Chapter 9

Impulse, Momentum, and Collisions

Systems

- When dealing with conservation laws, we need to define the system.
  - A quantity that is always conserved may still be transferred into or out of a system.
    - Work was defined as the transfer of energy into or out of the system
- For conservation of momentum we will need to know which forces are external or internal to the system.
  - In many cases, the best choice for the system will be clear.
  - A transfer of momentum into or out of the system is called an “impulse.”

Linear Momentum

$$\mathbf{p} = m\mathbf{v}$$

$$\sum \mathbf{F} = m\mathbf{a} = m\, d(\mathbf{v})/dt = d(m\mathbf{v})/dt = d\mathbf{p}/dt$$

$$\boxed{\sum \mathbf{F} = d\mathbf{p}/dt}$$

Newton's 2nd Law: *The rate of change of momentum of a body is proportional to the net force applied to it.*

This is a more general form of Newton's 2nd law for it includes both the case when mass and/or velocity changes.

Interactive Question

A rubber ball and a lump of putty have equal mass. They are thrown with equal speed against a wall. The ball bounces back with nearly the same speed with which it hit. The putty sticks to the wall. Which object experiences the greater momentum change?

A) The ball
B) The putty
C) Both experience the same momentum change
D) Cannot be determined from the information given

Interactive Question

A person attempts to knock down a large wooden bowling pin by throwing a ball at it. The person has two balls of equal size and mass, one made of rubber and the other of putty. The rubber ball bounces back, while the ball of putty sticks to the pin. Which ball is most likely to topple the bowling pin?

A) the rubber ball
B) the ball of putty
C) makes no difference
D) need more information

Consider a system of particles. Let **P** be the vector sum of the momentum of the entire system.

$$\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \ldots = \sum m_i\mathbf{v}_i = \sum \mathbf{p}_i$$

$$d\mathbf{P}/dt = \sum d\mathbf{p}_i/dt = \sum \mathbf{F}_i$$

where $\mathbf{F}_i$ is the net force on the *i*th object in the system.

$$d\mathbf{P}/dt = \sum \mathbf{F}_{i(int)} + \sum \mathbf{F}_{i(ext)} = 0 + \sum \mathbf{F}_{i(ext)}$$

$$\boxed{\sum \mathbf{F}_{ext} = d\mathbf{P}/dt}$$

This is the most important equation in the chapter and is a very general principle. Problems involving the conservation of momentum start here.

Conservation of Momentum

$$\sum \mathbf{F}_{ext} = d\mathbf{P}/dt$$

If the system is isolated, then $d\mathbf{P}/dt = 0$

The total momentum of an isolated system of bodies remains constant.

When, $\sum \mathbf{F}_{ext} = 0$, then $\mathbf{P}_{initial} = \mathbf{P}_{final}$.

Where $\mathbf{P} = \sum \mathbf{p}_i$

Problem: A 90-kg fullback attempts to dive over the goal line with a velocity of 6.00 m/s. He is met at the goal line by a 110-kg linebacker moving at 4.00 m/s in the opposite direction. They collide in mid-air and the linebacker holds on to the fullback. What happens?

External Forces and Collisions

In this problem, there are no external forces in the horizontal direction so momentum is conserved in our system in the horizontal direction. But what about the vertical direction where the force of gravity is an external force?

$$\sum \mathbf{F}_{\text{ext}} = d\mathbf{P}/dt$$
$$\sum \mathbf{F}_{\text{ext}}\, dt = d\mathbf{P}$$

If $\sum \mathbf{F}_{\text{ext}}\, dt$ is very small during the collision compared to $\sum \mathbf{F}_{\text{int}}\, dt$ then we can neglect the effects of the external forces. The system is effectively isolated even in the vertical direction.

Interactive Question

A car accelerates from rest. In doing so the car gains a certain amount of momentum and the Earth gains

A) more momentum.
B) the same amount of momentum.
C) less momentum.
D) the answer depends on how we define the system.

Interactive Question

Suppose the entire population of the world gathered together at one spot and everyone jumps up at the same time. While all the people are in the air, does the earth gain momentum in the opposite direction?

A) No; the inertial mass of the earth is so large that the planet's change in motion is imperceptible.

B) Yes; because of its much larger inertial mass, however, the change in momentum of the earth is much less than that of all the jumping people.

