Assignment #11

Chapter 10 Questions:

10.32 Once leaving the board, there is no external torque on the diver. By “tucking,” a diver decreases his moment of inertia and so his spin rate (angular velocity) increases to conserve angular momentum. When the diver straightens out, his moment of inertia increases dramatically and so the angular velocity also decreases dramatically, but does not become zero.

10.38 Using the CWE theorem, chose a coordinate system with **j** up and an origin at the level of the center of the circles when they are at the base of the inclined plane. Let the height of the plane be *h*. Since

$W = \Delta K + \Delta U$
$0 = (mv^2/2 + I_{cm}\omega^2/2 - 0) + (0 - mgh)$ J

Writing the moment of inertia as βmR^2,

$mgh = mv^2/2 + \beta mR^2v^2/2R^2$
$gh = (1+\beta)v^2/2$

The object with the smallest value of β has the largest velocity, v, since the left hand side of the equation is the same for all the objects.

Chapter 10 Problems:

10.27

a) Using the right hand rule, we find that the angular momentum $\vec{\mathbf{L}}$ is in the same direction as $\vec{\omega}$.

b) The gravitational force of the Earth on the disk produces a nonzero torque about the swivel. For the given orientation, the gravitational torque is directed perpendicular to and out of the page.

c) The torque of the gravitational force is in the same direction as the change in the angular momentum vector, so the precession is in a counterclockwise sense when viewed looking vertically downward from above the system.

d) The moment arm of the gravitational force is $b\sin\phi$, where ϕ is the angle between the symmetry axis and the direction of $\vec{\mathbf{g}}$. Hence, the magnitude of the torque of the gravitational force about the pivot is

$$\tau = mgb\sin\phi.$$

The magnitude of the torque also is equal to the magnitude of the time rate of change of the angular momentum,

$$\tau = \frac{dL}{dt}.$$

When viewed from above looking down, during a short time interval Δt, the component of the angular momentum in the horizontal plane ($L\sin\phi$) moves through a small angle $\Delta\theta$. Recalling the standard relation $\theta = s/r$, we can associate ΔL with s, $L\sin\phi$ with r and $\Delta\theta$ with θ, so

$$\Delta\theta = \frac{\Delta L}{L\sin\phi} \implies \frac{\Delta\theta}{\Delta t} = \frac{\Delta L}{\Delta t L\sin\phi}.$$

As Δt approaches zero, the left-hand side becomes the angular speed ω' of the precession, and on the right-hand side, the quantity $\Delta L/\Delta t$ becomes equal to the magnitude of the torque (which we evaluated above). Therefore,

$$\omega' = \frac{mgb\sin\phi}{L\sin\phi} = \frac{mgb}{L}.$$

But $L = I\omega$, so

$$\omega' = \frac{mgb}{I\omega}.$$

10.47

a) The moment of inertia of the tub is the sum of the moment of inertia of a cylindrical shell and that of a disk, both found in Table 10.1 on page 440 of the text,

$$I = I_{\text{cylindrical shell}} + I_{\text{disk}} = mR^2 + \frac{1}{2}m'R^2$$

$$\Longrightarrow\ I = (6.0\text{ kg})(0.25\text{ m})^2 + \frac{1}{2}(5.0\text{ kg})(0.25\text{ m})^2$$

$$\Longrightarrow\ I = 0.38\text{ kg}\cdot\text{m}^2 + 0.16\text{ kg}\cdot\text{m}^2$$

$$\Longrightarrow\ I = 0.54\text{ kg}\cdot\text{m}^2.$$

b) Convert the initial angular speed from rev / min to rad / s,

$$\omega_{\text{initial}} = \frac{(180\text{ rev / min})(2\pi\text{ rad / rev})}{60\text{ s / min}} = 18.8\text{ rad/s}.$$

The initial kinetic energy of the tub is

$$\text{KE}_{\text{initial}} = \frac{1}{2}I\omega_{\text{initial}}^2 = \frac{1}{2}(0.54\text{ kg}\cdot\text{m}^2)(18.8\text{ rad/s})^2 = 95\text{ J}.$$

c) Convert the final angular speed from rev / min to rad / s,

$$\omega_{\text{final}} = \frac{(60\text{ rev / min})(2\pi\text{ rad / rev})}{60\text{ s / min}} = 6.3\text{ rad/s}.$$

