Assignment #10

Chapter 10 Questions:

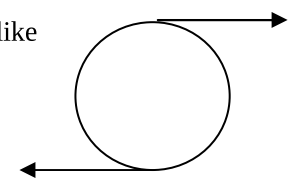
10.2 Since **L** is defined as the vector product of **r** and **p**, **L** will be perpendicular to both **r** and **p**.

10.3 If ω = 0 rad/s, this does not imply that the torque is zero because the angular acceleration may not be zero even if the angular velocity is zero. If the angular acceleration α is zero, then the torque must also be zero.

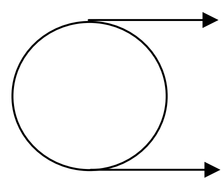
10.6 The system with the greatest (least) kinetic energy is that with the greatest (least) moment of inertia. Thus the greatest kinetic energy is that of the cylindrical hoop while the least is the sphere.

10.9 a) With $I_{CM} = \beta m R^2$, the ratio of $K_{Trans}/K_{Rot} = 1/\beta$.
b) The ratio is independent of speed.
c) The greatest ratio will occur for the object with the smallest β, which is a sphere. The ratio is equal to 1 for a thin cylindrical shell or hoop since β is equal to 1.

10.18 a) Two forces so that $F = 0$ but $\tau \neq 0$ would be like



b) Tow forces so that $F \neq 0$ but $\tau = 0$ would be like



10.19 If the system is rotating with constant angular velocity, then the angular acceleration is zero and so is the total torque on it.

Chapter 10 Problems:

10.2

a) The position vector of the bird when it is at point (a) is

$$\vec{r} = -(10.0\text{ m})\cos 20^\circ \hat{i} + (10.0\text{ m})\sin 20^\circ \hat{j} = (-9.4\text{ m})\hat{i} + (3.4\text{ m})\hat{j}.$$

The momentum of the bird at point (a) is

$$\vec{p} = m\vec{v} = (5.00\text{ kg})(6.0\text{ m/s})\hat{i} = (30\text{ kg}\cdot\text{m/s})\hat{i}.$$

The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = (-9.4\text{ m}\,\hat{i} + 3.4\text{ m}\,\hat{j}) \times (30\text{ kg}\cdot\text{m/s})\hat{i} = -(1.0 \times 10^2\text{ kg}\cdot\text{m}^2/\text{s})\hat{k}.$$

b) The position vector of the bird when it is at point (b) is

$$\vec{r} = (4.8 \text{ m}) \cos 45^\circ \hat{i} + (4.8 \text{ m}) \sin 45^\circ \hat{j} = 3.4 \text{ m}\,\hat{i} + 3.4 \text{ m}\,\hat{j}.$$

The momentum of the bird at point (b) is the same as at point (a):

$$\vec{p} = m\vec{v} = (5.00 \text{ kg})(6.0 \text{ m/s})\hat{i} = (30 \text{ kg}\cdot\text{m/s})\hat{i}.$$

The angular momentum is

$$\vec{L} = \vec{r} \times \vec{p} = (3.4 \text{ m}\,\hat{i} + 3.4 \text{ m}\,\hat{j}) \times (30 \text{ kg}\cdot\text{m/s})\hat{i} = -(1.0 \times 10^2 \text{ kg}\cdot\text{m}^2/\text{s})\hat{k}.$$

10.12 The position vector of the center of mass is found from the equation

$$\begin{aligned}
\vec{r}_{CM} &= \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3}{m_1 + m_2 + m_3} \\
&= \frac{(1.00 \text{ kg})(0 \text{ m})\hat{i} + (2.00 \text{ kg})(4.00 \text{ m})\hat{i} + (3.00 \text{ kg})(6.00 \text{ m})\hat{i}}{1.00 \text{ kg} + 2.00 \text{ kg} + 3.00 \text{ kg}} \\
&= (4.33 \text{ m})\hat{i}.
\end{aligned}$$

The moment of inertia of the system about an axis through the center of mass and parallel to the y-axis is found from the sum of the moments of inertia of each particle about this axis. The 1.00 kg mass is 4.33 m from this axis, the 2.00 kg mass is 0.33 m from the axis, and the 3.00 kg mass is 1.67 m from this axis. Hence, the moment of inertia is

$$\begin{aligned}
\sum_{i=1}^{3} m_i r_{i\perp}^2 &= (1.00 \text{ kg})(4.33 \text{ m})^2 + (2.00 \text{ kg})(0.33 \text{ m})^2 + (3.00 \text{ kg})(1.67 \text{ m})^2 \\
&= 18.7 \text{ kg}\cdot\text{m}^2 + 0.22 \text{ kg}\cdot\text{m}^2 + 8.37 \text{ kg}\cdot\text{m}^2 \\
&= 27.3 \text{ kg}\cdot\text{m}^2.
\end{aligned}$$

The system is not symmetric about this axis.

