Assignment #4

Chapter 5 Questions:

5.2 Third law force pairs act on different systems. Only one member of each third law force pair ever is used in Newton's second law.

5.3 Because of the rotation of the earth, the surface of the earth is only approximately an inertial reference frame.

5.5 The reading of the scale on the incline will be less than when the scale is on level ground. The scale measures the magnitude of the normal force of you on its surface.

5.10 There may be forces acting on the system, but the vector sum of the forces will be zero since the system is moving at constant velocity or is at rest.

5.18 No, the total force is parallel to the acceleration.

5.32 The magnitude of the force you exert on the earth is 600 N and the force is directed vertically upward at your location.

Chapter 5 Problems:

5.6

a) The acceleration of the system is in the same direction as the total force. Since the given force has both x and y components, while the acceleration has only an x component, there must be another force on the system.

b) Let $\vec{\mathbf{F}}$ be the other force on the system. Then from Newton's second law,

$$\vec{\mathbf{F}}_{\text{total}} = m\vec{\mathbf{a}}$$

$$\Longrightarrow \vec{\mathbf{F}} + (3.00\ \mathrm{N})\hat{\mathbf{i}} - (6.00\ \mathrm{N})\hat{\mathbf{j}} = (2.50\ \mathrm{kg})(4.00\ \mathrm{m/s^2})\hat{\mathbf{i}} = (10.00\ \mathrm{N})\hat{\mathbf{i}}$$

$$\Longrightarrow \vec{\mathbf{F}} = (10.00\ \mathrm{N})\hat{\mathbf{i}} - \Big((3.00\ \mathrm{N})\hat{\mathbf{i}} - (6.00\ \mathrm{N})\hat{\mathbf{j}}\Big) = (7.00\ \mathrm{N})\hat{\mathbf{i}} + (6.00\ \mathrm{N})\hat{\mathbf{j}}.$$

5.8 Choose $\hat{\mathbf{i}}$ to point in the direction the sprinter runs, and let the origin be at the starting line. Then

$$x_0 = 0\ \mathrm{m} \qquad \text{and} \qquad v_{x0} = 0\ \mathrm{m/s}\,,$$

so during the interval of constant acceleration,

$$x(t) = a_x \frac{t^2}{2}.$$

Now let t be the time that the acceleration ceases. At this time, the sprinter has covered 10.0 m, so $10.0\ \mathrm{m} = a_x \frac{t^2}{2}$, hence

$$a_x = \frac{20.0\ \mathrm{m}}{t^2}. \tag{1}$$

Since $v_{x0} = 0\text{ m/s}$, the velocity component at time t is

$$v_x = a_x t = \frac{20.0\text{ m}}{t^2} t = \frac{20.0\text{ m}}{t}.$$

The remaining 90.0 m of the track are run with this constant velocity component, over the remaining time $10.0\text{ s} - t$. Hence

$$90.0\text{ m} = \frac{20.0\text{ m}}{t}(10.0\text{ s} - t) = \frac{(20.0\text{ m})(10.0\text{ s})}{t} - 20.0\text{ m}.$$

So, solving for t,

$$t = \frac{(20.0\text{ m})(10.0\text{ s})}{90.0\text{ m} + 20.0\text{ m}} = 1.82\text{ s}.$$

Use this value of t in equation (1) to find a_x.

$$\frac{20.0\text{ m}}{(1.82\text{ s})^2} = 6.04\text{ m/s}^2.$$

Therefore, the magnitude of the total force on the sprinter during the interval of constant acceleration is

$$F_{\text{total}} = ma = (60.0\text{ kg})(6.04\text{ m/s}^2) = 362\text{ N}.$$

5.10

a) Choose $\hat{\mathbf{i}}$ to point in the direction of travel of the incoming ball and let $t = 0\text{ s}$ be the time that the ball hits the player's head. Then

$$v_{x0} = 15.0\text{ m/s}, \qquad \text{and} \quad v_x = -18.0\text{ m/s} \quad \text{when } t = 0.100\text{ s}.$$

Therefore, since the acceleration is constant over this 0.100 s interval,

$$-18.0\text{ m/s} = 15.0\text{ m/s} + a_x(0.100\text{ s}) \implies a_x = -330\text{ m/s}^2.$$

The magnitude of the acceleration is the absolute value of this single acceleration component.

