Assignment #1

Chapter 1 Problems:

1.18 Let r be the radius of the car tire, so $3r$ is the radius of the truck tire. The circumference of the car tire is $2\pi r$, while that of the truck tire is $2\pi(3r) = 3(2\pi r)$. Therefore, the truck tire goes farther by a factor of three.

1.47

a) Let ℓ be the length of a side of a square field with area one hectare. Then $\ell^2 = 10\,000\ \mathrm{m}^2$, so $\ell = 100.00\ \mathrm{m}$.

b) Let ℓ be the length of a side of a one acre square field. Then $\ell^2 = 43\,560\ \mathrm{ft}^2$, so $\ell = 208.71\ \mathrm{ft}$.

c) To find the number of acres per hectare, we first convert from square meters to square feet. One meter is 3.28 ft, so one square meter is $10.8\ \mathrm{ft}^2$. Therefore,

$$1.00 \text{ hectare} = 1.00 \times 10^4\ \mathrm{m}^2 \left(10.8 \frac{\mathrm{ft}^2}{\mathrm{m}^2}\right)\left(\frac{1.00 \text{ acre}}{43\,560\ \mathrm{ft}^2}\right) = 2.48 \text{ acre}.$$

d) One square kilometer is

$$(10^3\ \mathrm{m})^2 = 10^6\ \mathrm{m}^2 \left(\frac{1.00 \text{ hectare}}{10^4\ \mathrm{m}^2}\right) = 100 \text{ hectare}.$$

1.66 Assume about 1 container per person per day, or about $4 \times 10^2 \dfrac{\text{container}}{\text{person} \cdot \text{y}}$. The population of the United States is about 2.5×10^8 people. Hence the total number of containers per year is about

$$2.5 \times 10^8 \text{ person} \left(4 \times 10^2 \frac{\text{container}}{\text{person} \cdot \text{y}}\right) \approx 1 \times 10^{11} \frac{\text{container}}{\text{y}}.$$

1.76

a) a) For addition, you can only add numbers with the same precision, i.e., with the same number of decimal places

	5.6 m	
	0.22 m	the last "2" should be dropped.
	16.1 m	
+	0.04 m	the last "4" should be dropped.

Hence the addition should be performed as follows:

$$5.6\ \mathrm{m} + .02\ \mathrm{m} + 16.1\ \mathrm{m} + 0.0\ \mathrm{m} = 21.9\ \mathrm{m}.$$

b) The number of significant figures in a product the smallest number of significant figures among the individual factors. Thus, in the product $[3.46\ \mathrm{m}][5.891\ \mathrm{m}]$, we should show only three significant figures:

$$[3.46\ \mathrm{m}][5.891\ \mathrm{m}] = 20.4\ \mathrm{m}^2.$$

A similar rule holds for quotients.

c) Display π on your calculator, and round it to six significant figures: 3.14159.

d) You can keep only three significant figures in the product: $7.83\pi = 24.6$.

e) In the product $\sqrt{2} \times \pi$, each factor has an unlimited number of significant figures, so the product also has an unlimited number of significant figures.

f) Since 8.42 has only three significant figures, you can only retain three significant figures in π when adding it to 8.42. Hence $8.42 + \pi = 8.42 + 3.14 = 11.56$.

Chapter 2 Questions:

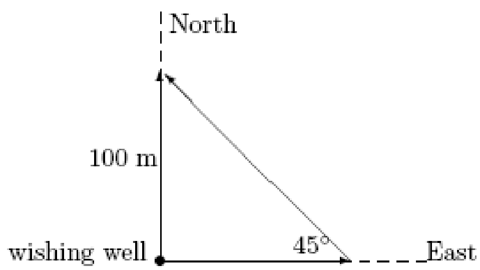
2.2 Vectors of magnitude 1 that form an equilateral triangle do the job.

2.17 The vector sum can only be the same as the scalar sum of their magnitudes when the vectors all point along the same direction. In general, they can not be the same

2.24 The magnitude of the vector is always greater than or equal to the absolute value of its smallest Cartesian coordinate. Since the coordinates are at right angle to each other , the magnitude $A = \{A_x^2 + A_y^2 + A_z^2\}^{1/2}$ so that $A > |A_x|$.

Chapter 2 Problems:

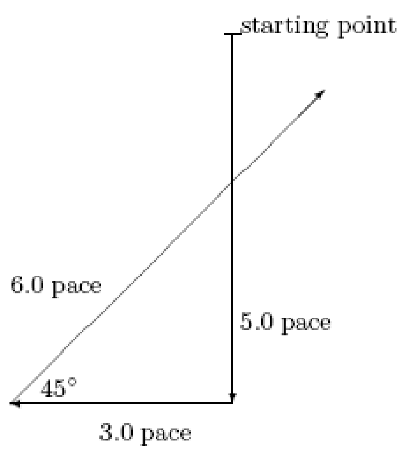
2.5 The stroll is sketched below.