The moment of inertia of the sand in the tub is that of a cylinder

$$I_{\text{sand}} = \frac{1}{2}m_{\text{sand}}R^2.$$

Apply the law of conservation of angular momentum to the tub-and-sand system, using the magnitudes of the vectors:

$$L_{\text{final}} = L_{\text{initial}}$$

$$\Longrightarrow\ (I + I_{\text{sand}})\omega = I\omega_{\text{initial}}$$

$$\Longrightarrow\ 0.54\text{ kg}\cdot\text{m}^2 + \frac{1}{2}m_{\text{sand}}(0.25\text{ m})^2(6.3\text{ rad/s}) = (0.54\text{ kg}\cdot\text{m}^2)(18.8\text{ rad/s})$$

$$\Longrightarrow\ m_{\text{sand}} = 35\text{ kg}.$$

d) The moment of inertia of the tub-and-sand system when the sand is rotating with the tub is

$$I' = I_{\text{tub}} + I_{\text{sand}} = 0.54 \text{ kg}\cdot\text{m}^2 + \frac{1}{2}(35 \text{ kg})(0.25 \text{ m})^2 = 1.63 \text{ kg}\cdot\text{m}^2.$$

The kinetic energy of the tub-and-sand system is

$$\text{KE}_{\text{final}} = \frac{1}{2}I'\omega_{\text{final}}^2 = \frac{1}{2}(1.63 \text{ kg}\cdot\text{m}^2)(6.3 \text{ rad/s})^2 = 32 \text{ J}.$$

e) The fraction is

$$\frac{\text{KE}_{\text{final}}}{\text{KE}_{\text{initial}}} = \frac{32 \text{ J}}{95 \text{ J}} = 0.34.$$

10.50 Use the CWE theorem. The work done by you is $W_{\text{nonconservative}}$. There is no work done by the force of static friction on the tire since it is a zero-work force. Choose a coordinate system with origin at the bottom of the incline, with $\hat{\imath}$ in the horizontal and $\hat{\jmath}$ in the vertical direction. With these choices,

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE}) = \left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh\right) - (0 \text{ J} + 0 \text{ J})$$

The rolling constraint implies that the speed and angular speed of the tire are related by

$$v = R\omega.$$

Also, the moment of inertia of the disk-like tire is

$$I = \frac{1}{2}mR^2.$$

Substitute for ω and I in the CWE theorem,

$$W_{\text{nonconservative}} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + mgh = \frac{3}{4}mv^2 + mgh.$$

10.54 The angular momentum of the merry-go-round-kid system is conserved because there are no torques acting on it. Therefore, using the magnitudes of the vectors, we have

$$L_{\text{final}} = L_{\text{initial}}$$

$$\Longrightarrow\ I_{\text{final}}\omega_{\text{final}} = I_{\text{initial}}\omega_{\text{initial}}$$

$$\Longrightarrow\ (I_{\text{disk}} + I_{\text{kid}})\omega_{\text{final}} = I_{\text{disk}}\omega_{\text{initial}}$$

$$\Longrightarrow\ \omega_{\text{final}} = \frac{I_{\text{disk}}\omega_{\text{initial}}}{(I_{\text{disk}} + I_{\text{kid}})}.$$

The moment of inertia of the merry-go-round is that of a disk, while the moment of inertia of the child is that of a particle in orbital motion. The moment of inertia of the disk is

$$I_{\text{disk}} = \frac{1}{2}m_{\text{disk}}R^2 = \frac{1}{2}(150 \text{ kg})(2.00 \text{ m})^2 = 300 \text{ kg}\cdot\text{m}^2.$$

The moment of inertia of the kid is

$$I_{\text{kid}} = m_{\text{kid}}R^2 = (30 \text{ kg})(2.00 \text{ m})^2 = 1.2 \times 10^2 \text{ kg}\cdot\text{m}^2.$$