C) Yes; the earth recoils, like a rifle firing a bullet, with a change in momentum equal and opposite to that of the people

Interactive Question

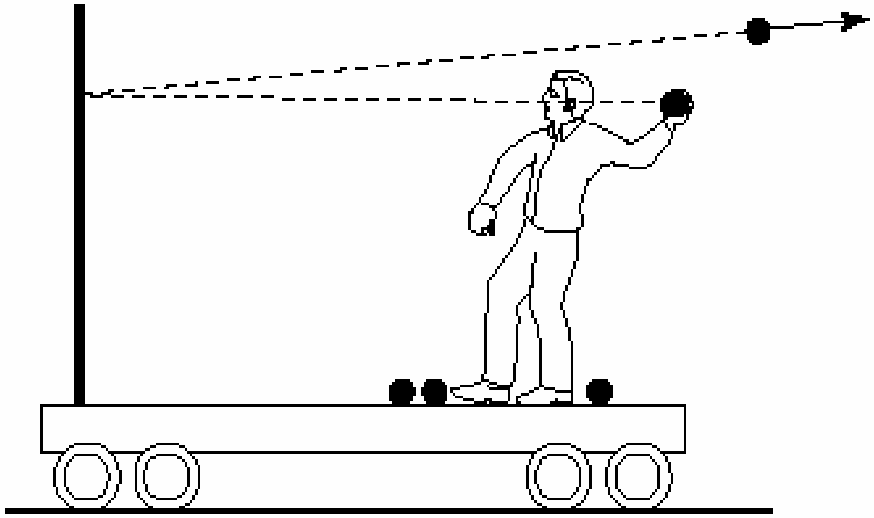
Suppose the entire population of the world gathered together at one spot and everyone jumps up at the same time. When the 5 billion people land back on the ground, the earth's momentum is

A) the same as what is was before the people jumped.

B) different from what is was before the people jumped.

Interactive Question

You are on a cart initially at rest on a track with no friction. You throw balls at a partition that is rigidly mounted on the cart. If the balls bounce straight back as shown, is the cart put in motion?



A) Yes, it moves to the right.
B) Yes, it moves to the left.
C) No, it remains in place.

Problem: A firecracker weighing 100 g, initially at rest, explodes into 3 parts. One part with a mass of 25 g moves along the x axis at 75 m/s. One part with mass of 34 g moves along the y axis at 52 m/s. What is the velocity of the third part?

Interactive Question

Consider two objects with different masses. Is it possible for the two objects to simultaneously have the same momentum and kinetic energy?

A) No, this is never possible.
B) Yes, this is possible for a certain non-zero velocity.
C) Yes, this is possible for many velocities.
D) Yes, but only if their velocities are both zero.

The relationship of momentum to kinetic energy

$$K = (1/2)mv^2 = (mv)^2/(2m) = p^2/(2m)$$

Interactive Question

A car accelerates from rest. In doing so the car gains a certain amount of kinetic energy and the Earth gains

A) more kinetic energy.
B) the same amount of kinetic energy.
C) less kinetic energy.

Impulse

$\sum \mathbf{F}_{ext} = d\mathbf{P}/dt$

$\int \mathbf{F}_{ext} dt = \int d\mathbf{P}$

$\mathbf{I} = \Delta\mathbf{P}$

where $\mathbf{I} = \int \mathbf{F}_{ext} dt$

I is called the impulse.

$$\mathbf{I} = \int \mathbf{F}_{\text{ext}}\,dt = \Delta\mathbf{P}$$

This equation plays the same role in our study of momentum, as the work-energy theorem played in our study of energy:

1) It is the starting point for most problems that involve momentum.
2) If there are no external forces, then $\mathbf{I} = \Delta\mathbf{P} = 0$, so $\mathbf{P}_i = \mathbf{P}_f$.
3) Although momentum is always conserved (like energy), the value of **I** will depend on how the system is defined.

Interactive Question

Suppose a ping-pong ball and a bowling ball are rolling toward you. Both have the same momentum, and you exert the same force to stop each. How do the time intervals to stop them compare?

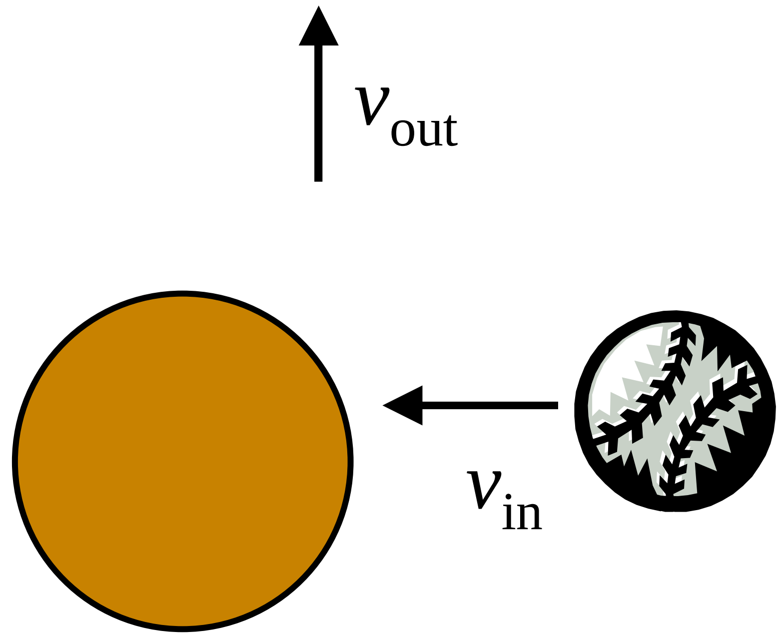
A) It takes less time to stop the ping-pong ball.

