x(t) = 0\text{ m/s}^2.$$

e) The coefficient of $x(t)$ in the differential equation for a simple harmonic oscillator is the square of the angular frequency. Hence

$$\omega^2 = \frac{k_1 + k_2}{m} \implies \omega = \sqrt{\frac{k_1 + k_2}{m}}.$$

The frequency ν is $\dfrac{\omega}{2\pi}$, so the period T is

$$T = \frac{1}{\nu} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\dfrac{k_1 + k_2}{m}}} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}.$$

7.59 First we need to find the magnitude of the acceleration due to gravity on the surface of such a white dwarf star. Let M be the mass of the star, and R its radius. The star exerts a gravitational force on a mass m near its surface whose magnitude is $\frac{GMm}{R^2}$. Assume this is the only force on m, and let g_{star} be the magnitude of the acceleration of m due to gravity. Then

$$\frac{GMm}{R^2} = mg_{\text{star}} \implies g_{\text{star}} = \frac{GM}{R^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(1.99 \times 10^{30}\ \mathrm{kg})}{(6.37 \times 10^{6}\ \mathrm{m})^2} = 3.27 \times 10^{6}\ \mathrm{m/s^2}\,.$$

The period T_{star} of a simple pendulum of length $\ell = 1.00\ \mathrm{m}$ on the surface of the star is

$$T_{\text{star}} = 2\pi\sqrt{\frac{\ell}{g_{\text{star}}}} = 2\pi\sqrt{\frac{1.00\ \mathrm{m}}{3.27 \times 10^{6}\ \mathrm{m/s^2}}} = 3.47 \times 10^{-3}\ \mathrm{s}\,.$$

Its frequency is

$$\nu_{\text{star}} = \frac{1}{T_{\text{star}}} = \frac{1}{3.47 \times 10^{-3}\ \mathrm{s}} = 288\ \mathrm{Hz}\,.$$