Use these moments of inertia in the equation above for ω_{final}:

$$\omega_{\text{final}} = \frac{(300\ \text{kg}\cdot\text{m}^2)(20\ \text{rev}/\text{min})}{(300\ \text{kg}\cdot\text{m}^2 + 1.2\times 10^2\ \text{kg}\cdot\text{m}^2)} \implies \omega_{\text{final}} = 14\ \text{rev}/\text{min}.$$

10.66 Use conservation of angular momentum for the magnitudes of the vectors:

$$L_{\text{final}} = L_{\text{initial}}$$

$$\implies (I_1 + I_2)\omega = I_1\omega_0$$

$$\implies \omega = \frac{I_1}{I_1 + I_2}\omega_0.$$

10.67

a) Use the parallel axis theorem to determine the moment of inertia of the hoop about the peg:

$$I = I_{\text{CM}} + md^2.$$

Here, $d = R$, and we can find I_{CM} from Table 10.1 on page 440 of the text:

$$I = mR^2 + mR^2 = 2mR^2.$$

b) Refer to Figure P.67 on page 483 of the text. The forces on the hoop are its weight $\vec{\text{w}}$, and the force of the peg on the hoop. The force of the peg on the hoop produces no torque about the peg since its line of action passes through the peg. Hence, the torque is caused only by the weight.

Choose $\hat{\text{k}}$ into the page. The moment arm of the weight is $R\sin\theta$. Hence, the magnitude of the torque resulting from the weight is

$$\tau = mg(\text{moment arm}) = mgR\sin\theta.$$

The direction of the torque is along $\hat{\text{k}}$, which we choose to point into the page. Notice in Figure P.67 that the torque always tends to decrease θ . Hence, the angular acceleration has a negative component of magnitude α. Therefore,

$$\vec{\tau} = I\vec{\alpha}$$

$$\implies mgR\sin\theta\hat{\text{k}} = (2mR^2)(-\alpha\hat{\text{k}})$$

$$\implies 2R\alpha + g\sin\theta = 0\ \text{m/s}^2$$

$$\implies 2R\frac{d^2\theta}{dt^2} + g\sin\theta = 0\ \text{m/s}^2$$

$$\implies \frac{d^2\theta}{dt^2} + \frac{g}{2R}\sin\theta = 0\ \text{rad/s}^2.$$

For small angles, $\sin\theta \approx \theta$, so

$$\frac{d^2\theta}{dt^2} + \frac{g}{2R}\theta = 0\ \text{rad/s}^2.$$

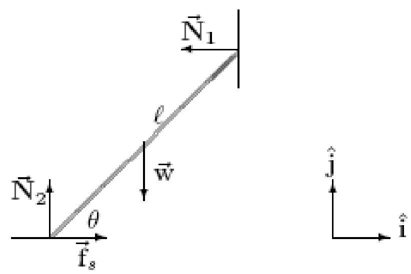
This differential equation is the equation for simple harmonic oscillation. The square of the angular frequency ω of the oscillation is the coefficient of θ in the differential equation. Hence,

$$\omega^2 = \frac{g}{2R} \implies \omega = \sqrt{\frac{g}{2R}}.$$

The frequency ν of the oscillation is

$$\nu = \frac{\omega}{2\pi} = \frac{1}{T} \implies T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{2R}{g}}.$$

10.73 The forces on the ladder are its weight $\vec{w}$, the normal force $\vec{N}_1$ of the vertical, frictionless wall on the ladder, the normal force $\vec{N}_2$ of the horizontal surface, and the force of static friction $\vec{f}_s$. The second law force diagram and an appropriate coordinate choice are shown below.



