Assignment #12

Chapter 12 Questions:

12.10 The wire is wrapped around the base strings to increase the mass per unit length and lower the fundamental frequency for a fixed length and tension.

12.13 The speed of each wave is the product of the frequency and wavelength. For sound and light waves of the same wavelength, the light wave has the greater frequency because the speed of light is so much greater than the speed of sound.

12.24 This is the third harmonic

12.39 The frequency increases because the wavelength decreases as the length of the resonating air column decreases, and the speed of the wave stays the same, $v = \lambda\nu$.

Chapter 12 Problems:

12.26 The speed v of waves on a wire with a tension T is

$$v = \sqrt{\frac{T}{\mu}} \implies T = \mu v^2,$$

where μ is the mass of a one meter length of the (cylindrical) wire. Since we are given v, to find T we just need to find μ.

The mass per unit length is defined as

$$\mu = \frac{m}{\ell} = \frac{\rho V}{\ell} = \frac{\rho \pi r^2 \ell}{\ell} = \rho\,\pi r^2.$$

Putting in the numbers, we have

$$\mu = (8.50 \times 10^3\ \mathrm{kg/m^3})[\pi(0.1016 \times 10^{-2}\ \mathrm{m})^2] = (8.50 \times 10^3\ \mathrm{kg/m^3})(3.243 \times 10^{-6}\ \mathrm{m^2}) = 2.76 \times 10^{-2}\ \mathrm{kg/m}\,.$$

The tension T of the string is thus

$$T = \mu v^2 = (2.76 \times 10^{-2}\ \mathrm{kg/m})(125\ \mathrm{m/s})^2 = 431\ \mathrm{N}\,.$$

12.27 The tension (of magnitude T) in the two ropes is the same since, if it weren't, there would be a nonzero total force on pieces of the each rope which would make them accelerate. The speed v_1 of the wave in the rope with mass per unit length μ_1 is

$$v_1 = \sqrt{\frac{T}{\mu_1}},$$

and the speed v_2 of the wave in the rope with mass per unit length μ_2 is

$$v_2 = \sqrt{\frac{T}{\mu_2}}.$$

The ratio of the speeds is

$$\frac{v_1}{v_2} = \frac{\sqrt{\frac{T}{\mu_1}}}{\sqrt{\frac{T}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}}.$$

12.35 The luminosity L of the Sun is its power output. This power is spread uniformly over the surface of an imaginary sphere of radius r centered on the Sun. The power per square meter on the surface of this imaginary sphere is the solar constant,

$$1.36 \times 10^3 \text{ W/m}^2 = \frac{L}{4\pi r^2},$$

where r is the distance between the Earth and the Sun (1.496×10^{11} m). Solve for L:

$$L = 4\pi r^2 (1.36 \times 10^3 \text{ W/m}^2) = 4\pi(1.496 \times 10^{11} \text{ m})^2(1.36 \times 10^3 \text{ W/m}^2) = 3.82 \times 10^{26} \text{ W}.$$

(This is *a lot* of light bulbs!)

12.45 The sound level is

$$\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right)$$

where $I_0 = 10^{-12}$ W/m^2 is the threshold of hearing. Use the given information to find the intensity at the given distance:

$$120 \text{ dB} = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right) \implies 12.0 = \log\left(\frac{I}{I_0}\right) \implies \frac{I}{I_0} = 10^{12.0} \implies I = (10^{12} \text{ W/m}^2)10^{-12.0} = 1.00 \text{ W/m}^2.$$

The intensity is related to the power P of the source and the distance r from it by

$$I = \frac{P}{4\pi r^2} \implies P = 4\pi r^2 I = 4\pi(6.0 \text{ m})^2(1.00 \text{ W/m}^2) = 4.5 \times 10^2 \text{ W}.$$

Now use the power to find the intensity I' at a distance 30.0 m from the source:

$$I' = \frac{P}{4\pi r^2} = \frac{4.5 \times 10^2 \text{ W}}{4\pi(30.0 \text{ m})^2} = 4.0 \times 10^{-2} \text{ W/m}^2.$$