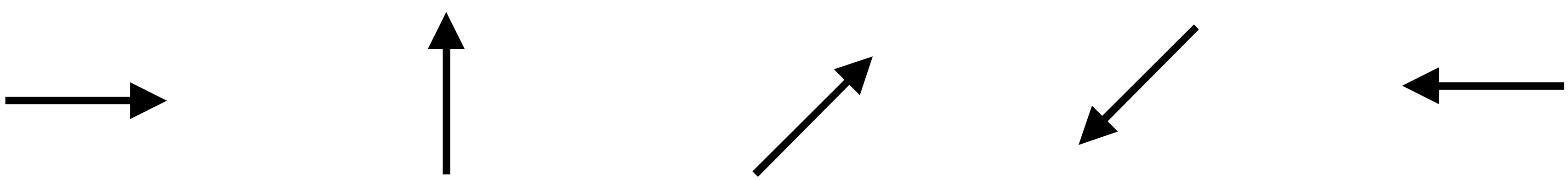
B) Both take the same time.

C) It takes more time to stop the ping-pong ball.

Interactive Question

A pitched baseball is popped straight up by the bat as shown in the figure at the right. In which direction is the impulse provided by the bat?





(A) (B) (C) (D) (E)

Problem: A 450-g baseball is moving at a speed of 35 m/s at an angle of 10° below the horizontal when it is hit by a bat. The ball is deformed by about 2.0 cm during the time of collision and the ball leaves the collision with a velocity of 50 m/s at an angle of 45° above the horizontal.

(a) What is the impulse during the collision?

(b) Estimate the time for the collision?

(c) Estimate the average force during the collision?

Note the similarities and differences

$$W_{\text{net}} = \int \mathbf{F}_{\text{net}} \cdot d\mathbf{s} = \Delta K$$

When only external forces do work on the system, then

$$W_{\text{ext}} = \int \mathbf{F}_{\text{ext}} \cdot d\mathbf{s} = \Delta K$$

$$\mathbf{I} = \int \mathbf{F}_{\text{ext}} dt = \Delta \mathbf{P}$$

Interactive Question

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the momentum of the light cart is

A) four times
B) twice
C) equal to
D) one-half
E) one-quarter

the momentum of the heavy cart.

Interactive Question

Consider two carts, of masses *m* and 2*m*, at rest on an air track. If you push first one cart for 3 s and then the other for the same length of time, exerting equal force on each, the kinetic energy of the light cart is

A) larger than

B) equal to

C) smaller than

the kinetic energy of the heavy cart.

Interactive Question

Two objects are sitting on a “frictionless” air hockey table. Object *A* has twice the mass of object *B*. Both objects are pushed with the same force for the same distance. Which statement is true?

A) The objects have the same momentum but different kinetic energies.

B) The objects have the same momentum and the same kinetic energies.

C) The objects have different momentum and different kinetic energies.

D) The objects have different momentum but the same kinetic energies.

Problem: A 100-g ball is dropped from 2.00 m above the ground. It rebounds to a height of 1.50 m. What was the average force exerted by the floor if the ball was in contact with the floor for 1.00×10^{-2} s.

Interactive Question

A small car meshes with a large truck in a head-on collision. Which of the following statements concerning the magnitude of the collision force is correct?

A) The truck experiences the greater average force.
B) The small car experiences the greater average force.
C) The small car and the truck experience the same average force.
D) It is impossible to tell since the masses and velocities are not given.

Interactive Question

A compact car and a large truck collide head on and stick together. Which undergoes the larger momentum change?

A) car

B) truck

C) the momentum change is the same for both

D) you can't tell without knowing the final velocity and combined mass.

Consider the collision of two objects,

Before Collision

After Collision

$$\mathbf{I}_1 = \int \mathbf{F}_{12}\, dt = \Delta \mathbf{p}_1$$

$$\mathbf{I}_2 = \int \mathbf{F}_{21}\, dt = \Delta \mathbf{p}_2$$

Since $\mathbf{F}_{12} = -\mathbf{F}_{21}$, $\Delta \mathbf{p}_1 = -\Delta \mathbf{p}_2$

Interactive Question

A compact car and a large truck collide head on and stick together. Which undergoes the larger acceleration during the collision?