When the ladder is ready to slip, the force of static friction has its maximum magnitude

(1) $$f_{s\ \max} = \mu_s N_2.$$

Since the ladder is equilibrium, the sum of the forces along each coordinate axes must be zero. Therefore,

$$F_{x\ \text{total}} = 0\ \text{N} \implies f_{s\ \max} - N_1 = 0\ \text{N}$$

so

(2) $$f_{s\ \max} = N_1.$$

Also

$$F_{y\ \text{total}} = 0\ \text{N} \implies N_2 - mg = 0\ \text{N}$$

so

(3) $$N_2 = mg.$$

Since the ladder is in equilibrium, the sum of the torques taken about any point must be zero. Take torques about the base of the ladder. The forces $\vec{N}_2$ and $\vec{f}_{s\ \max}$ each produce zero torque about this point, since their lines of action pass through the point. Set the sum of the remaining torques to zero, so

$$0\ \text{N·m} = (mg\frac{\ell}{2}\cos\theta)(-\hat{k}) + (N_1\ell\sin\theta)\hat{k} \implies N_1 = \frac{mg\cos\theta}{2\sin\theta} = \frac{mg}{2}\cot\theta.$$

Use equation (2) for $f_{s\ \max}$

$$f_{s\ \max} = \frac{mg}{2}\cot\theta,$$

and then use equations (1) and (3) to find θ:

$$\mu_s mg = \frac{mg}{2}\cot\theta \implies \cot\theta = 2\mu_s \implies \tan\theta = \frac{1}{2\mu_s} = \frac{1}{2\times 0.30} = 1.7 \implies \theta = 59^\circ.$$

So, at any angle less than 59°, the ladder will slip.

10.77 Choose a coordinate system with origin at the edge of the roof, $\hat{\mathbf{i}}$ pointing horizontally to the right, and $\hat{\mathbf{j}}$ pointing up. Let x be the maximum distance you can walk from the edge of the building without tipping the plank. When the plank is ready to rotate about the origin (the edge of the roof), the forces on the system are:

1. the weight $\vec{\mathbf{w}}_1$ of the plank, directed downward. This may be thought of as being applied at the center of mass of the plank, at the point $(-0.50\text{ m})\hat{\mathbf{i}}$;
2. your weight $\vec{\mathbf{w}}_2$, directed downward. This is applied at the point $x\hat{\mathbf{i}}$;
3. the force $\vec{\mathbf{F}}$ directed upward of the edge of the building on the plank. This is applied at the origin.

Take torques about the origin. Then force $\vec{\mathbf{F}}$ produces zero torque, since its line of action passes through the origin. Since the system is in equilibrium, the total torque on the system is zero, so

$$\begin{aligned}\vec{\tau}_{\text{total}} = 0\text{ N·m} &\implies (100\text{ kg})(9.81\text{ m/s}^2)(0.50\text{ m}))\hat{\mathbf{k}} + (70.0\text{ kg})(9.81\text{ m/s}^2)x(-\hat{\mathbf{k}}) = \mathbf{0}\text{ N·m} \\ &\implies (100\text{ kg})(9.81\text{ m/s}^2)(0.50\text{ m}) - (70.0\text{ kg})(9.81\text{ m/s}^2)x = 0\text{ N·m} \implies x = 0.71\text{ m}.\end{aligned}$$

Chapter 12 Questions:

12.14 On a local scale, the speed of light can be considered to be essentially infinite compared with the speed of sound, v. The sound of thunder travels a distance d in a time t, d = vt.
If d is in kilometers, the d = (0.34 km/s) t
Hence the distance in kilometers is about one third the time in seconds between seeing the lightning and hearing the thunder. The distance in miles is about 1/5 the time in seconds. So a difference between the lightning and thunder of 5 seconds meant the lighting struck about one mile away.

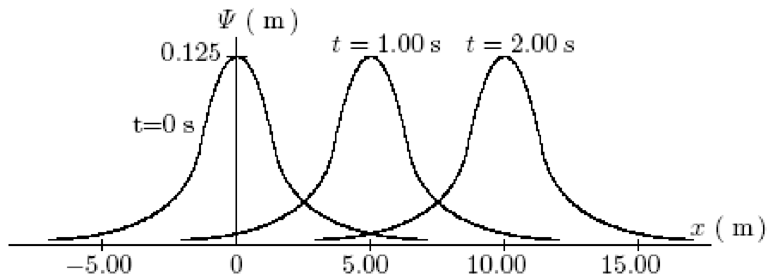
12.15 The acceleration is greatest at the maximum amplitudes ±A. The acceleration is zero when the transverse position is zero (the equilibrium position). This is also the position of greatest speed. The speed is zero at the maximum amplitude, ±A.

