Assignment #3

Chapter 3 Problems:

3.34 Choose the origin on the floor with $\hat{\mathrm{i}}$ pointing up. Initially, $x_0 = 1.2\ \mathrm{m}$ and $\vec{\mathrm{a}} = (-9.81\ \mathrm{m/s^2})\hat{\mathrm{i}}$. Thus the velocity is $\vec{\mathrm{v}}(t) = \vec{\mathrm{v}}_0 + (-9.81\ \mathrm{m/s^2})t\hat{\mathrm{i}}$ and the position is $\vec{\mathrm{r}}(t) = (1.2\ \mathrm{m})\hat{\mathrm{i}} + t\vec{\mathrm{v}}_0 + (-9.81\ \mathrm{m/s^2})\frac{t^2}{2}\hat{\mathrm{i}}$.

When the ring reaches maximum height, its velocity is $\mathbf{0}$ m/s and its position is $(3.0\ \mathrm{m})\hat{\mathrm{i}}$. Hence the velocity and position equations become

$$0\ \mathrm{m/s} = \vec{\mathrm{v}}_0 - (9.81\ \mathrm{m/s^2})t\hat{\mathrm{i}}, \qquad \text{and} \qquad (3.0\ \mathrm{m})\hat{\mathrm{i}} = (1.2\ \mathrm{m})\hat{\mathrm{i}} + t\vec{\mathrm{v}}_0 - (9.81\ \mathrm{m/s^2})\frac{t^2}{2}\hat{\mathrm{i}},$$

giving two equations in two unknowns.

a) Solve the first equation for $\vec{\mathrm{v}}_0 = (9.81\ \mathrm{m/s^2})t\hat{\mathrm{i}}$ and substitute into the second:

$$(3.0\ \mathrm{m})\hat{\mathrm{i}} = (1.2\ \mathrm{m})\hat{\mathrm{i}} + (9.81\ \mathrm{m/s^2})t^2\hat{\mathrm{i}} - (9.81\ \mathrm{m/s^2})\frac{t^2}{2}\hat{\mathrm{i}}.$$

Since the ring reaches maximum height *after* it is launched, use the *positive* root: $t = 0.61\ \mathrm{s}$.

b) Use this value of t to find $\vec{\mathrm{v}}_0 = (9.81\ \mathrm{m/s^2})(0.61\ \mathrm{s})\hat{\mathrm{i}} = (6.0\ \mathrm{m/s})\hat{\mathrm{i}}$.

c) When the ring hits the floor its position is the zero vector 0 m/s, so from the position equation:

$$0\ \mathrm{m} = (1.2\ \mathrm{m})\hat{\mathrm{i}} + (6.0\ \mathrm{m/s})t\hat{\mathrm{i}} + (-9.81\ \mathrm{m/s^2})\frac{t^2}{2}\hat{\mathrm{i}},$$

which implies that $(-9.81\ \mathrm{m/s^2})\frac{t^2}{2} + (6.0\ \mathrm{m/s})t + 1.2\ \mathrm{m} = 0\ \mathrm{m}$. The quadratic formula gives two roots, $t = -0.17\ \mathrm{s}$ and $t = 1.4\ \mathrm{s}$. Since the ring lands *after* it was launched, $t = 1.4\ \mathrm{s}$.

d) Use $t = 1.4\ \mathrm{s}$ in the velocity equation to find $\vec{\mathrm{v}} = (-7.7\ \mathrm{m/s})\hat{\mathrm{i}}$. So $v = 7.7\ \mathrm{m/s}$.

3.57 Choose a coordinate system with $\hat{\mathrm{i}}$ pointing up and with origin on the ground. Then $v_{x0} = 3.0\ \mathrm{m/s}$ and $a_x = -9.81\ \mathrm{m/s^2}$. The equation for the x-component of the position vector is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = x_0 + (3.0\ \mathrm{m/s})t + (-9.81\ \mathrm{m/s^2})\frac{t^2}{2}.$$

When $t = 1.5\ \mathrm{s}$, the pine cone is at the origin, so

$$0\ \mathrm{m} = x_0 + (3.0\ \mathrm{m/s})(1.5\ \mathrm{s}) + (-9.81\ \mathrm{m/s^2})\frac{(1.5\ \mathrm{s})^2}{2}.$$

Solving for x_0, $x_0 = 6.5\ \mathrm{m}$.