10.13 The total moment of inertia is the sum of the moments of inertia of the various parts about the same axis, so

$$I = I_{\text{disk}} + I_{\text{rim}} + I_{\text{point particles}} = \frac{mR^2}{2} + mR^2 + 4\left(\frac{m}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{7}{4}mR^2.$$

10.18

a) The total torque is the vector sum of the torque of the individual forces about the center of mass of the text. To find these torques, we first need to find the perpendicular distance of the line of action of each force from the center of mass.

To do this, first note that the angle θ which the diagonal makes with the horizontal can be found from

$$\tan\theta = \frac{(30.0\text{ cm}/2)}{(20.0\text{ cm}/2)} = \frac{15.0\text{ cm}}{10.0\text{ cm}} = 1.5 \implies \theta = 56.3^\circ,$$

which is exactly the angle the line of action of the upper force of 5.0 N makes with the horizontal. Hence, the line of action of the upper force passes through the center of mass of the text, which means that the moment arm of this force is zero and that the force produces zero torque about the center of mass.

The moment arm r of the force on the lower right hand corner can be found from the Pythagorean theorem. Since the force is perpendicular to the diagonal,

$$r^2 = \left(\frac{20\text{ cm}}{2}\right)^2 + \left(\frac{30\text{ cm}}{2}\right)^2 \implies r = 18.0\text{ cm}.$$

The magnitude of the torque of the second force is thus

$$\tau = F(\text{moment arm}) = (5.0\text{ N})(0.180\text{ m}) = 0.90\text{ N·m},$$

and this is the magnitude of the total torque on the text.

b) The moment of inertia of a rectangular plate of dimensions a and b about an axis perpendicular to the plate and passing through the center of mass is found in Table 10.1 on page 440 of the text to be

$$I = \frac{1}{12}m(a^2+b^2) = \frac{1}{12}(2.50\text{ kg})[(0.200\text{ m})^2 + (0.300\text{ m})^2] = 2.71\times 10^{-2}\text{ kg·m}^2.$$

The magnitude of the total torque τ and the magnitude α of the angular acceleration are related by the moment of inertia I:

$$\tau = I\alpha \implies \alpha = \frac{\tau}{I} = \frac{0.90\text{ N·m}}{2.71\times 10^{-2}\text{ kg·m}^2} = 33\text{ rad/s}^2.$$

10.22

a) The apparent gravitational acceleration is provided by the centripetal acceleration, so

$$a_{\text{centripetal}} = r\omega^2 \implies 9.81\text{ m/s}^2 = (1.50\times 10^3\text{ m})\omega^2 = 8.09\times 10^{-2}\text{ rad/s}$$

Convert the angular speed ω from rad/s to rev/min:

$$\omega = \frac{(8.09\times 10^{-2}\text{ rad/s})(60\text{ s/min})}{2\pi\text{ rad / rev}} = 0.773\text{ rev/min}.$$

b) The space station is a cylindrical shell spinning about the axis of the cylinder. The moment of inertia of such a system is $I = mR^2$. The rotational kinetic energy is

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(mR^2)\omega^2$$

$$\implies \text{KE}_{\text{rot}} = \frac{1}{2}[(1.20\times 10^{13}\text{ kg})(1.50\times 10^3\text{ m})^2](8.09\times 10^{-2}\text{ rad/s})^2$$

$$\implies \text{KE}_{\text{rot}} = 8.84\times 10^{16}\text{ J}.$$

c) First determine the magnitude of the angular acceleration of the space station during the year it begins spinning. To do this, use the equation appropriate for a constant angular acceleration,

$$\omega_z(t) = \omega_{z0} + \alpha_z t.$$

After one year (3.156×10^7 s), the angular velocity component is 8.09×10^{-2} rad/s. Since the space station began spinning from rest, we have

$$8.09 \times 10^{-2} \text{ rad/s} = 0 \text{ rad/s} + \alpha_z(3.16 \times 10^7 \text{ s}) \implies \alpha_z = 2.56 \times 10^{-9} \text{ rad/s}^2$$