Chapter 12 Problems:

12.2

a) Mathematical functions representing traveling waves only contain the variables x and t in the combination $(x - vt)$ or $(x + vt)$, where v is the speed of the wave. So, in the expression given, the coefficient of t is $v = 5.00\ \mathrm{m/s}$.

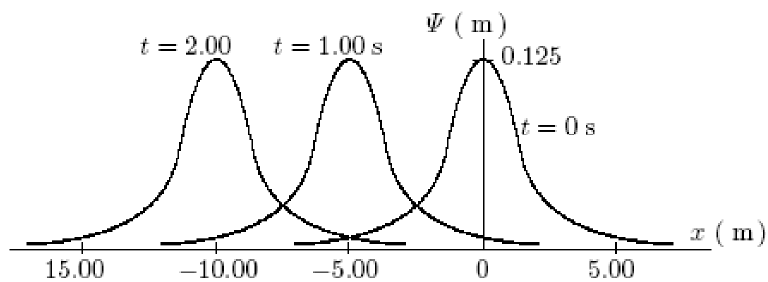
b) Here are plots of the waveform at the specified times.



c) The wave disturbance $\Psi(x,t)$ is a maximum at the values of x and t which make the denominator a minimum. This happens whenever $x - (5.00\ \mathrm{m/s})t = 0\ \mathrm{m}$. We are *not* asked to find the values of x and t for which this occurs; rather, we only are asked to find the value of $\Psi(x,t)$ *when* it occurs. Thus, the maximum value of the wave disturbance is

$$\Psi_{\max} = \frac{0.250\ \mathrm{m}^3}{2.00\ \mathrm{m}^2 + 0\ \mathrm{m}^2} = 0.125\ \mathrm{m}\,.$$

d) Here are the plots of the waveform at the specified times.



12.16

a) The amplitude A of the wave is the coefficient of the cosine term, so $A = 0.300\ \mathrm{m}$.

b) The angular wavenumber k is the coefficient of x in the argument of the cosine, so $k = 12.57\ \mathrm{rad/m}$.

c) The wavelength λ is found from

$$k = \frac{2\pi}{\lambda} \implies \lambda = \frac{2\pi}{k} = \frac{2\pi}{12.57\ \mathrm{rad/m}} = 0.5000\ \mathrm{m}\,.$$

d) The angular frequency ν of the wave is the coefficient of t in the argument of the cosine, $\omega = 251.3\ \mathrm{rad/s}$.

e) The frequency ν and angular frequency ω are related by

$$\omega = 2\pi\nu \implies \nu = \frac{\omega}{2\pi} = \frac{251.3\ \mathrm{rad/s}}{2\pi} = 40.00\ \mathrm{Hz}\,.$$

f) The period T of the wave is

$$T = \frac{1}{\nu} = \frac{1}{40.00\ \mathrm{Hz}} = 2.500 \times 10^{-2}\ \mathrm{s}\,.$$

g) The speed v of the wave is

$$v = \nu\lambda = (40.00\ \mathrm{Hz})(0.5000\ \mathrm{m}) = 20.00\ \mathrm{m/s}\,.$$

h) To find the wave disturbance Ψ at $x = 3.00$ m when $t = 1.50 \times 10^{-3}$ s, substitute these numerical values into the wavefunction:

$$\begin{aligned}\Psi &= (0.300\ \mathrm{m})\cos[(12.57\ \mathrm{rad/m})(3.00\ \mathrm{m}) - (251.3\ \mathrm{rad/s})(1.50 \times 10^{-3}\ \mathrm{s})]\\ &= (0.300\ \mathrm{m})\cos[37.7\ \mathrm{rad} - 0.377\ \mathrm{rad}]\\ &= (0.300\ \mathrm{m})\cos(37.3\ \mathrm{rad}) = (3)(0.931)\\ &\Longrightarrow\ \Psi = 0.276\ \mathrm{m}\,.\end{aligned}$$