The sound level at 30.0 m is

$$\begin{aligned}\beta' &= (10 \text{ dB}) \log\left(\frac{I'}{I_0}\right) = (10 \text{ dB}) \log\left(\frac{4.0 \times 10^{-2} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) \\ &= (10 \text{ dB}) \log(4.0 \times 10^{10}) = (10 \text{ dB}) \log(4.0) + (10 \text{ dB})10 = 6.0 \text{ dB} + 100 \text{ dB} = 106 \text{ dB}.\end{aligned}$$

12.52

a) Since the source is moving away from you, the frequency you hear will be less than the "true" frequency, so the note will sound "flat."

b) The beat frequency is the difference between the two frequencies. The frequency of the moving violin, heard by the stationary observer, thus is 261.6 Hz $-$ 2.0 Hz $=$ 259.6 Hz. Since the source is moving, we have

$$\nu' = \nu \frac{v}{v + v_{\text{source}}} \implies 259.6\ \text{Hz} = 261.6\ \text{Hz}\, \frac{343\ \text{m/s}}{343\ \text{m/s} + v_{\text{source}}}$$

$$\implies \frac{261.6}{259.6} = \frac{343\ \text{m/s} + v_{\text{source}}}{343\ \text{m/s}}$$

$$\implies 343\ \text{m/s} + v_{\text{source}} = (343\ \text{m/s})\left(\frac{261.6}{259.6}\right)$$

$$\implies 343\ \text{m/s} + v_{\text{source}} = 346\ \text{m/s}$$

$$\implies v_{\text{source}} = 3\ \text{m/s}.$$

12.58 The speed of the aircraft can be found from the Mach number, since

$$\text{Mach number} = \frac{v_{\text{source}}}{v} \implies v_{\text{source}} = \text{Mach number} \times v.$$

Putting in the numbers, we have

$$v_{\text{source}} = (1.30)\,(343\ \text{m/s}) = 446\ \text{m/s}.$$

The plane travels a distance d from when it was directly overhead to where it is located when you hear the sonic boom, and it takes twelve seconds to travel this distance. Hence,

$$d = v_{\text{source}} t = (446\ \text{m/s})\,(12.0\ \text{s}) = 5.35 \times 10^3\ \text{m},\ \text{or}\ 5.35\ \text{km}.$$

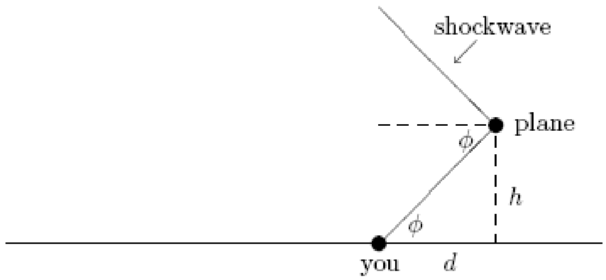
Next we use this distance to find the altitude of the plane. Consider Figure 12.47, and the discussion below it, on page 557 of the text. It is shown there that the half-angle ϕ of the Mach cone is

$$\sin\phi = \frac{1}{\text{Mach number}} = \frac{1}{1.30} = 0.769$$

which means that

$$\sin\phi = 50.3°.$$

The geometry is shown in the sketch below:



Referring to the sketch, the altitude h of the aircraft is found from

$$\tan\phi = \frac{h}{5.35\times10^3\ \text{m}} \implies h = (5.35\times10^3\ \text{m})\tan 50.3^\circ = 6.44\times10^3\ \text{m or } 6.44\ \text{km}.$$

12.66

a) Start with Equation 12.49 on page 561 of the text:

$$\nu_n = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}$$

where T is the tension in the string, μ is the string's mass per unit length, ℓ its length, and n the number of the harmonic. In this problem we are only concerned with the fundamental frequency of each string, so $n = 1$ in everything that follows. Write the above equation for the fundamental frequency of each string and then take the ratio:

$$\frac{\nu_A}{\nu_B} = \frac{\dfrac{1}{2\ell_A}\sqrt{\dfrac{T_A}{\mu_A}}}{\dfrac{1}{2\ell_B}\sqrt{\dfrac{T_B}{\mu_B}}}$$

$$= \frac{\ell_B}{\ell_A}\sqrt{\frac{T_A\mu_B}{T_B\mu_A}}$$

$$= \frac{\ell_B}{\ell_A}\sqrt{\frac{T_A\ell_A}{T_B\ell_B}}$$

$$= \left(\frac{\ell_B}{\ell_A}\right)^{1/2}\sqrt{\frac{T_A}{T_B}},$$

since $\mu = m/\ell$ and the strings have equal mass.