A) car

B) truck

C) both experience the same acceleration

D) you can’t tell without knowing the final velocity and combined mass.

Elastic Collisions

If a system is isolated, then all collisions will conserve momentum. Most collisions do not conserve kinetic energy. But if they do, they are called “elastic collisions.”

Problem: A ball with a mass of 1.2 kg moving to the right at 2.0 m/s collides head on with a ball of mass 1.8 kg moving at 1.5 m/s to the left. If the collision is an elastic collision, what are the velocities of the balls after the collision?

In this problem we had two equations and two unknown quantities. What would have happened in three dimensions?

Before Collision

After Collision

$m_1 = 1.2$ kg $\quad m_2 = 1.8$ kg.

$v_{1\mathrm{i}} = 2.0$ m/s $\quad v_{2\mathrm{i}} = -1.5$ m/s

$v_{1\mathrm{f}x} = ?\quad v_{2\mathrm{f}x} = ?$

$v_{1\mathrm{f}y} = ?\quad v_{2\mathrm{f}y} = ?$

$P_{\mathrm{i}x} = P_{\mathrm{f}x}$

$P_{\mathrm{i}y} = P_{\mathrm{f}y}$

$K_{\mathrm{i}} = K_{\mathrm{f}}$

We would need one more constraint to solve this problem.

Interactive Question

A golf ball is fired at a bowling ball initially at rest and bounces back elastically. Compared to the bowling ball, the golf ball after the collision has

A) more momentum but less kinetic energy.
B) more momentum and more kinetic energy.
C) less momentum and less kinetic energy.
D) less momentum but more kinetic energy.
E) none of the above.

Inelastic Collisions

Most collisions do not conserve kinetic energy. An **inelastic collision** is one in which kinetic energy is not conserved. (All real macroscopic collisions are inelastic). A collision that conserves the least amount of kinetic energy is called a **completely inelastic collision**. Such collisions occur when the objects stick together after colliding. Of course, linear momentum is still conserved in inelastic collisions when the system is isolated.

Coefficient of Restitution (ε)

$$\varepsilon = \frac{|v_{2f} - v_{1f}|}{|v_{2i} - v_{1i}|}$$

For elastic collisions, $\varepsilon = 1$

For inelastic collisions, $\varepsilon = 0$

For other collisions, $0 < \varepsilon < 1$

Problem: Your friend has been in a traffic accident and she believes the other car was speeding and is at fault. She was traveling north through an intersection and was hit by a car traveling east. The two cars remained stuck together and left skid marks for 56 feet at an angle of 30º north of east. The speed limit was 50 mph for cars traveling north and east. From the make and model of each car you find that your friend's car has a weight of 2600 lbs, and the other car has a weight of 2200 lbs. The coefficient of kinetic friction for rubber tire skidding on dry pavement is 0.80. Who was at fault in this accident?

Interactive Question

While driving on a one way street you notice an identical car coming at you at the same speed as you are going. You can either hit the car head on or swerve and hit a massive concrete wall, also head on. In the split second before impact you decide to

A) hit the other car.

B) hit the wall.

C) hit either one, it makes no difference.

D) consult your lecture notes.

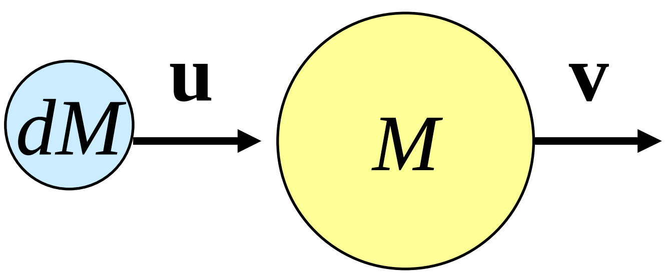
Problem: The ballistic pendulum is used to determine the velocity of a projectile. A projectile is fired into a pendulum and sticks to the bottom of the pendulum. The pendulum then swings up a certain angle. Suppose we have a bullet with mass m and velocity v_{1b}, and a pendulum with mass M. The bullet hits the pendulum and sticks to it. The pendulum then rises a distance h from its original position. We want to determine the velocity of the initial projective.

Let's go a step farther. Suppose the bullet is now fired horizontally from a height = y, above the ground how far will the bullet travel horizontally when it hits the ground?