3.60

a) Choose a coordinate system with $\hat{\imath}$ pointing up and the origin at the point of release. Then, for the first rock $x_0 = 0$ m, $v_{x0} = 0$ m/s, and $a_x = -9.81$ m/s^2. The position equation is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = (-9.81\ \text{m/s}^2)\frac{t^2}{2}.$$

When the rock hits the water, its position is $x = -269$ m, so $-269\ \text{m} = (-9.81\ \text{m/s}^2)\frac{t^2}{2}$. Solve for t to find $t = \pm 7.41$ s. Since impact occurs *after* $t = 0$ s, we use the positive root: $t = 7.41$ s.

b) Use the same spatial coordinate system for the second rock, but let $t = 0$ s be the time of the second rock's release. Its flight time is 1.00 s less than the first rock's, or 6.41 s. For the second rock we also have $x_0 = 0$ m and $a_x = -9.81$ m/s^2. Its position equation is

$$x(t) = x_0 + v_{x0}t + a_x\frac{t^2}{2} = v_{x0}t + (-9.81\ \text{m/s}^2)\frac{t^2}{2}.$$

The second rock hits the water when $t = 6.41$ s and $x = -269$ m. Hence,

$$-269\ \text{m} = v_{x0}(6.41\ \text{s}) - (9.81\ \text{m/s}^2)\frac{(6.41\ \text{s})^2}{2}.$$

Solve for v_{x0}: $v_{x0} = -10.5$ m/s. Thus, the initial velocity of the second rock is $(-10.5\ \text{m/s})\hat{\imath}$. The *speed* is the magnitude of the velocity, so the initial speed is $v = 10.5$ m/s.

3.65 Take $\hat{\imath}$ to point down and let the origin be at the top of the well. Let the (unknown) depth of the well be d. For a rock falling from rest we have $x_0 = 0$ m, $v_{x0} = 0$ m/s, and $a_x = g$. Using these initial values, the x-component of the position vector of the ball at the time t when it reaches the bottom of the well is $d = g\frac{t^2}{2}$, so

$$t = \sqrt{\frac{2d}{g}}.$$

The time t' for the sound of the splash to propagate from the bottom to the top of the well is $t' = \frac{d}{v_s}$ where v_s is the speed of sound. Thus, the total time T, from the release of the rock until the splash is heard at the top of the well, is

$$T = t + t' = \sqrt{\frac{2d}{g}} + \frac{d}{v_s}.$$

The time T is given as 3.00 s, hence

$$3.00\ \text{s} = \sqrt{\frac{2d}{9.81\ \text{m/s}^2}} + \frac{d}{343\ \text{m/s}}$$

Let $\alpha = \sqrt{d}$. Then in terms of α

$$3.00\ \text{s} = \left(0.452\frac{\text{s}}{\text{m}^{1/2}}\right)\alpha + \frac{\alpha^2}{343\ \text{m/s}}.$$

Clear fractions and transpose:

$$\alpha^2 + (155\ \mathrm{m}^{1/2})\alpha - 1.03 \times 10^3\ \mathrm{m} = 0\ \mathrm{m}\,.$$

This is quadratic in α. Use the quadratic formula to solve for the positive root:

$$\alpha = 6.5\ \mathrm{m}^{1/2}.$$

Since $\alpha^2 = d$, the depth of the well is

$$d = (6.5\ \mathrm{m}^{1/2})^2 = 42\ \mathrm{m}\,.$$

Chapter 4 Questions:

4.15

a) All the fleas have the same initial vertical velocity component v_{y0} since they all rise to the same height. Flea C has the greatest initial horizontal velocity component v_{x0} (that remains constant during the motion.) Hence flea C has the greatest initial speed v_0 since $v_0 = \{v_{x0}^2 + v_{y0}^2\}^{1/2}$.

