Assignment #5

Chapter 5 Questions:

5.62 The passenger experiences a centripetal acceleration due to the forces of the side of the car on their body. From Newton's third law, the body matches this force in magnitude, exerting a force on the side of the car that is directed away from the center of the turn. The force of the car on the body is toward the center of the turn.

Chapter 5 Problems:

5.39 Consider the 75.0 kg mass on the scale to be the system. When the scale reads zero, the forces on the 75.0 kg system are:

1. its weight $\vec{\mathrm{w}}$; and
2. the force $\vec{\mathrm{T}}$ of the rope on the system.

The 75.0 kg mass is not accelerating. Apply Newton's second law to this system, taking $\hat{\jmath}$ to point up.

$$F_{y\ \mathrm{total}} = ma_y \implies -mg + T = 0\ \mathrm{N} \implies T = mg = (75.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 736\ \mathrm{N}.$$

Now consider the gymnast of mass m' as the system. The forces on this system are:

1. the weight $\vec{\mathrm{w}}'$ of the gymnast; and
2. the upward force $\vec{\mathrm{T}}$ of the rope on the gymnast, of magnitude 736 N, as determined above.

Apply Newton's second law to the gymnast system.

$$F_{y\ \mathrm{total}} = m'a_y \implies -m'g + T = m'a_y \implies$$
$$a_y = \frac{-m'g + T}{m'} = \frac{(-50.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) + 736\ \mathrm{N})}{50.0\ \mathrm{kg}} = 4.91\ \mathrm{m/s^2}.$$

5.60

a) Take the textbook as the system. The forces on the text are:

1. its weight $\vec{\mathrm{w}}$, directed down;
2. the normal force $\vec{\mathrm{N}}$ of the reading surface on the book, directed perpendicularly to the surface at a 35° angle to the straight up direction;
3. the applied force $\vec{\mathrm{F}}$ along and up the incline, of magnitude 20 N; and
4. the kinetic frictional force $\vec{\mathrm{f}}_k$, directed opposite to the velocity and $\vec{\mathrm{F}}$, with magnitude $f_k = \mu_k N$.

b) Introduce a coordinate system with $\hat{\imath}$ pointing in the direction of the applied force $\vec{\mathrm{F}}$, and $\hat{\jmath}$ perpendicular to the surface and in the same direction as $\vec{\mathrm{N}}$. Let $\theta = 35°$. Apply Newton's second law to each coordinate direction. Since the velocity is constant, the acceleration is zero, so the total force is zero.

x direction

$$F_{x\ \mathrm{total}} = 0\ \mathrm{N} \implies 20\ \mathrm{N} - \mu_k N - mg\sin\theta = 0\ \mathrm{N}$$
$$\implies \mu_k = \frac{20\ \mathrm{N} - mg\sin\theta}{N}.$$

y direction

$$F_{y\ \mathrm{total}} = ma_y \implies N - mg\cos\theta = 0\ \mathrm{N}$$
$$\implies N = mg\cos\theta.$$

Substitute the expression for N from the y equation into the x equation.

$$\mu_k = \frac{20\ \text{N} - mg\sin\theta}{mg\cos\theta} = \frac{20\ \text{N} - (2.50\ \text{kg})(9.81\ \text{m/s}^2)\sin 35°}{(2.50\ \text{kg})(9.81\ \text{m/s}^2)\cos 35°} = 0.30.$$

c) Since the book is sliding, the coefficient is that of kinetic friction.

5.64

a) First convert the speed from km/h to m/s

$$v = 35\ \text{km/h} = (35\ \text{km/h})\left(\frac{10^3\ \text{m}}{\text{km}}\right)\left(\frac{\text{h}}{3600\ \text{s}}\right) = 9.7\ \text{m/s}$$

The magnitude of the centripetal acceleration of the car is

$$a_{\text{centripetal}} = \frac{v^2}{r} = \frac{(9.7\ \text{m/s})^2}{150\ \text{m}} = 0.63\ \text{m/s}^2.$$

b) Since the car travels around the curve at constant speed, the tangential acceleration of the car is $0\ \text{m/s}^2$.

c) The force providing the centripetal acceleration of the car is found from Newton's second law. Use the magnitudes of the vectors.

$$F_{\text{total}} = ma = (1250\ \text{kg})(0.63\ \text{m/s}^2) = 7.9 \times 10^2\ \text{N}.$$

This force is produced by the static force of friction between the tires and the road.

d) The forces acting on the car are:

1. its weight $\vec{\text{w}}$, directed downward;
2. the normal force $\vec{\text{N}}$ of the surface on the car, directed up; and
3. the static force of friction $\vec{\text{f}}_s$ between the tires and the roadway, directed horizontally towards the center of the turn (in the same direction as the centripetal acceleration).