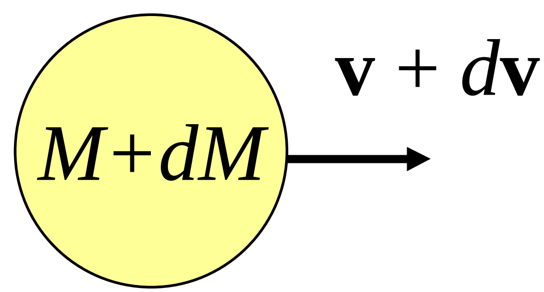
Varying Mass

Consider an object with mass M moving at a speed $\mathbf{v}$ and another very small object with mass dM moving in the same direction with a speed $\mathbf{u}$. (For a rocket ejecting hot gas, u and dM would be negative numbers). When the small mass collides with the larger mass, the speed of the larger mass becomes $\mathbf{v} + d\mathbf{v}$.

Before



After



$$\sum \mathbf{F}_{\text{ext}} = d\mathbf{P}/dt = (\mathbf{P}_\text{f} - \mathbf{P}_\text{i})/dt$$
$$= \{(M + dM)(\mathbf{v} + d\mathbf{v}) - (dM\mathbf{u} + M\mathbf{v})\}/dt$$
$$= \{M\mathbf{v} + M\,d\mathbf{v} + dM\mathbf{v} + dM\,d\mathbf{v} - dM\mathbf{u} - M\mathbf{v}\}/dt$$
$$= \{M\,d\mathbf{v} + dM\mathbf{v} + dM\,d\mathbf{v} - dM\mathbf{u}\}/dt$$
$$= \{M\,d\mathbf{v} + dM(\mathbf{v} - \mathbf{u}) + dM\,d\mathbf{v}\}/dt$$
$$= \{M\,d\mathbf{v} + dM(\mathbf{v} - \mathbf{u})\}/dt$$

Let's define the relative velocity of the two objects before the collision as $\mathbf{v}_{\text{rel}} = \mathbf{u} - \mathbf{v}$

$$\sum \mathbf{F}_{\text{ext}} + \mathbf{v}_{\text{rel}}\,(dM/dt) = M(d\mathbf{v}/dt)$$

$$\sum \mathbf{F}_{ext} + \mathbf{v}_{rel} \, (dM/dt) = M(d\mathbf{v}/dt)$$

The term on the right is the mass times the acceleration of the object. This equation holds at the instant that the mass of the object is equal to M.

For a rocket climbing straight up, $\sum \mathbf{F}_{ext} = Mg$

For an object with no external forces acting on it, like maybe a rocket in deep space, (in one dimension)

$M(dv/dt) = v_{rel} \, (dM/dt)$.

$dv = v_{rel} \, dM/M$

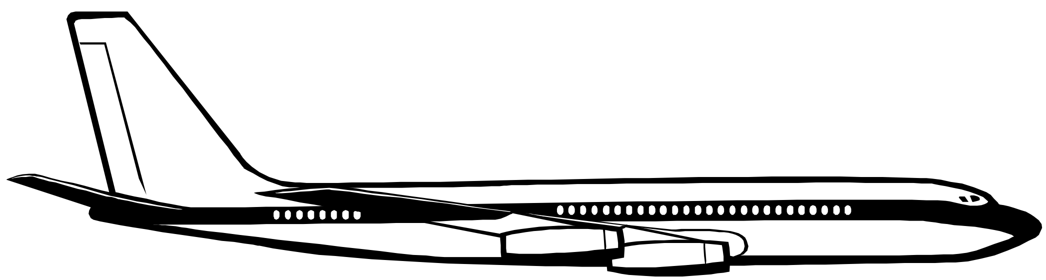
$v - v_0 = v_{rel} \ln(M/M_0)$

Problem: The jet engine of an airplane takes in 100 kg of air per second, which is burned with 4.2 kg of fuel per second. The burned gases leave the plane at a speed of 550 m/s (relative to the plane). If the plane is traveling at a constant speed of 270 m/s, determine

(a) the thrust due to the ejected fuel,

(b) the thrust due to the accelerated air

(c) the force of air resistance on the airplane.



Center of Mass

Consider the complicated motion of an object that may be spinning, or wobbling, like a tossed frisbee. How do we describe the motion of that object? The motion of most of the points on the object is quite complicated. However, there is one point of the object that behaves in the same way as a single particle would move subject to the same forces. That point of the body is called the center of mass. Even with rotating, and spinning, the center of mass moves in the same way that a single particle would move given the same external forces.