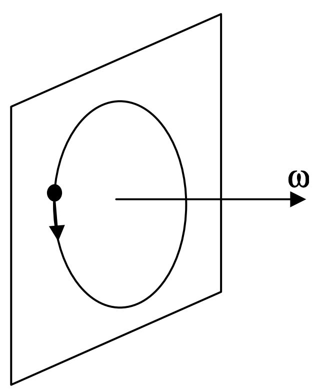
b) They all are in the air for the same time interval since they all rise to the same height.

c) All have the same magnitude of acceleration during their flights (g).

d) Flea C has the greatest horizontal velocity component since it traveled the greatest horizontal distance.

e) They all have the same vertical velocity component that is greatest at the instant of launch.

4.33



Chapter 4 Problems:

4.6

a) Choose the coordinate system with $\hat{\imath}$ pointed to the right, $\hat{\jmath}$ pointed up, and the origin at ground level directly under the point from which the boulder is ejected.

b) Use the coordinate system from part a) and make the following identifications

$$\begin{aligned} y_0 &= 2.5\times10^3 \text{ m} & x_0 &= 0 \text{ m}\\ v_{y0} &= v_0 \sin 40^\circ & v_{x0} &= v_0\cos 40^\circ\\ a_y &= -g & a_x &= 0 \text{ m/s}^2\,. \end{aligned}$$

The equations for the velocity components are then

$$v_y(t) = v_{y_0} + a_y t = v_0 \sin 40^\circ - gt \qquad v_x(t) = v_{x0} + a_x t = v_0 \cos 40^\circ.$$

The equations for the position coordinates are

$$\begin{aligned} y(t) &= y_0 + v_{y0}t + a_y\frac{t^2}{2} & x(t) &= x_0 + v_{x0}t + a_x\frac{t^2}{2}\\ &= 2.5\times10^3 \text{ m} + v_0(\sin 40^\circ)t - g\frac{t^2}{2} & &= v_0(\cos 40^\circ)t. \end{aligned}$$

c) Impact occurs where $x = 6.0\times10^3$ m and $y = 0$ m. The equation for x becomes

$$6.0\times10^3 \text{ m} = v_0(\cos 40^\circ)t.$$

Solve for t to find $t = \dfrac{6.0\times10^3 \text{ m}}{v_0\cos 40^\circ}$.

The equation for y becomes

$$0 \text{ m} = 2.5\times10^3 \text{ m} + v_0(\sin 40^\circ)t - g\frac{t^2}{2}.$$

Substitute the value for t and find

$$0 \text{ m} = 2.5\times10^3 \text{ m} + v_0(\sin 40^\circ)\frac{6.0\times10^3 \text{ m}}{v_0\cos 40^\circ} - \frac{\left(\dfrac{6.0\times10^3 \text{ m}}{v_0\cos 40^\circ}\right)^2}{2}.$$

Solve this for v_0,

$$v_0 = 2.0\times10^2 \text{ m/s}.$$

With the initial speed known, we can find the time of flight,

$$t = \frac{6.0\times10^3 \text{ m}}{v_0\cos 40^\circ} = \frac{6.0\times10^3 \text{ m}}{(2.0\times10^2 \text{ m/s})\cos 40^\circ} = 39 \text{ s}.$$

4.9

a) Choose a coordinate system with $\hat{\imath}$ pointing right, $\hat{\jmath}$ pointing up, and origin at the launch point of the soccer ball.

b) In this coordinate system,

$$\begin{aligned} y_0 &= 0\text{ m} \\ v_{y0} &= (20.0\text{ m/s})\sin 30.0^\circ \\ &= 10.0\text{ m/s} \\ a_y &= -g \end{aligned} \qquad \begin{aligned} x_0 &= 0\text{ m} \\ v_{x0} &= (20.0\text{ m/s})\cos 30.0^\circ \\ &= 17.3\text{ m/s} \\ a_x &= 0\text{ m/s}^2\,. \end{aligned}$$

The equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= 10.0\text{ m/s} - gt \\ y(t) &= (10.0\text{ m/s})t - g\frac{t^2}{2} \end{aligned} \qquad \begin{aligned} v_x(t) &= 17.3\text{ m/s} \\ x(t) &= (17.3\text{ m/s})t. \end{aligned}$$

c) The soccer ball impacts when $y(t) = -40.0$ m. Substitute this into the equation for $y(t)$,

$$-40.0\text{ m} = (10.0\text{ m/s})t - g\frac{t^2}{2},$$

and then use the quadratic formula to solve for t. The two roots are $t = 4.05$ s and $t = -2.01$ s. Since impact occurs after $t = 0$ s (the time when the ball is kicked), choose the positive root. Thus, the flight time is 4.05 s.

d) The x-coordinate of the impact point is determined from the equation for $x(t)$ with $t = 4.05$ s.

$$x(t) = (17.3\text{ m/s})t = (17.3\text{ m/s})(4.05) = 70.1\text{ m}.$$

The coordinates of the impact point are therefore $x = 70.1$ m and $y = -40.0$ m.