We are told that string A has twice the length and twice the tension of string B, hence

$$\frac{\nu_A}{\nu_B} = \left(\frac{\ell_B}{2\ell_B}\right)^{1/2}\sqrt{\frac{2T_B}{T_B}} = \left(\frac{1}{2}\right)^{1/2}\sqrt{2} = 1$$

b) Start with the expression derived in part a):

$$\frac{\nu_A}{\nu_B} = \left(\frac{\ell_B}{\ell_A}\right)^{1/2} \sqrt{\frac{T_A}{T_B}},$$

put in the given information, and solve for T_A:

$$\frac{\nu_A}{\nu_B} = \left(\frac{\ell_B}{\ell_A}\right)^{1/2} \sqrt{\frac{T_A}{T_B}} \implies \frac{1}{2} = \left(\frac{\ell_B}{2\ell_B}\right)^{1/2} \sqrt{\frac{T_A}{T_B}}$$

$$\implies \frac{1}{2} = \left(\frac{1}{2}\right)^{1/2} \sqrt{\frac{T_A}{T_B}}$$

$$\implies \frac{1}{\sqrt{2}} = \sqrt{\frac{T_A}{T_B}}$$

$$\implies T_A = \frac{T_B}{2}.$$

So the tension in string A must be 1/2 that in string B in order for the fundamental frequency of A to be half the fundamental frequency of string B.

12.70

a) A wave of the same amplitude, frequency, and wavelength, but traveling in the opposite direction will form a standing wave with the given one. The given wavefunction is traveling toward increasing values of x, so the wavefunction of the additional wave is one traveling toward decreasing values of x:

$$\Psi(x,t) = (2.00 \times 10^{-2}\ \mathrm{m}) \cos[(18.0\ \mathrm{rad/m})x + (24.0\ \mathrm{rad/s})t].$$

b) The distance between the nodes is half the wavelength. The wavelength is found from the angular wavenumber:

$$k = \frac{2\pi}{\lambda} \implies \lambda = \frac{2\pi}{k} = \frac{2\pi}{18.0\ \mathrm{rad/m}} = 0.349\ \mathrm{m}\,.$$

The distance between nodes then is

$$\frac{0.349\ \mathrm{m}}{2} = 0.175\ \mathrm{m}\,.$$

c) The distance between the antinodes is the same as the distance between the nodes: half the wavelength, or $0.175\ \mathrm{m}$.

d) The distance between a node and the nearest antinode is one-fourth the wavelength,

$$\frac{\lambda}{4} = \frac{0.349\ \mathrm{m}}{4} = 0.0873\ \mathrm{m}\,.$$

12.72 From page 563 of the text, the fundamental frequency of an open organ pipe is

$$\nu = \frac{v}{2\ell} \implies \ell = \frac{v}{2\nu} = \frac{343 \text{ m/s}}{2(440 \text{ Hz})} = 0.390 \text{ m}.$$

The second harmonic of the open pipe is twice the fundamental frequency (the first harmonic), or 880 Hz. For a closed organ pipe, the fundamental frequency is derived on 563 of the text,

$$\nu = \frac{v}{4\ell} \implies \ell = \frac{v}{4\nu} = \frac{343 \text{ m/s}}{(4)\,(880 \text{ Hz})} = 9.74 \times 10^{-2} \text{ m}.$$

12.73 The wavelength λ of the fundamental vibration of a string of length ℓ is twice the length of the string,

$$\lambda = 2\ell.$$

The speed of the waves on the string is

$$v = \sqrt{\frac{T}{\mu}}.$$

The speed is also the product of the frequency and wavelength,

$$v = \nu\lambda \implies \nu = \frac{1}{\lambda}v = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}.$$

If the string is shortened to a length $\alpha\ell$, then the corresponding fundamental frequency is reduced to

$$\nu' = \frac{1}{2\alpha\ell}\sqrt{\frac{T}{\mu}}.$$

Therefore, dividing the expression for ν' by the expression for ν,

$$\frac{\nu'}{\nu} = \frac{1}{\alpha} \implies \nu' = \frac{\nu}{\alpha}.$$