The moment of inertia of the cylindrical shell of a space station is (from Table 10.1 on page 440 of the text)

$$I = mR^2 = (1.20 \times 10^{13} \text{ kg})(1.50 \times 10^3 \text{ m})^2 = 2.70 \times 10^{19} \text{ kg}\cdot\text{m}^2.$$

The magnitude of the torque needed to provide the angular acceleration is

$$\tau = I|\alpha_z| = (2.70 \times 10^{19} \text{ kg}\cdot\text{m}^2)(2.56 \times 10^{-9} \text{ rad/s}^2) = 6.91 \times 10^{10} \text{ N}\cdot\text{m}.$$

The torque is provided by the total force acting tangential to a circular cross section, so the magnitude of the torque is

$$\tau = F(\text{moment arm}),$$

where the moment arm is the radius of the space station. Hence

$$6.91 \times 10^{10} \text{ N}\cdot\text{m} = F(1.50 \times 10^3 \text{ m}) \implies F = 4.61 \times 10^7 \text{ N}.$$

Each thrusting rocket provides a force of magnitude 1000 N, so the number of rockets needed is

$$\text{number of rockets} = \frac{4.61 \times 10^7 \text{ N}}{1000 \text{ N}} = 4.61 \times 10^4.$$

which is over forty thousand rockets!

10.26

a) The angular momentum is defined as the vector product of the position vector with the momentum,

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}},$$

so in order to find $\vec{\mathbf{L}}$ we need to find $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$. For motion in a circle of radius r', the velocity and angular velocity are related by

$$\vec{\mathbf{v}} = \vec{\omega} \times \vec{\mathbf{r}}\,'$$

$$= (\omega\hat{\mathbf{k}}) \times (r\sin\theta\hat{\mathbf{i}})$$

$$= \omega r \sin\theta\hat{\mathbf{j}}.$$

Now use these results to evaluate $\vec{\mathbf{L}}$:

$$\vec{\mathbf{L}} = \vec{\mathrm{r}} \times \vec{\mathrm{p}} = (r \sin\theta \hat{\mathrm{i}} + r\cos\theta \hat{\mathrm{k}}) \times (m\omega r \sin\theta \hat{\mathrm{j}})$$

$$= mr^2\omega \sin^2\theta \hat{\mathrm{k}} - mr^2\omega \sin\theta\cos\theta \hat{\mathrm{i}}$$

$$= -mr^2\omega \sin\theta\cos\theta \hat{\mathrm{i}} + mr^2\omega \sin^2\theta \hat{\mathrm{k}}.$$

b) The angular momentum $\vec{\mathbf{L}}$ is not in the same direction as the angular velocity $\vec{\omega}$ because the rotational axis is not coincident with the symmetry axis of the system. The angular momentum is not parallel to the angular velocity vector at any time during the motion.

10.32

a) Table 10.1 on page 440 of the text gives the moment of inertia of such a thin rod about an axis perpendicular to the rod and passing through its center of mass,

$$I_{\mathrm{CM}} = \frac{1}{12} m\ell^2$$

Use the parallel axis theorem (Equation 10.29 on page 452) to determine the moment of inertia about a parallel axis through one end of the rod, a distance d away,

$$I = I_{\mathrm{CM}} + md^2.$$

Here $d = \dfrac{\ell}{2}$, so

$$I = \frac{1}{12} m\ell^2 + \frac{1}{4} m\ell^2 = \frac{1}{3} m\ell^2$$

b) The magnitude of the initial torque due to the weight is

$$\tau = F(\text{moment arm}) = \frac{mg\ell}{2}$$

Introduce a coordinate system with $\hat{\mathrm{i}}$ in the horizontal and $\hat{\mathrm{j}}$ in the vertical direction, and with origin on the pivot point. In this system,

$$\vec{\tau} = -mg\frac{\ell}{2}\hat{\mathrm{k}}.$$

c) The force of the pivot on the rod is the other force acting on the system. This force produces no torque about the axis, since its line of action passes through the axis and thus has a moment arm equal to zero.