4.21

a) Choose a coordinate system with $\hat{\imath}$ pointing to the right, $\hat{\jmath}$ pointing up, and origin on the ground directly below the launch point.

In this coordinate system

$$\begin{aligned} y_0 &= 15\text{ m} \\ v_{y0} &= v_0 \sin 45^\circ = \frac{v_0}{\sqrt{2}} \\ a_y &= -g \end{aligned} \qquad \begin{aligned} x_0 &= 0\text{ m} \\ v_{x0} &= v_0 \cos 45^\circ = \frac{v_0}{\sqrt{2}} \\ a_x &= 0\text{ m/s}^2\,. \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= \frac{v_0}{\sqrt{2}} - gt \\ y(t) &= 15\text{ m} + \frac{v_0}{\sqrt{2}}t - g\frac{t^2}{2} \end{aligned} \qquad \begin{aligned} v_x(t) &= \frac{v_0}{\sqrt{2}} \\ x(t) &= \frac{v_0}{\sqrt{2}}t. \end{aligned}$$

The x-coordinate of the upright piano's impact point is 140 m. Use this information in the equation for $x(t)$ at the time of impact,

$$140\text{ m} = \frac{v_0}{\sqrt{2}}t$$

and solve for the impact time:

$$t = \frac{(140\text{ m})\sqrt{2}}{v_0}.$$

The y-coordinate of the impact point is 0 m. Hence, the equation for $y(t)$ yields

$$0 \text{ m} = 15 \text{ m} + \frac{v_0}{\sqrt{2}}t - g\frac{t^2}{2}.$$

Substitute the expression for the impact time into this equation,

$$0 \text{ m} = 15 \text{ m} + \frac{v_0}{\sqrt{2}}\left(\frac{(140 \text{ m})\sqrt{2}}{v_0}\right) - g\frac{\left(\frac{(140 \text{ m})\sqrt{2}}{v_0}\right)^2}{2} = 155 \text{ m} - g\frac{\left(\frac{(140 \text{ m})\sqrt{2}}{v_0}\right)^2}{2},$$

and solve for v_0:

$$v_0 = 35 \text{ m/s}.$$

b) The average acceleration is the change in velocity divided by the time required to make that change:

$$\vec{a}_{\text{ave}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{(35 \text{ m/s})(\cos 45°)\hat{\imath} + (35 \text{ m/s})(\sin 45°)\hat{\jmath} - \mathbf{0} \text{ m/s}}{1.5 \text{ s}} = (16 \text{ m/s}^2)\hat{\imath} + (16 \text{ m/s}^2)\hat{\jmath}.$$

The magnitude of the average acceleration is

$$v_{\text{ave}} = \sqrt{(16 \text{ m/s}^2)^2 + (16 \text{ m/s}^2)^2} = 23 \text{ m/s}^2.$$

4.28 Choose a coordinate system, with the origin at the point where the ball was hit (one meter above home plate), $\hat{\imath}$ pointing from the origin to the point where the ball was caught, and $\hat{\jmath}$ pointing up. Then

$$\begin{aligned} y_0 &= 0 \text{ m} & x_0 &= 0 \text{ m} \\ v_{y0} &= v_0 \sin\theta & v_{x0} &= v_0\cos\theta \\ a_y &= -g & a_x &= 0 \text{ m/s}^2. \end{aligned}$$

So, the equations for the velocity and position components are

$$\begin{aligned} v_y(t) &= v_0 \sin\theta - gt & v_x(t) &= v_0\cos\theta \\ y(t) &= (v_0\sin\theta)t - g\frac{t^2}{2} & x(t) &= (v_0\cos\theta)t \end{aligned}$$

a) The ball is caught at $x = 90.0$ m, when $t = 3.00$ s. Use this in the equation for x.