Let $\hat{\text{i}}$ point toward the center of the turn, and $\hat{\text{j}}$ point up. There is no acceleration along the $\hat{\text{j}}$ direction, so the total force component in this direction is 0 N. Thus

$$F_{y\ \text{total}} = 0\ \text{N} \implies N - mg = 0\ \text{N} \implies N = mg.$$

The maximum magnitude $f_{s\,\text{max}}$ of the static force of friction is related to the magnitude of the normal force by

$$f_{s\,\text{max}} = \mu_s N = \mu_s mg.$$

The static force of friction provides the centripetal acceleration, so

$$7.9 \times 10^2\ \text{N} = \mu_s mg = \mu_s(1250\ \text{kg})(9.81\ \text{m/s}^2) \implies \mu_s = 0.064.$$

5.65

a) If the hanging mass is zero, the force of the cord on you is zero. In this case, the forces acting on you are:

1. your weight $\vec{w}$, acting downward;
2. the normal force $\vec{N}$ of the surface on you, directed perpendicular to the surface; and
3. a force of friction $\vec{f}$, directed parallel to the surface. (If you do not slip, this is a static force of friction.)

You will slip down the plane if the component of your weight down the plane exceeds the maximum magnitude of the static force of friction up the plane. The component of your weight down the plane is

$$m'g\sin\theta = (70\text{ kg})(9.81\text{ m/s}^2)\sin 40^\circ = 4.4\times 10^2\text{ N}.$$

There is zero acceleration of the system perpendicular to the plane. Choosing a coordinate system with $\hat{j}$ in this direction we have

$$F_{y\text{ total}} = m'a_y \implies N - m'g\cos\theta = m'(0\text{ m/s}^2) \implies N = m'g\cos\theta.$$

The maximum magnitude of the static force of friction is

$$f_{s\text{ max}} = \mu_s N = \mu_s m'g\cos\theta = 0.40(70\text{ kg})(9.81\text{ m/s}^2)\cos 40^\circ = 2.1\times 10^2\text{ N}.$$

Since the component of your weight down the incline is greater than the maximum magnitude of the static force of friction, you will slide down the plane.

b) Your weight $\vec{w}$ and the normal force of the inclined surface on you are unchanged. Since we need to determine the mass m that enables you to accelerate up the inclined plane, the force of friction is the kinetic force of friction directed opposite to your velocity. There is an additional force acting on you too, the force of the cord.

Two forces act on the hanging mass m: its weight, which points down, and the tension of the cord, which points up. Choose $\hat{j}$ to point down. Then

$$mg - T = ma \implies T = m(g-a).$$

Now apply Newton's second law to the mass m' on the inclined plane, choosing a coordinate system with $\hat{i}$ along the cord and $\hat{j}$ along $\vec{N}$. In this case,

x direction

$$\begin{aligned} F_{x\text{ total}} &= m'a_x \\ &\implies T - m'g\sin\theta - f_k = m'a \\ &\implies T - m'g\sin\theta - \mu_k N = m'a. \end{aligned}$$

y direction

$$\begin{aligned} F_{y\text{ total}} &= m'a_y \\ &\implies N - m'g\cos\theta = m'(0\text{ m/s}^2) \\ &\implies N = m'g\cos\theta. \end{aligned}$$

Substitute for T and N in the x-direction equation:

$$m(g-a) - m'g\sin\theta - \mu_k m'g\cos\theta = m'a$$

Solve for m:

$$\begin{aligned} m &= m'\left(\frac{g\sin\theta + \mu_k g\cos\theta + a}{g-a}\right) \\ &= 70\text{ kg}\left(\frac{(9.81\text{ m/s}^2)\sin 40^\circ + 0.35(9.81\text{ m/s}^2)\cos 40^\circ + 1.50\text{ m/s}^2}{9.81\text{ m/s}^2 - 1.50\text{ m/s}^2}\right) \\ &= 88\text{ kg} \end{aligned}$$

c) From b)

$$T = m(g-a) = (88\text{ kg})(9.81\text{ m/s}^2 - 1.50\text{ m/s}^2) = 7.3\times 10^2\text{ N}.$$

5.67

a) The location where the mass will first slip is at the lowest point in its circular path. At that location the static force of friction and the component of the weight along the incline are in opposite directions. Their difference must provide the total force toward the center of the circular motion that causes the centripetal acceleration.

b) The forces on the mass are:

1. the weight $\vec{w}$ of the particle, directed downward;
2. the normal force $\vec{N}$ of the surface on the particle, directed outward perpendicularly from the surface; and
3. the static force of friction $\vec{f}_s$ on the particle, acting in a direction to oppose slippage.