d) The initial angular acceleration of the rod is found from

$$\vec{\tau} = I\vec{\alpha} \implies \vec{\alpha} = \frac{\vec{\tau}}{I}$$

$$\implies \vec{\alpha} = \frac{-\dfrac{mg\ell\hat{\mathrm{k}}}{2}}{m\ell^2/3}$$

$$\implies \vec{\alpha} = -\frac{3g}{2\ell}\hat{\mathrm{k}}.$$

e) When the rod is vertical, the line of action of the gravitational force passes through the axis. Thus, the moment arm of this force is zero, and the torque resulting from it is zero. Since the torque is zero, the angular acceleration also is zero (because $\vec{\tau} = I\vec{\alpha}$).

f) There are no nonconservative forces, so their work is zero. Using the coordinate system in part b), the CWE theorem becomes

$$W_{\text{nonconservative}} = \Delta(\text{KE} + \text{PE})$$

$$\Longrightarrow\ 0\ \text{J} = \Delta(\text{KE} + \text{PE})_{\text{final}} - \Delta(\text{KE} + \text{PE})_{\text{initial}}$$

$$\Longrightarrow\ 0\ \text{J} = [\frac{1}{2}I\omega^2 + (-mg\frac{\ell}{2})] - [0\ \text{J} + 0\ \text{J}]$$

$$\Longrightarrow\ \omega = \sqrt{\frac{mg\ell}{I}} = \sqrt{\frac{mg\ell}{m\ell^2/3}}$$

$$\Longrightarrow\ \omega = \sqrt{\frac{3g}{\ell}}.$$

Determine the direction of the angular velocity vector using the circular motion right-hand rule:

$$\vec{\omega} = -\sqrt{\frac{3g}{\ell}}\hat{\text{k}}.$$

g) The angular momentum of the rod is

$$\vec{\mathbf{L}} = I\vec{\omega} = \left(\frac{m\ell^2}{3}\right)\left(-\sqrt{\frac{3g}{\ell}}\hat{\text{k}}\right)$$

$$\Longrightarrow\ \vec{\mathbf{L}} = m\sqrt{\frac{g\ell^3}{3}}\hat{\text{k}}.$$

10.52

a) The forces on the politician are

1. The weight $\vec{\mathrm{w}}$, of magnitude mg, located at the center of mass and pointing down; and
2. The force $\vec{\mathrm{T}}$ of the cord, located at a point P on the rim, pointing up.

b) Calculate the torque about the point P at which the cord leaves the cylinder. The torque from $\vec{\mathrm{T}}$ is zero, since its line of action passes through the point about which the torque is calculated. The magnitude of the torque resulting from the weight is

$$\tau = mg(\text{moment arm}) = mgR.$$

The moment of inertia of the cylinder about point P is found from the parallel axis theorem,

$$I = I_{\mathrm{CM}} + md^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2.$$

Since the cord rolls off the cylinder, we can use the torque equation

$$\tau = I\alpha \Longleftrightarrow \alpha = \frac{\tau}{I}$$

in the rolling constraint

$$a = \alpha R$$

to find the magnitude a of the acceleration:

$$a = \frac{\tau}{I}R = \frac{(mgR)R}{\left(\dfrac{3}{2}mR^2\right)} = \frac{2g}{3} = 6.54\ \mathrm{m/s^2}\,.$$

c) The magnitude α of the angular acceleration is

$$\alpha = \frac{a}{R} = \frac{6.54\ \mathrm{m/s^2}}{0.200\ \mathrm{m}} = 32.7\ \mathrm{rad/s^2}\,.$$

d) Use the kinematic equation for motion with a constant angular acceleration. Take $\hat{\mathrm{k}}$ in the direction determined by the circular motion right-hand rule. With this choice,

$$\omega_z = \omega_{z0} + \alpha_z t = 0\ \mathrm{rad/s} + (32.7\ \mathrm{rad/s^2})(1.50\ \mathrm{s}) = 49.1\ \mathrm{rad/s}\,.$$

e) The rolling constraint implies that the speed of the center of mass is

$$v = \omega R \implies v = (49.1\ \mathrm{rad/s})(0.200\ \mathrm{m}) = 9.82\ \mathrm{m/s}\,.$$