We find the center of mass of a group of objects by using the equation

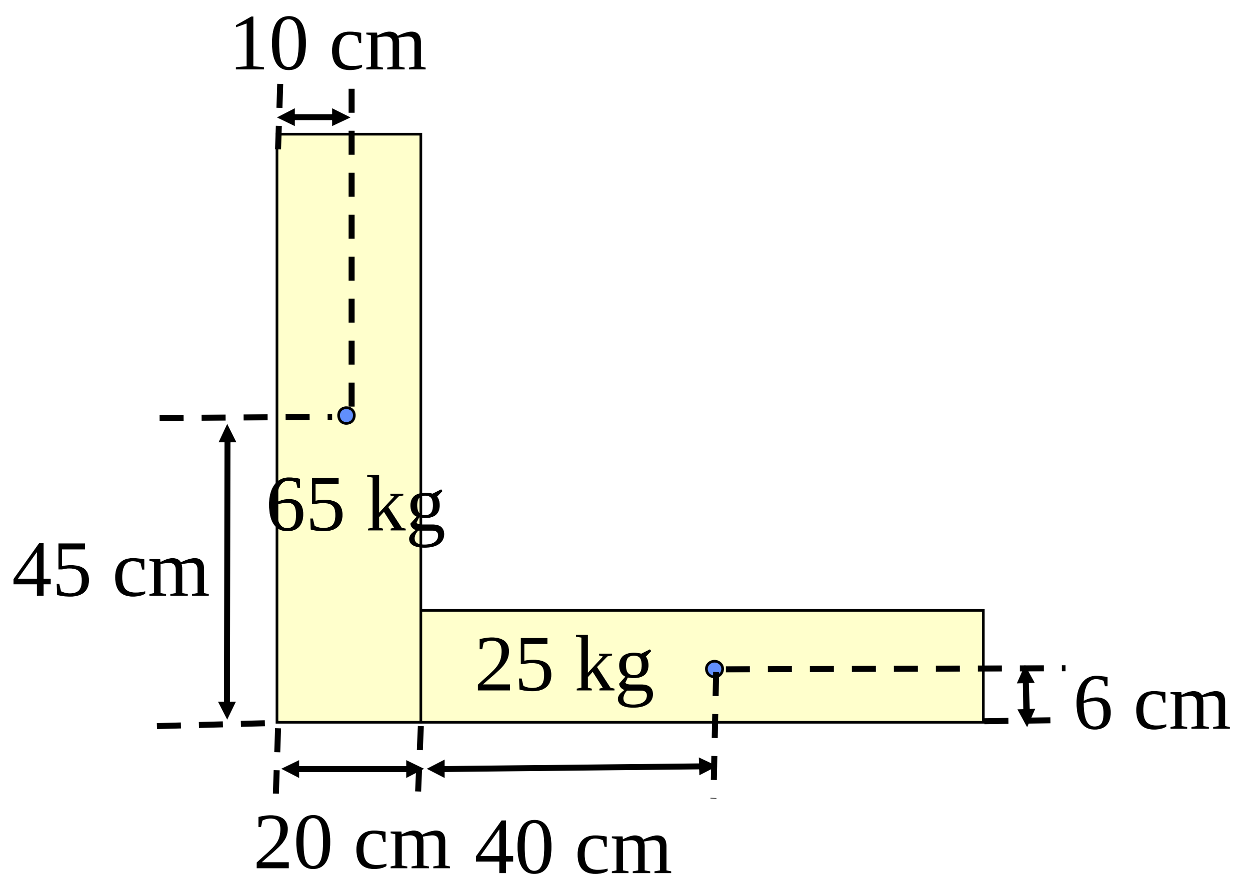
$$\mathbf{r}_{\mathrm{CM}} = (\Sigma m_i \mathbf{r}_i)/\Sigma m_i$$

Of course this is a shorthand way of writing the center of mass separately along the *x*, *y*, and *z* axes.

$$x_{\mathrm{CM}} = (\Sigma m_i x_i)/\Sigma m_I$$
$$y_{\mathrm{CM}} = (\Sigma m_i y_i)/\Sigma m_i$$
$$z_{\mathrm{CM}} = (\Sigma m_i z_i)/\Sigma m_i$$

The center of mass of a symmetric object is clearly at the midpoint of the symmetry.

Problem: The mass distribution of a person sitting down with his legs outstretched can be approximated by two rectangles as shown. Where is the person's center of mass?



1) Where is the center of mass of a doughnut?
   The center of mass does not have to be inside the physical mass of the object.
2) If two object have the same mass, do they have the same center of mass?
   The center of mass depends on the shape of the object. When a person stands up, the center of mass changes from that calculated in the previous problem.