At the bottom of the circular path, $\vec{f}_s$ points up the incline and parallel to it towards the center of the circle.

Choose a coordinate system with $\hat{\imath}$ directed parallel to $\vec{f}_s$ when the particle is at the bottom of its circular path. Thus $\hat{\imath}$ points from the bottom of the circular path up the slope and parallel to it. Let $\hat{\jmath}$ point outward perpendicular to the surface and parallel to $\vec{N}$. Let $\theta = 30.0°$ be the angle of the incline, which is also the angle that both $\hat{\jmath}$ and $\vec{N}$ make with $-\vec{w}$. Write Newton's second law for each coordinate direction, and consider the mass about to slip so $f_{s\,\max} = \mu_s N$. Then the total force in the x direction creates the centripetal acceleration, while the total force in the y direction is zero.

x direction

$$F_{x\ \mathrm{total}} = ma_x \implies f_{s\max} - mg\sin\theta = \frac{mv^2}{r}$$
$$\implies \mu_s N - mg\sin\theta = \frac{mv^2}{r}$$

y direction

$$F_{y\ \mathrm{total}} = 0\ \mathrm{N} \implies N - mg\cos\theta = 0\ \mathrm{N}$$
$$\implies N = mg\cos\theta.$$

Substitute the expression for N from the y equation into the x equation.

$$\mu_s mg\cos\theta - mg\sin\theta = m\frac{v^2}{r} \implies \mu_s = \frac{mg\sin\theta + m\frac{v^2}{r}}{mg\cos\theta} = \tan\theta + \frac{v^2}{rg\cos\theta}.$$

So

$$\mu_s = \tan\theta + \frac{v^2}{rg\cos\theta}. \qquad (1)$$

Note that the result in independent of the mass. To evaluate this expression, you need to find the speed of the particle. First find its angular speed ω in rad/s.

$$\omega = 33\ \mathrm{rev/min} = (33\ \mathrm{rev/min})\left(\frac{2\pi\ \mathrm{rad}}{\mathrm{rev}}\right)\left(\frac{\mathrm{min}}{60\ \mathrm{s}}\right) = 3.5\ \mathrm{rad/s}.$$

The speed of the particle in circular motion is

$$v = r\omega = (0.15\ \mathrm{m})(3.5\ \mathrm{rad/s}) = 0.53\ \mathrm{m/s}.$$

Now substitute this value for v into equation (1).

$$\mu_s = \tan 30.0° + \frac{(0.53\ \mathrm{m/s})^2}{(0.15\ \mathrm{m})(9.81\ \mathrm{m/s^2})\cos 30.0°} = 0.80.$$

5.71

a) The forces on the kid are:

1. the kid's weight $\vec{w}$, directed downward;
2. the normal force $\vec{N}$ of the surface on the kid;
3. the kinetic force of friction $\vec{f}_k$ on the kid, acting in a direction opposite to the velocity of the kid; and
4. the applied force $\vec{F}$ of the parent, directed at an angle of 45° to the surface.

Choose a coordinate system with $\hat{i}$ in the direction of motion, opposite $\vec{f}_k$, and with $\hat{j}$ pointing up, parallel to $\vec{N}$. Let $\theta = 45°$

b) The child is moving at constant velocity, so its acceleration is zero, and therefore the total force is zero.

x direction

$$F_{x\ \text{total}} = 0\ \text{N} \implies F\cos\theta - f_k = 0\ \text{N} \implies f_k = F\cos\theta.$$

y direction

$$F_{y\ \text{total}} = 0\ \text{N} \implies F\sin\theta + N - mg = 0\ \text{N} \implies N = mg - F\sin\theta.$$