Since north and east are perpendicular to each other, the two strolls and their vector sum form a 45° right triangle. Such a triangle has two equal sides. Hence, the vector to the east has magnitude 100 m . The vector to the northwest is the hypotenuse of the right triangle, and so has magnitude $\sqrt{(100\text{ m})^2 + (100\text{ m})^2} = 141\text{ m}$.

2.6

a) Here is a scale diagram of the walk.



b) From the geometry, the distance south of the starting point is

$$5.0 \text{ pace} - (6.0 \text{ pace}) \sin 45^\circ = 0.8 \text{ pace}.$$

c) From the geometry, the distance to the east of the starting point is

$$(6.0 \text{ pace}) \cos 45^\circ - 3.0 \text{ pace} = 1.2 \text{ pace}.$$

d) Using the results in parts b) and c) together with the Pythagorean theorem, The distance from the starting point is

$$\sqrt{(0.8 \text{ pace})^2 + (1.2 \text{ pace})^2} = 1.4 \text{ pace}.$$

e) Since vectors can be added in any order, the final position does not depend upon the sequence of steps taken; hence, the answers to parts (b), (c), and (d), are unaffected by the order of the sequence of steps.

f) From parts b) and c) you are 0.8 pace south and 1.2 pace east of the starting place. Let θ be the angle south of east from your starting point. Then

$$\tan\theta = \frac{0.8 \text{ pace}}{1.2 \text{ pace}} = 0.7 \implies \theta \approx 30^\circ.$$

2.10 Each choice can be tested with the right hand rule. Point the index finger of your right hand along the x-axis and the right-hand middle finger along the y-axis. If your extended right-hand thumb lies along the z-axis, the coordinate system is right-handed (RH); otherwise, the coordinate system is left-handed (LH). The results are

a) RH

b) LH

c) RH

d) LH

e) RH

f) LH

2.15

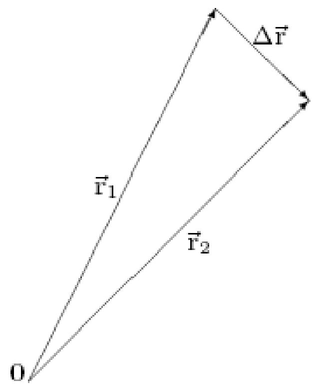
a) Since the z-axis is vertical, the difference in the heights of the two birds above the ground is the absolute value of the difference in the z-components of their respective position vectors.

$$|5.0 \text{ m} - 3.0 \text{ m}| = 2.0 \text{ m}.$$

b) Since the position vectors originate at the origin, the distance of either bird from the origin is the magnitude of the respective position vectors. For the turkey, this is

$$r_1 = \sqrt{x^2 + y^2 + z^2} = \sqrt{(3.0 \text{ m})^2 + (4.0 \text{ m})^2 + (5.0 \text{ m})^2} = 7.1 \text{ m}.$$

c) A sketch of the situation appears below.



Note that $\vec{r}_1 + \Delta\vec{r} = \vec{r}_2$, which means that the distance between the birds is the magnitude of the vector

$$\begin{aligned}\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 &= (-4.0\text{ m} - 3.0\text{ m})\hat{i} + (-2.0\text{ m} - 4.0\text{ m})\hat{j} + (3.0\text{ m} - 5.0\text{ m})\hat{k} \\ &= (-7.0\text{ m})\hat{i} - (6.0\text{ m})\hat{j} - (2.0\text{ m})\hat{k}.\end{aligned}$$

The magnitude of $\Delta\vec{r}$ is

$$|\Delta\vec{r}| = \sqrt{(-7.0\text{ m})^2 + (-6.0\text{ m})^2 + (-2.0\text{ m})^2} = 9.4\text{ m}.$$

2.24 Use the definition of the scalar product to obtain an expression for $\cos\theta$ where θ is the angle between the vectors with their tails at a common point.

$$\vec{A} \bullet \vec{B} = AB\cos\theta \implies \cos\theta = \frac{\vec{A} \bullet \vec{B}}{AB}.$$

Evaluate the numerator of this expression using the Cartesian form for the vectors.

$$\vec{A} \bullet \vec{B} = A_xB_x + A_yB_y + A_zB_z = (1.0)(-1.0) + (2.0)(3.0) + (-1.0)(5.0) = 0.0$$

Since the scalar product is zero, and neither vector is the zero vector, we must have $\cos\theta = 0 \implies \theta = 90°$ which means that the two vectors are perpendicular. The angle between them is 90°.