$$90.0 \text{ m} = (v_0\cos\theta)(3.00 \text{ s}),$$

so

(1) $$v_0\cos\theta = 30.0 \text{ m/s}.$$

Also, when the ball is caught the y-coordinate is 0 m and $t = 3.00$ s. Hence, the equation for y at $t = 3.00$ s is

$$0 \text{ m} = (v_0\sin\theta)(3.00 \text{ s}) - (9.81 \text{ m/s}^2)\frac{(3.00 \text{ s})^2}{2},$$

which reduces to

(2) $$v_0\sin\theta = 14.7 \text{ m/s}.$$

Divide equation (2) by equation (1) to eliminate v_0.

$$\frac{v_0\sin\theta}{v_0\cos\theta} = \frac{14.7 \text{ m/s}}{30.0 \text{ m/s}} \implies \tan\theta = 0.490 \implies \theta = 26.1°.$$

b) Now that we know $\theta = 26.1°$, we may find the initial speed of the ball from either equation (1) or (2). Using equation (2),

$$v_0 \sin 26.1° = 14.7 \text{ m/s} \implies v_0 = \frac{14.7 \text{ m/s}}{\sin 26.1°} = 33.4 \text{ m/s}.$$

c) At maximum height, $v_y = 0$ m/s, so from the equation for v_y,

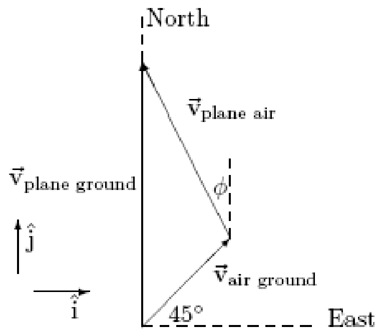
$$0 \text{ m/s} = (33.4 \text{ m/s}) \sin 26.1° - (9.81 \text{ m/s}^2)t \implies t = \frac{(33.4 \text{ m/s}) \sin 26.1°}{9.81 \text{ m/s}^2} = 1.50 \text{ s}.$$

Use this time in the equation for y to determine the maximum value of y.

$$y_{\text{max}} = (33.4 \text{ m/s})(\sin 26.1°)(1.50 \text{ s}) - (9.81 \text{ m/s}^2)\frac{(1.50 \text{ s})^2}{2} = 11.0 \text{ m}.$$

Since the origin of our coordinate system is 1.00 m above the ground, the actual maximum height of the ball above the ground is 11.0 m + 1.00 m = 12.0 m.

4.39 The geometry and a coordinate system are shown below.



Let v be the speed of the plane with respect to the ground. Then

$$\vec{v}_{\text{plane ground}} = v\hat{j}.$$

The velocity of the wind is

$$\vec{v}_{\text{air ground}} = (100 \text{ km/h}) \cos 45° \hat{i} + (100 \text{ km/h}) \sin 45° \hat{j} = (71 \text{ km/h})\hat{i} + (71 \text{ km/h})\hat{j}.$$

The three velocities are related by the relative velocity addition equation.

$$\vec{v}_{\text{plane ground}} = \vec{v}_{\text{plane air}} + \vec{v}_{\text{air ground}}.$$

Rearrange this slightly.

$$\vec{v}_{\text{plane air}} = \vec{v}_{\text{plane ground}} - \vec{v}_{\text{air ground}} = v\hat{j} - \left((71 \text{ km/h})\hat{i} + (71 \text{ km/h})\hat{j}\right) = (-71 \text{ km/h})\hat{i} + (v - 71 \text{ km/h})\hat{j}.$$

We know the speed of the plane with respect to the air. It is just the magnitude of $\vec{v}_{\text{plane air}}$. Use the equation above to evaluate

$$\vec{v}_{\text{plane air}} \bullet \vec{v}_{\text{plane air}} = v^2_{\text{plane air}} = (700 \text{ km/h})^2 \implies (700 \text{ km/h})^2 = (-71 \text{ km/h})^2 + (v - 71 \text{ km/h})^2.$$

Compute the square on the far right, and then rearrange the terms.