From the x direction equation,

$$f_k = F\cos\theta = (80\ \text{N})\cos 45° = 57\ \text{N}.$$

c) From the y direction equation,

$$N = mg - F\sin\theta.$$

Which says that the magnitude of the normal force is smaller than the magnitude of the weight of the child by an amount equal to the upward component of the force of the parent on the child. Putting in the values,

$$N = mg - F\sin\theta = (20\ \text{kg})(9.81\ \text{m/s}^2) - (80\ \text{N})\sin 45° = 1.4 \times 10^2\ \text{N}.$$

d) The magnitude of the force of kinetic friction is

$$f_k = \mu_k N \implies \mu_k = \frac{f_k}{N} = \frac{57\ \text{N}}{1.4 \times 10^2\ \text{N}} = 0.41.$$

Chapter 6 Questions:

6.5
a) Objects in orbit are not weightless. They have a force of gravity acting on tem
b) No mass is beyond the influence of gravity of the earth unless it is an infinite distance away. An object in orbit has a nonzero gravitational force acting on it that is responsible for its acceleration.

6.4

a) The magnitude of the gravitational force of Jupiter on the astrologer is

$$F = \frac{GMm}{r^2} = \frac{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(1.90 \times 10^{27}\ \mathrm{kg})(70.0\ \mathrm{kg})}{(5.90 \times 10^{11}\ \mathrm{m})^2} = 2.55 \times 10^{-5}\ \mathrm{N}$$

b) Assume both masses are point-like or appropriate spheres. In this case, we know the magnitude of the force and want to determine a mass. So

$$F = \frac{GMm}{r^2} \implies M = \frac{Fr^2}{Gm} = \frac{(2.55 \times 10^{-5}\ \mathrm{N})(10.0\ \mathrm{m})^2}{(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(70.0\ \mathrm{kg})} = 5.46 \times 10^5\ \mathrm{kg}.$$

6.15 The magnitude of the gravitational force of the planet of mass M on the astronaut of mass m is $F = \dfrac{GMm}{r^2}$ where r is the distance of the astronaut from the center of the planet. In this case r is the radius R of the planet, so

$$F = \frac{GMm}{R^2}.$$

The mass of the spherical planet is equal to the product of its average density ρ and its volume: $M = \rho\dfrac{4\pi R^3}{3}$. Therefore,

$$F = \frac{G\left(\rho\dfrac{4\pi R^3}{3}\right)m}{R^2} = \frac{4\pi G\rho Rm}{3} \implies$$

$$R = \frac{3F}{4\pi G\rho m} = \frac{3(1.00\ \mathrm{N})}{4\pi(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(5.00 \times 10^3\ \mathrm{kg/m^3})(80.0\ \mathrm{kg})} = 8.95 \times 10^3\ \mathrm{m} = 8.95\ \mathrm{km}.$$

6.20

a) Write Kepler's third law of planetary motion for each satellite:

$$T_1^2 = \frac{4\pi^2 r_1^3}{GM} \quad \text{and} \quad T_2^2 = \frac{4\pi^2 r_2^3}{GM}.$$

Divide the first equation by the second, simplify, and take square roots

$$\frac{T_1^2}{T_2^2} = \frac{\dfrac{4\pi^2 r_1^3}{GM}}{\dfrac{4\pi^2 r_2^3}{GM}} = \frac{r_1^3}{r_2^3} \implies \frac{T_1}{T_2} = \sqrt{\frac{r_1^3}{r_2^3}}.$$

b) If $r_2 = 2r_1$ then

$$\frac{T_1}{T_2} = \sqrt{\frac{r_1^3}{(2r_1)^3}} = \sqrt{\left(\frac{1}{2}\right)^3} = 0.354,$$

so $T_2 = \dfrac{1}{0.354}T_1 = 2.82T_1$.

6.21 Consider the mass m in free-fall near the surface of Mars. Let R be the radius of Mars and M_{Mars} its mass. The only force on m is the gravitational force of Mars on it. Therefore

$$F_{\text{total}} = ma \implies \frac{GM_{\text{Mars}}m}{R^2} = ma \implies M_{\text{Mars}} = \frac{aR^2}{G} = \frac{(3.776\ \text{m/s}^2)(3.37\times 10^6\ \text{m})^2}{6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 6.43\times 10^{23}\ \text{kg}.$$

Then the ratio

$$\frac{M_{\text{Mars}}}{M_{\text{Earth}}} = \frac{6.43\times 10^{23}\ \text{kg}}{5.98\times 10^{24}\ \text{kg}} = 0.108.$$

The mass of Mars is only about 11% the mass of the Earth.