Group Problem

Your friend who is studying film has decided that he going to modify the opening scene of the movie *Raiders of the Lost Ark*. In his version of the opening scene, a 100 kg spherical boulder with a radius of 1.0 m will start from rest at a height of 5.0 meters above Indiana Jones. The boulder will roll down a ramp onto level ground where Indiana is standing. Indiana knows that he can not outrun the boulder unless he can slow it down. He pulls out his pistol and fires a bullet toward the boulder while the boulder is rolling along the level ground. The bullet hits the boulder at its uppermost point. The bullet becomes imbedded in the boulder and the boulder continues to roll. Indiana knows that his bullets have a mass of 50 g and that the muzzle velocity of his gun is 700 m/s. Your friend says that the actor playing Indiana can run up at a speed of 8.0 m/s. He wants to know if Indiana will be able to outrun the boulder.

Hints: Consider the system to be the bullet and the boulder when the two collide and consider the collisions time to be just before the bullet strikes the boulder until just after the boulder is again rolling without slipping. In the "Approach" section, describe why it is very difficult to use conservation of linear momentum for this collision. However, if you choose to look at the rotation around a particular point it is quite simple to use conservation of angular momentum. Describe what point this is and why, for rotation around that point, $\sum \tau_{\text{ext}} = 0$ so that $\Delta L = 0$?

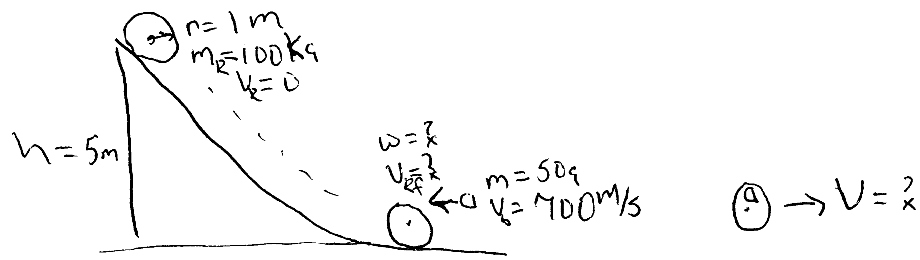
Date:________ Discussion Section:________ Group Number:________

Name: KEY Name:____________

Name:____________ Name:____________

FOCUS the PROBLEM

Draw a picture of the situation including ALL the information given in the problem.



Question(s): What is the problem asking you to find?

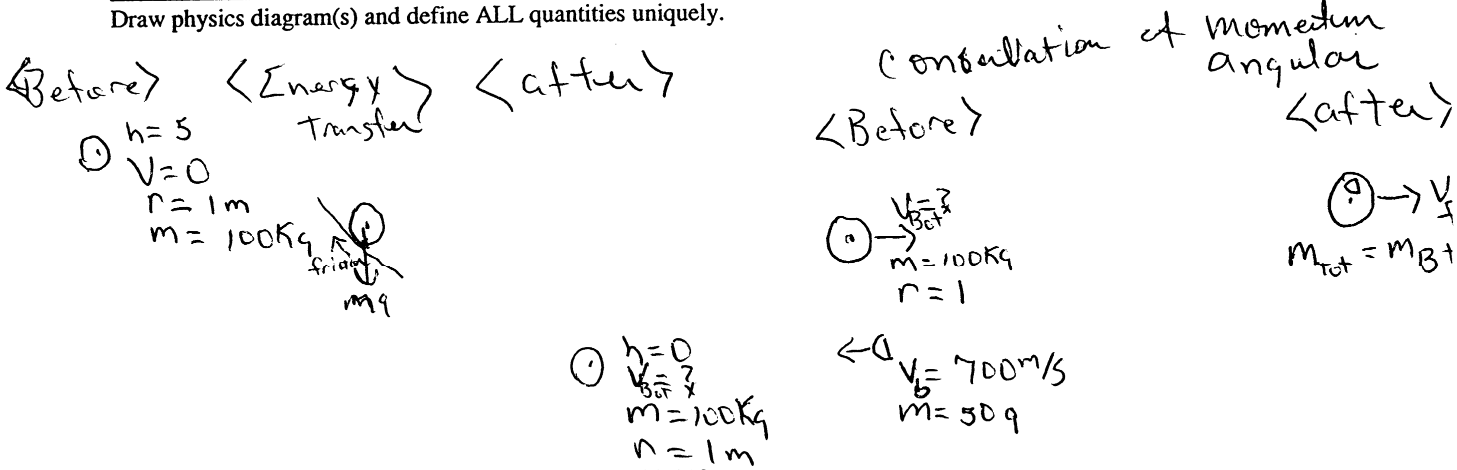
will Indiana survive, will he be able to outrun the rock

Approach: Outline the approach you will use.