$$v^2 - (142 \text{ km/h})v - 4.80 \times 10^5 (\text{ km/h})^2 = 0(\text{ km/h})^2.$$

Use the quadratic formula to solve for v and, since v must be positive, take the positive root. Thus, the ground speed of the plane is

$$v = 7.7 \times 10^2 \text{ km/h}.$$

The velocity of the plane with respect to the air then is

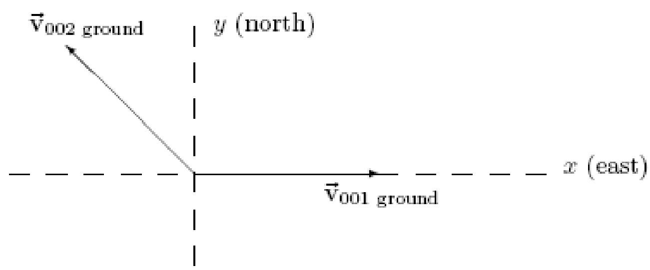
$$\vec{\mathbf{v}}_{\text{plane air}} = (-71 \text{ km/h})\hat{\mathbf{i}} + (7.7 \times 10^2 \text{ km/h} - 71 \text{ km/h})\hat{\mathbf{j}} = (-71 \text{ km/h})\hat{\mathbf{i}} + (7.0 \times 10^2 \text{ km/h})\hat{\mathbf{j}}.$$

From the picture at the beginning of the problem, the heading ϕ of the plane with respect to the direction north satisfies

$$\tan\phi = \left|\frac{v_x}{v_y}\right| = \frac{71 \text{ km/h}}{7.0 \times 10^2 \text{ km/h}} = 0.10,$$

so $\phi = 5.7°$.

4.45 The situation and a coordinate system are indicated below:



a) Flight 001 has a velocity with respect to the ground of

$$\vec{\mathbf{v}}_{001 \text{ ground}} = (600 \text{ km/h})\hat{\mathbf{i}}.$$

Flight 002 has a velocity with respect to the ground of

$$\vec{\mathbf{v}}_{002 \text{ ground}} = ((-700 \text{ km/h})\cos 45°)\hat{\mathbf{i}} + ((700 \text{ km/h})\sin 45°)\hat{\mathbf{j}} = (-495 \text{ km/h})\hat{\mathbf{i}} + (495 \text{ km/h})\hat{\mathbf{j}}.$$

The velocity of flight 002 with respect to flight 001 is found from the relative velocity addition equation:

$$\vec{\mathbf{v}}_{002\ 001} = \vec{\mathbf{v}}_{002 \text{ ground}} + \vec{\mathbf{v}}_{\text{ground } 001}.$$

But

$$\vec{\mathbf{v}}_{\text{ground } 001} = -\vec{\mathbf{v}}_{001 \text{ ground}}.$$

Hence, the relative velocity addition equation becomes

$$\vec{\mathbf{v}}_{002\ 001} = (-495 \text{ km/h})\hat{\mathbf{i}} + (495 \text{ km/h})\hat{\mathbf{j}} + (-600 \text{ km/h})\hat{\mathbf{i}} = (-1095 \text{ km/h})\hat{\mathbf{i}} + (495 \text{ km/h})\hat{\mathbf{j}}.$$

b) The speed is the magnitude of the vector $\vec{\mathbf{v}}_{002\ 001}$:

$$v_{002\ 001} = \sqrt{(-1095 \text{ km/h})^2 + (495 \text{ km/h})^2} = 1.20 \times 10^3 \text{ km/h}.$$

4.55

a) Assume the orbit is circular. The speed with which the Moon circles the Earth is

$$v = \frac{2\pi r}{\Delta t} = \frac{2\pi(3.84 \times 10^5 \text{ km})}{27.322 \text{ d}} = \left(\frac{2\pi(3.84 \times 10^5 \text{ km})}{27.322 \text{ d}}\right)\left(\frac{10^3 \text{ m}}{\text{km}}\right)\left(\frac{\text{d}}{8.6400 \times 10^4 \text{ s}}\right) = 1.02 \times 10^3 \text{ m/s}\,.$$