In general,

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\int \mathbf{r}\,dm$$

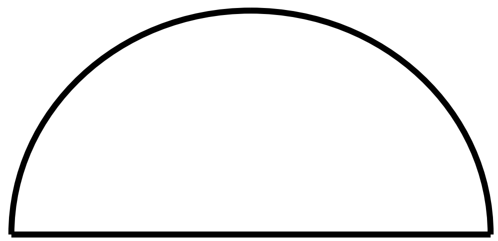
For an object with a constant density,

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{M}\int \mathbf{r}\,dm$$

$$= \frac{1}{M}\int \mathbf{r}\frac{dm}{dV}dV = \frac{1}{M}\int \mathbf{r}\rho\,dV = \frac{1}{M}\int \mathbf{r}\frac{M}{V}dV$$

$$\mathbf{r}_{\mathrm{CM}} = \frac{1}{V}\int \mathbf{r}\,dV$$

<u>Problem:</u> Where is the center of mass of a thin semi-circular disk of uniform density?



Let’s show that the center of mass can be considered as the point where all the external forces are acting

$$\mathbf{r}_{CM} = (\sum m_i \mathbf{r}_i)/\Sigma m_i$$
$$M\mathbf{r}_{CM} = \sum m_i \mathbf{r}_i$$

Take the derivative with respect to time twice for this equation to get,

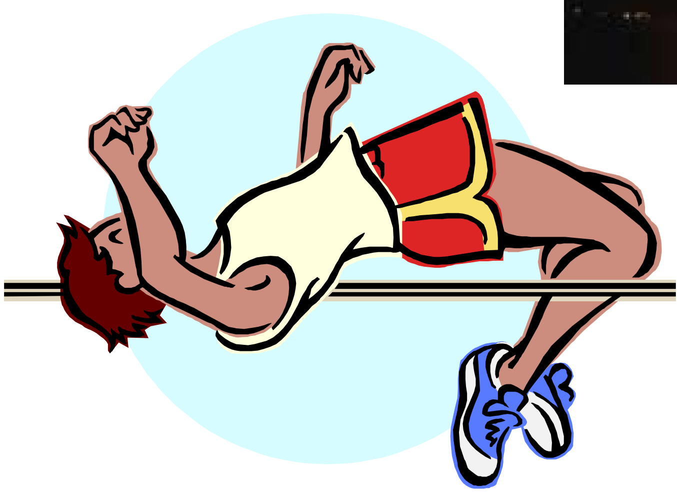
$$M\mathbf{a}_{CM} = \sum m_i \mathbf{a}_i = \sum \mathbf{F}_i = \sum \mathbf{F}_{ext} + \sum \mathbf{F}_{int}$$

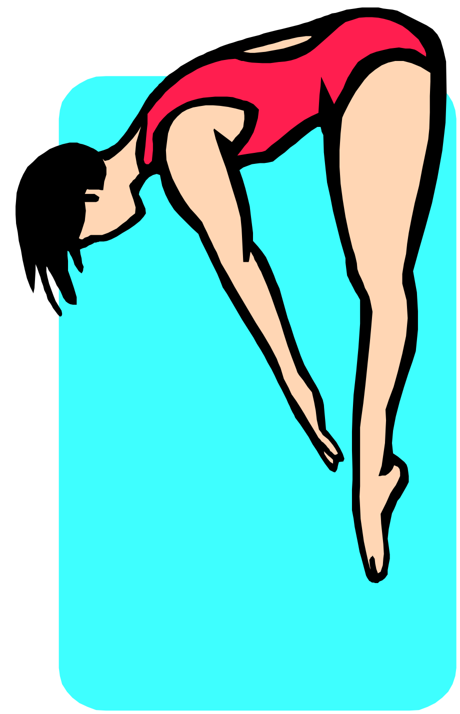
By Newton’s third law, all of the internal forces cancel and we just get

$$M\mathbf{a}_{CM} = \sum \mathbf{F}_{ext}$$

which is what we wanted to prove.

This leads to some interesting consequences.





Problem: Consider a rocket that is shot from the ground and is flying through the air. At the top of its flight it explodes into two parts with $m_{\mathrm{II}} = 3m_{\mathrm{I}}$. One part with mass m_{I} falls straight down. Where does the other part land?

Interactive Question

A plane, flying horizontally, releases a bomb, which explodes before hitting the ground. Neglecting air resistance, the center of mass of the bomb fragments, just after the explosion

A) is zero
B) moves horizontally
C) moves vertically
D) moves along a parabolic path
E) not enough information to determine the path

Interactive Question

A plane, flying horizontally, releases a bomb, which explodes before hitting the ground. What can you say about the motion of each of the fragments of the bomb?

A) Nothing.
B) None of them move along a parabolic path.
C) Some, but not all move along a parabolic path.
D) All of them move along a parabolic path.

Momentum and Reference Frames

From point of view of person watching, $v_1' = v$, $v_2' = -v$
From point of view of 1st car, $v_1 = 0$, $v_2 = -2v$

Person watching view: $P_i = 0$, $P_f = 0$

1st car view: $P_i = -2mv$

$$P_f = (2m)(-v) = -2mv$$

Momentum is conserved in both frames of reference.