Use conservation of Energy including rotation to find V_{Rf} at bottom before bullet hits. then use conservation of angular momentum about the bottom point to find final velocity after collision

DESCRIBE the PHYSICS

Draw physics diagram(s) and define ALL quantities uniquely.



Which of your defined quantities is your Target variable(s)?

V_f

Quantitative Relationships: Write equations you will use to solve this problem.

$W_{ext} = \Delta K + \Delta U$

$\Delta L = 0$

$I_{sphere} = \frac{2}{5}MR^2$

$V = \frac{\omega}{R}$

$I_{\parallel} = I_{cm} + mh^2$

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

$$W_{ext} = \Delta K + \Delta U$$

$$0 = mg(h_2 - h_1) + \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}I\omega_2^2 - \tfrac{1}{2}I\omega_1^2$$

$$mgh = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}I\omega^2 \qquad I = \tfrac{2}{5}MR^2$$

$$\omega = \frac{v}{R}$$

$$mgh = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\left(\frac{v_2}{R}\right)^2$$

$$mgh = \tfrac{1}{2}mv_2^2 + \tfrac{1}{5}mv_2^2$$

$$gh = \frac{7}{10}v_2^2$$

$$v_2 = \sqrt{\frac{10}{7}gh} \qquad v_2 = v_{bottom}$$

Now only at the bottom of the Rock is conservation of angular momentum conserved!

Thus we have

$$L_i = L_f$$

$$L_i = \underbrace{-2mv_bR}_{Bullet} + \underbrace{I_1\omega_1}_{Rock}$$

$$I_{1\,Rock} = \left(\tfrac{2}{5}MR^2 + MR^2\right) \qquad \omega_1 = \frac{v_2}{R}$$

↑ use parallel axis THM.

$$L_i = -2mv_bR + \left(\tfrac{2}{5}MR^2 + MR^2\right)\frac{v_2}{R}$$

$$L_f = I_2\omega_f\ ; \quad I_2 = I_{1\,Rock} + \underbrace{2mR^2}_{bullet}$$

Check Units

$$\frac{Kg\,\frac{m^2}{s} + Kg\,\frac{m^2}{s}}{Kg\,m^2 + Kg\,m^2} = \frac{1}{s} = \frac{rad}{s}$$ ✓ OK!

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$\omega_f = \frac{-2(.05)(700)(1) + \frac{7}{5}(100)(1)\sqrt{\frac{10}{7}(9.8)(5)}}{7/5(100)(1)^2 + 2(.05)(1)^2}$$

$$\omega_f = 7.86\ \frac{rad}{sec}$$

$$v_f = 7.86\ m/s$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes with proper units

Is Answer Unreasonable?

No, just slower than before collision.

Is Answer Complete?

No, Indiana will be able to out run the bolder

(extra space if needed)

$$\rightarrow L_f = \left(I_{1\,Rock} + 2mR^2\right)\omega_f$$

$$L_i = L_f$$

$$-2mv_bR + \left(\tfrac{2}{5}MR^2 + MR^2\right)\frac{v_2}{R} = \left(\tfrac{2}{5}MR^2 + MR^2 + 2mR^2\right)\omega_f$$

$$\Downarrow$$

$$\omega_f = \frac{-2mv_bR + \frac{7}{5}MRv_2}{7/5\,MR^2 + 2mR^2}$$

$$v_2 = \sqrt{\frac{10}{7}gh}$$

$$\omega_f = \frac{-2mv_bR + \frac{7}{5}MR\sqrt{\frac{10}{7}gh}}{7/5\,MR^2 + 2mR^2}$$

$$v_f = \omega_f$$