The magnitude of the centripetal acceleration is

$$a_{\text{centripetal}} = \frac{v^2}{r} = \frac{(1.02 \times 10^3 \text{ m/s})^2}{3.84 \times 10^8 \text{ m}} = 2.71 \times 10^{-3} \text{ m/s}^2\,.$$

b) The ratio is

$$\frac{a_{\text{centripetal}}}{g} = \frac{2.71 \times 10^{-3} \text{ m/s}^2}{9.81 \text{ m/s}^2} = 2.76 \times 10^{-4} \approx \frac{1}{3600}.$$

4.62

a) The tip of the minute hand travels one circumference during one hour. Hence

$$v = \frac{2\pi r}{\Delta t} = \frac{2\pi(4.30 \text{ m})}{3600 \text{ s}} = 7.50 \times 10^{-3} \text{ m/s}\,.$$

b) The magnitude of the centripetal acceleration is

$$a_{\text{centripetal}} = \frac{v^2}{r} = \frac{(7.50 \times 10^{-3} \text{ m/s})^2}{4.30 \text{ m}} = 1.31 \times 10^{-5} \text{ m/s}^2\,.$$

c) A year of 365 days has $(365 \text{ d})\left(\frac{24 \text{ h}}{\text{d}}\right) = 8760$ hours. Each hour, the tip of the hand travels one circumference of radius 4.30 m. Hence the total distance traveled by the tip of the minute hand in a year is

$$(8760 \text{ h})(2\pi)(4.30 \text{ m/h}) = 2.37 \times 10^5 \text{ m} = 237 \text{ km}\,.$$

4.75

a) Choose $\hat{\text{k}}$ to be the parallel to the angular velocity vector $\vec{\omega}$, as determined from the right-hand rule. With this choice, $\hat{\text{k}}$ is directed into the page in Figure P.75 on page 165 of the text.

b) The magnitude of the centripetal acceleration is the magnitude of the component of the total acceleration directed toward the center of the circle.

$$a_{\text{centripetal}} = (40.0 \text{ m/s}^2)\cos 20.0° = 37.6 \text{ m/s}^2\,.$$

c) The magnitude of the tangential acceleration is the magnitude of the component of the total acceleration tangent to the circle.

$$a_{\text{tangential}} = (40.0 \text{ m/s}^2)\sin 20.0° = 13.7 \text{ m/s}^2\,.$$

d) The magnitude of the centripetal acceleration is

$$a_{\text{centripetal}} = r\omega^2.$$

In part b) we found that $a_{\text{centripetal}} = 37.6 \text{ m/s}^2$ and $r = 4.00$ m. Therefore,

$$a_{\text{centripetal}} = r\omega^2 \implies 37.6 \text{ m/s}^2 = (4.00 \text{ m})\omega^2 \implies \omega = 3.07 \text{ rad/s} \implies \vec{\omega} = (3.07 \text{ rad/s})\hat{\text{k}}.$$

e) The magnitude of the tangential acceleration is

$$a_{\text{tangential}} = r\alpha,$$

where α is the magnitude of the angular acceleration and r is the radius of the circle. Hence, using the result from part c),

$$13.7\ \text{m/s}^2 = (4.00\ \text{m})\alpha \implies \alpha = 3.43\ \text{rad/s}^2\,.$$

The tangential acceleration is opposite in direction to the velocity of the particle, so it is slowing down. The angular acceleration vector is therefore opposite to the direction of $\vec{\omega}$, and therefore in the direction $-\hat{\text{k}}$. Hence

$$\vec{\alpha} = -(3.43\ \text{rad/s}^2)\hat{\text{k}}.$$

f) The particle is slowing down in its circular motion.

g) The tangential acceleration has constant magnitude since it depends only upon α and r, both of which are constant. The centripetal acceleration depends upon both r and the angular speed. Since the angular speed is changing with time, then the magnitude of the centripetal acceleration also changes with time.

h) The angular velocity component is

$$\omega_z(t) = \omega_{z0} + \alpha_z t \implies \omega_z(t) = 3.07\ \text{rad/s} - (3.43\ \text{rad/s}^2)t.$$

When the particle comes to rest, its angular velocity component is zero, so

$$0\ \text{rad/s} = 3.07\ \text{rad/s} - (3.43\ \text{rad/s}^2)t. \implies t = 0.895\ \text{s}\,.$$