$$a = 330\text{ m/s}^2.$$

b) The magnitude of the total force on the ball is

$$F_{\text{total}} = ma = (0.430\text{ kg})(330\text{ m/s}^2) = 142\text{ N}.$$

5.12

a) If the scalar product of two vectors is zero, then they are perpendicular. Hence, to show the forces are perpendicular to one another, take their scalar products with one another.

$$\vec{\mathbf{F}}_1 \bullet \vec{\mathbf{F}}_2 = \Big((3.0\text{ N})\hat{\mathbf{i}} + (2.0\text{ N})\hat{\mathbf{j}}\Big) \bullet (2.0\text{ N})\hat{\mathbf{k}} = 0\text{ N}^2.$$
$$\vec{\mathbf{F}}_1 \bullet \vec{\mathbf{F}}_3 = \Big((3.0\text{ N})\hat{\mathbf{i}} + (2.0\text{ N})\hat{\mathbf{j}}\Big) \bullet \Big(2.0\text{ N})\hat{\mathbf{i}} - (3.0\text{ N})\hat{\mathbf{j}})\Big) = 6.0\text{ N}^2 - 6.0\text{ N}^2 = 0\text{ N}^2.$$
$$\vec{\mathbf{F}}_2 \bullet \vec{\mathbf{F}}_3 = (2.0\text{ N})\hat{\mathbf{k}} \bullet \Big((2.0\text{ N})\hat{\mathbf{i}} - (3.0\text{ N})\hat{\mathbf{j}})\Big) = 0\text{ N}^2.$$

b) First find the acceleration of the system from Newton's second law.

$$\begin{aligned}
&\vec{\mathbf{F}}_{\text{total}} = m\vec{\mathbf{a}} \\
&\quad\implies \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 = m\vec{\mathbf{a}} \\
&\quad\implies \Big((3.0\text{ N})\hat{\mathbf{i}} + (2.0\text{ N})\hat{\mathbf{j}}\Big) + (2.0\text{ N})\hat{\mathbf{k}} + \Big((2.0\text{ N})\hat{\mathbf{i}} - (3.0\text{ N})\hat{\mathbf{j}})\Big) = (20\text{ kg})\vec{\mathbf{a}} \\
&\quad\implies \vec{\mathbf{a}} = (0.25\text{ m/s}^2)\hat{\mathbf{i}} - (0.050\text{ m/s}^2)\hat{\mathbf{j}} + (0.10\text{ m/s}^2)\hat{\mathbf{k}}.
\end{aligned}$$

This is a constant acceleration. The velocity components are found from the one-dimensional kinematic equations

$$v_x(t) = v_{x0} + a_x t, \qquad v_y(t) = v_{y0} + a_y t, \qquad \text{and} \quad v_z(t) = v_{z0} + a_z t.$$

The initial velocity components v_{x0}, v_{y0}, and v_{z0} are all zero. Hence, when $t = 3.0\,\text{s}\,(\text{s})$,

$$v_x = 0.75\ \text{m/s}\,, \qquad v_y = -0.15\ \text{m/s}\,, \qquad \text{and} \quad v_z = 0.30\ \text{m/s}\,,$$

so

$$\vec{\mathbf{v}} = (0.75\ \text{m/s})\hat{\mathbf{i}} - (0.15\ \text{m/s})\hat{\mathbf{j}} + (0.30\ \text{m/s})\hat{\mathbf{k}}.$$

The speed is the magnitude of the velocity vector, so

$$v = \sqrt{(0.75\ \text{m/s})^2 + (-0.15\ \text{m/s})^2 + (0.30\ \text{m/s})^2} = 0.82\ \text{m/s}\,.$$

5.26 Refer to Figure P.26 on page 219 of the text.

a) The magnitude of the total force on the crate is

$$F_{\text{total}} = ma = (51.0\ \text{kg})(0.100\ \text{m/s}^2) = 5.10\ \text{N}\,.$$

The direction of the total force is in the same direction as the acceleration — along and up the inclined plane.