2.48 To construct a vector perpendicular to the two given vectors, form their vector product.

$$(2.0\hat{i} + 1.5\hat{j}) \times (2.0\hat{j} + 3.2\hat{k}) = (2.0)(2.0)\hat{k} + (2.0)(3.2)(-\hat{j}) + 0 + (1.5)(3.2)\hat{i} = 4.8\hat{i} - 6.4\hat{j} + 4.0\hat{k}.$$

This vector is not one unit long. In fact, its magnitude is

$$\sqrt{4.8^2 + (-6.4)^2 + (4.0)^2} = 8.9.$$

To form a unit vector in the same direction as $4.8\hat{i} - 6.4\hat{j} + 4.0\hat{k}$ divide the vector by its magnitude.

$$\frac{4.8\hat{i} - 6.4\hat{j} + 4.0\hat{k}}{8.9} = 0.54\hat{i} - 0.72\hat{j} + 0.45\hat{k}.$$

There is one (and only one) other solution, the negative of the vector above:

$$-0.54\hat{i} + 0.72\hat{j} - 0.45\hat{k}.$$

2.55

a)

$$\vec{p}_1 + \vec{p}_2 = \left(2.1\hat{i} - 5.3\hat{j} + 3.4\hat{k}\right) + \left(3.6\hat{i} + 2.8\hat{j} - 0.9\hat{k}\right) = 5.7\hat{i} - 2.5\hat{j} + 2.5\hat{k}$$

b)

$$\vec{p}_1 - \vec{p}_2 = \left(2.1\hat{i} - 5.3\hat{j} + 3.4\hat{k}\right) - \left(3.6\hat{i} + 2.8\hat{j} - 0.9\hat{k}\right) = -1.5\hat{i} - 8.1\hat{j} + 4.3\hat{k}.$$

c) The magnitude of $\vec{p}_1$ is

$$p_1 = \sqrt{(2.1)^2 + (-5.3)^2 + (3.4)^2} = 6.6.$$

d) The magnitude of $\vec{p}_2$ is

$$p_2 = \sqrt{(3.6)^2 + (2.8)^2 + (-0.9)^2} = 4.7.$$

e) The scalar product of $\vec{p}_1$ and $\vec{p}_2$ is

$$\vec{p}_1 \bullet \vec{p}_2 = \left(2.1\hat{i} - 5.3\hat{j} + 3.4\hat{k}\right) \bullet \left(3.6\hat{i} + 2.8\hat{j} - 0.9\hat{k}\right) = (2.1)(3.6) + (-5.3)(2.8) + (3.4)(-0.9) = -10.3.$$

f) The vector product of $\vec{p}_1$ with $\vec{p}_2$is

$$\begin{aligned}\vec{p}_1 \times \vec{p}_2 &= \left(2.1\hat{i} - 5.3\hat{j} + 3.4\hat{k}\right) \times \left(3.6\hat{i} + 2.8\hat{j} - 0.9\hat{k}\right)\\ &= 0 + (2.1)(2.8)\hat{k} + (2.1)(-0.9)(-\hat{j})\\ &\quad + (-5.3)(3.6)(-\hat{k}) + 0 + (-5.3)(-0.9)\hat{i}\\ &\quad + (3.4)(3.6)\hat{j} + (3.4)(2.8)(-\hat{i}) + 0\\ &= -4.8\hat{i} + 14.1\hat{j} + 25.0\hat{k}.\end{aligned}$$

g) To find the angle θ between $\vec{p}_1$ and $\vec{p}_2$, use the definition of the scalar product.

$$\vec{p}_1 \bullet \vec{p}_2 = p_1 p_2 \cos\theta.$$

Evaluate the left-hand side with the Cartesian form of each vector [see part e) above] and put in the magnitudes of the two vectors on the right-hand side.

$$-10.3 = (6.6)(4.7)\cos\theta \implies \cos\theta = -0.34 \implies \theta = (1.1 \times 10^2)^\circ.$$

2.63 Since we consider the magnitude and direction of each of the unit vectors to be fixed, they can be treated as constants when differentiating. Hence to differentiate a vector, differentiate each of its components.

$$\frac{d\vec{A}}{dt} = \frac{d}{dt}(3t^2)\hat{i} + \frac{d}{dt}(2t)\hat{j} + \frac{d}{dt}(8)\hat{k} = 6t\hat{i} + 2\hat{j} + 0\hat{k} = 6t\hat{i} + 2\hat{j}$$