Interactive Question

You are standing on a boat which is sitting still in the water. You pick up a rock and throw it away from you at a speed of v_0 relative to you. What is the speed of the rock relative to the shore?

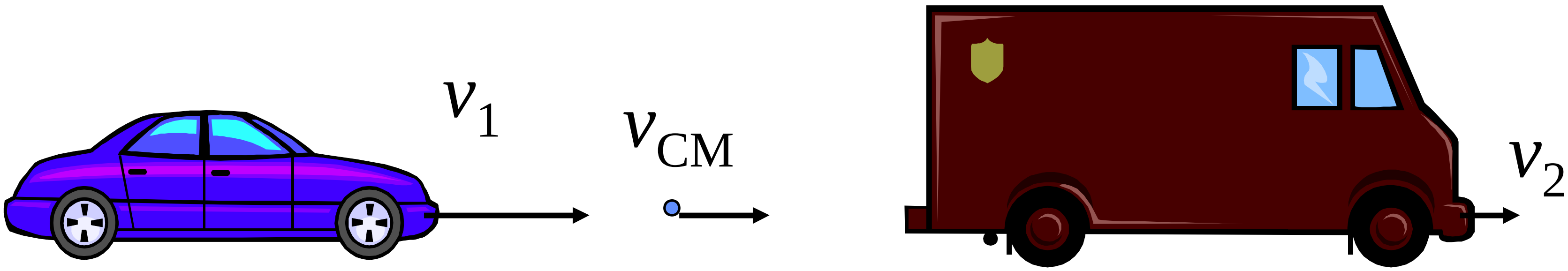
A) More than v_0

B) v_0

C) Less than v_0

D) It depends on whether you throw the rock straight out or at an angle upward and outward.

Suppose one object is more massive than the other.

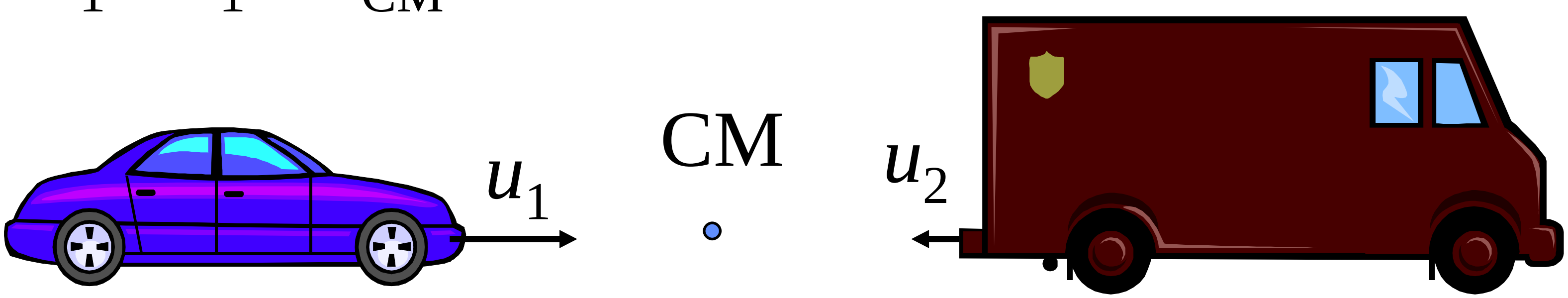


$$\mathbf{r}_{CM} = (m_1\mathbf{r}_1 + m_2\mathbf{r}_2)/(m_1 + m_2)$$
$$\mathbf{v}_{CM} = (m_1\mathbf{v}_1 + m_2\mathbf{v}_2)/(m_1 + m_2)$$

Or in the center of mass frame:

$u_2 = v_2 - v_{CM}$

$u_1 = v_1 - v_{CM}$



In CM frame of reference: $\mathbf{p}_1 = -\mathbf{p}_2$

$$m_1\mathbf{u}_1 = -m_2\mathbf{u}_2$$

Interactive Question

When a small ball collides elastically with a more massive ball, initially at rest, the massive ball tends to remain nearly at rest, whereas the small ball bounces back at almost its original speed. Now consider a massive ball of mass M at speed v striking a small ball of mass m initially at rest. The change in the small ball's momentum is

A) Mv
B) $2Mv$
C) mv
D) $2mv$
E) none of the above.

Interactive Question

A tennis ball is put on top of a basketball and the combination is dropped from a certain height. Compared to the speed it has just before the basketball hits the ground, the speed with which the tennis ball rebounds is

A) the same.
B) twice as large.
C) three times as large
D) four times as large
E) none of the above.