b) The magnitude of the weight of the crate is

$$w = mg = (51.0\ \text{kg})(9.81\ \text{m/s}^2) = 500\ \text{N}\,.$$

c) Let $\vec{\mathbf{F}}_{\text{h}}$ be the horizontal force applied. Use a coordinate system with $\hat{\mathbf{i}}$ in the direction of motion of the crate, and $\hat{\mathbf{j}}$ perpendicular to the plane, so that $\hat{\mathbf{j}}$ makes a $20.0°$ angle with the straight upwards direction. Then in this coordinate system the normal force of the plane is entirely in the $\hat{\mathbf{j}}$ direction, so the only forces with non-zero x components are $\vec{\mathbf{F}}_{\text{h}}$, with $F_{x\ \text{h}} = F_{\text{h}} \cos 20.0°$, and $\vec{\mathbf{w}}$, with $w_x = -mg \sin 20.0°$. Hence

$$\begin{aligned} F_{x\ \text{total}} = ma_x &\implies F_{\text{h}} \cos 20.0° - mg \sin 20.0° = ma_x \\ &\implies F_{\text{h}} = \frac{m(a_x + g \sin 20.0°)}{\cos 20.0°} = \frac{(51.0\ \text{kg})[(0.100\ \text{m/s}^2) + (9.81\ \text{m/s}^2) \sin 20.0°]}{\cos 20.0°} = 188\ \text{N}\,. \end{aligned}$$

d) Note that all three forces $\vec{\mathbf{F}}_{\text{h}}$, $\vec{\mathbf{w}}$, and $\vec{\mathbf{N}}$, have non-zero y components. These components are $F_{\text{h}} \sin 20.0°$, $-mg \cos 20.0°$, and N respectively. Since the acceleration is zero in the y direction, we have

$$\begin{aligned} F_{y\ \text{total}} &= ma_y \\ &\implies -F_{\text{h}} \sin 20.0° - mg \cos 20.0° + N = m(0\ \text{m/s}^2) = 0\ \text{N} \\ &\implies N = mg \cos 20.0° + F_{\text{h}} \sin 20.0° = (51.0\ \text{kg})(9.81\ \text{m/s}^2) \cos 20.0° + (188\ \text{N}) \sin 20.0° = 534\ \text{N}\,. \end{aligned}$$

5.35

a) The forces on the meteorite system are:

1. its weight $\vec{\mathbf{w}}$, of magnitude mg, directed down;
2. the force $\vec{\mathbf{T}}_1$ of the horizontal cable on the meteorite directed along the cable and toward the wall;
3. the force $\vec{\mathbf{T}}_2$ of the other cable on the meteorite, directed along the cable towards the point where it is fastened to the ceiling.

b) Choose a coordinate system with $\hat{\mathbf{i}}$ pointing to the right and $\hat{\mathbf{j}}$ pointing up. Then

$$\vec{\mathbf{w}} = -mg\hat{\mathbf{j}}, \qquad \vec{\mathbf{T}}_1 = -T_1\hat{\mathbf{i}}, \qquad \text{and} \quad \vec{\mathbf{T}}_2 = T_2(\sin 30.0^\circ)\hat{\mathbf{i}} + T_2(\cos 30.0^\circ)\hat{\mathbf{j}}.$$

Apply Newton's second law in each direction, recognizing that the acceleration of the meteorite is zero. In the $\hat{\mathbf{j}}$ direction we have

$$F_{y\ \text{total}} = ma_y \implies -mg + T_2\cos 30.0^\circ = 0\ \text{N} \implies T_2 = \frac{mg}{\cos 30.0^\circ} = \frac{(200\ \text{kg})(9.81\ \text{m/s}^2)}{0.866} = 2.27\times 10^3\ \text{N}.$$

In the $\hat{\mathbf{i}}$ direction we have

$$F_{x\ \text{total}} = ma_x \implies -T_1 + T_2\sin 30.0^\circ = 0\ \text{N} \implies T_1 = T_2\sin 30.0^\circ = (2.27\times 10^3\ \text{N})(0.500) = 1.13\times 10^3\ \text{N}.$$

5.37

a) Consider each mass to be a separate system. The forces on each are:

1. its weight; and
2. the force of the string on it.

Note that since the string is ideal, the force of the string on each mass has the same magnitude T.

Assume m_2 accelerates downward while m_1 simultaneously goes upward with an acceleration of the same magnitude a. Let $\hat{\mathbf{j}}$ point up. Apply Newton's second law to the motion of each mass. Then, for m_1,

$$F_{y\ \text{total}} = m_1a_y \implies T - m_1g = m_1a \implies T = m_1(g+a). \tag{1}$$

For m_2

$$F_{y\ \text{total}} = m_2a_y \implies T - m_2g = m_2(-a). \tag{2}$$

Substitute the expression for T from the last equation in (1) into the last equation in (2):

$$m_1(g+a) - m_2g = m_2(-a) \implies (m_1+m_2)a = (m_2-m_1)g \implies a = \left(\frac{m_2-m_1}{m_1+m_2}\right)g.$$

Therefore, the magnitude of the acceleration is

$$a = \left(\frac{|m_2-m_1|}{m_1+m_2}\right)g$$

b) From the last equation in (1) in part a) we have.

$$T = m_1(g+a) = m_1g + m_1a.$$

Substitute the expression found in part a) for a.

$$T = m_1g + m_1\left(\frac{m_2-m_1}{m_1+m_2}\right)g = \left(\frac{2m_1m_2}{m_1+m_2}\right)g.$$

5.38

a) The external forces on the system of two climbers are:

1. their weights $\vec{\mathbf{w}}_1$ and $\vec{\mathbf{w}}_2$, directed downward; and
2. the normal force $\vec{\mathbf{N}}$ of the inclined surface on the 60.0 kg mass, directed perpendicularly away from the inclined surface.

Assume, for the moment, that the acceleration of the system is such that the 50.0 kg mass is moving down. If the system actually moves the other way, then the acceleration component we find will turn out to be negative. Let $\hat{\mathbf{j}}$ point down. The problem is essentially one-dimensional, since as the 50.0 kg mass falls, the 60.0 kg mass slides up the plane. The forces on the (combined) system along the line of motion are the downward weight of the 50.0 kg mass and the component of weight of the 60.0 kg parallel to the inclined surface. Apply Newton's second law to the combined system.

$$m_2 g - m_1 g \sin 30.0^\circ = (m_1 + m_2)a \implies$$
$$a = \frac{m_2 g - m_1 g \sin 30.0^\circ}{m_1 + m_2} = \frac{(50.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) - (60.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2})\sin 30.0^\circ}{50.0\ \mathrm{kg} + 60.0\ \mathrm{kg}} = 1.78\ \mathrm{m/s^2}.$$

Since the acceleration component is positive, the system moves in the direction we originally assumed.

b) The forces on each climber considered as an individual system include the forces on each mentioned in part a) as well as the force $\vec{\mathbf{T}}$ of the rope on each.

c) Consider the 50.0 kg climber since it has fewer forces acting on it. Apply Newton's second law to this climber.

$$F_{y\ \mathrm{total}} = m_2 a_y \implies m_2 g - T = m_2 a \implies T = m_2(g - a).$$

In part a) we found $a = 1.78\ \mathrm{m/s^2}$. Hence

$$T = (50.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2} - 1.78\ \mathrm{m/s^2}) = 402\ \mathrm{N}.$$

5.46

a) Choose the lower gargoyle as the system. The forces on this system are:

1. its weight $\vec{\mathbf{w}}$, of magnitude mg, directed down; and
2. the force $\vec{\mathbf{T}}$ of the string on it, directed up.

Choose $\hat{\mathbf{j}}$ to point upward. Apply Newton's second law to this system. Since the acceleration is downward of magnitude $2.0\ \mathrm{m/s^2}$, then $a_y = -2.0\ \mathrm{m/s^2}$.

$$F_{y\ \mathrm{total}} = ma_y \implies T - mg = ma_y \implies T = m(g + a_y) = (2.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2} - 2.0\ \mathrm{m/s^2}) = 16\ \mathrm{N}.$$

b) Choose both gargoyles as the system. The forces on this system are:

1. its weight $\vec{\mathbf{w}}'$, of magnitude $m_{\mathrm{total}} g$, directed downward; and
2. the force $\vec{\mathbf{T}}'$ of the string on it, directed upward.

Choose $\hat{\mathbf{j}}$ to point upward. Apply Newton's second law to this system. Since the acceleration is downward and of magnitude $2.0\ \mathrm{m/s^2}$, then $a_y = -2.0\ \mathrm{m/s^2}$. Thus,

$$F_{y\ \mathrm{total}} = ma_y \implies T' - m_{\mathrm{total}} g = m_{\mathrm{total}} a_y \implies$$
$$T' = m_{\mathrm{total}}(g + a_y) = (2.00\ \mathrm{kg} + 2.00\ \mathrm{kg})(9.81\ \mathrm{m/s^2} - 2.0\ \mathrm{m/s^2}) = 31\ \mathrm{N}.$$