Chapter 8

Work, Energy, and the CWE Theorem

Scalar ("Dot") Product Revisited

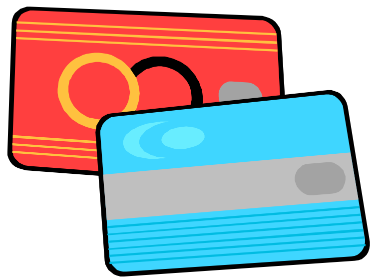
$$\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$$
$$= A_xB_x + A_yB_y + A_zB_z = AB\cos\theta$$

$\mathbf{A}\cdot\mathbf{B}$ is the magnitude of **B** in the direction of **A** multiplied by the magnitude of **A**, (or completely equivalently, the magnitude of **A** in the direction of **B** multiplied by the magnitude of **B**.)

Before discussing the concept of energy it is insightful to talk about money.

Define “money”

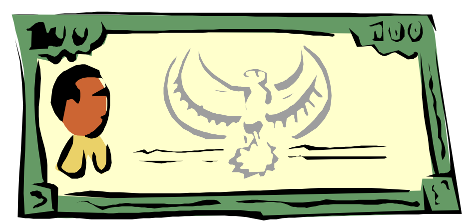
Cash



Credit

Savings

Land



Stocks

The total amount of “money” doesn’t change, but it can be transferred from one “system” to another.

How much money a system has depends on the definition of the system.

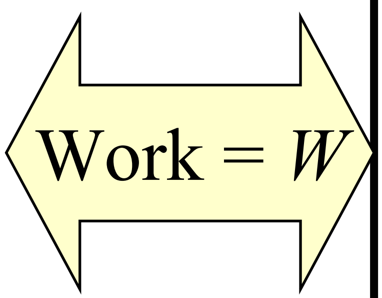
Is this a transfer of money from one system to another or not?

Define "energy"

It is hard to define energy. But we can say that the total amount of energy never changes. It is conserved. Energy can be transferred from one system to another.

Environment

Energy transfer



System

- Motional energy
  = kinetic energy = K
- Stored energy
  = potential energy = U
- Internal energy (not discussed too much this semester)

Mechanical energy = $E = U + K$

To understand this, we must develop the above concepts .

Energy Transfer and Conservation

(Initial energy of system) + (Energy transferred into system) =
(Final energy of system) + (Energy transferred out of system)

- or -

(Initial energy of system) – (Final energy of system) =
(Net Energy transfer)

How would you write this if your system is the whole universe?

(Initial energy of system) – (Final energy of system) = 0

-or -

(Initial energy of system) = (Final energy of system)

Energy is conserved

The previous page can be expressed as the Work-Energy Theorem: $\Delta K = W_{\text{net}}$

$$\Delta K = W_{\text{C}} + W_{\text{NC}} + W_{\text{ext}} = -\Delta U - \Delta E_{\text{int}} + W_{\text{ext}}$$
$$E_{\text{int}} = E_{\text{ther}} + E_{\text{chem}} + E_{\text{nucl}} + E_{\text{sound}} + E_{\text{light}} + \ldots$$

We will find next semester that to make this complete, we must add one more term, energy transferred by heat (Q)

$$\Delta K = Q + W_{\text{net}}$$
$$\Delta K + \Delta U + \Delta E_{\text{int}} = Q + W_{\text{ext}}$$

But for this semester, we will set $E_{\text{int}} = 0$ and Q=0, so we will write the conservation of energy as
$\Delta K = W_{\text{net}}$ or, more often:

$$\Delta K + \Delta U = W_{\text{ext}}$$

An “isolated” system has no external forces acting on it. A “closed” system does not allow any energy or matter to be transferred into or out of the system.

The universe is both a closed and isolated system.

If energy is always conserved, what do we mean when we say that you should turn the lights off or drive a fuel efficient vehicle to “conserve” energy?

Definition of Work

Work is the product of the component of force along the direction of displacement times the magnitude of the distance.

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

where W is the work done by the force F acting on an object, when the object moves a distance $d\mathbf{s}$.

If $\mathbf{F}$ is the net force acting on the object, then we have,

$$W_{\mathrm{net}} = \int \mathbf{F}_{\mathrm{net}} \cdot d\mathbf{r}$$

The SI unit of work is N·m which is given the name Joule.
1 N·m = 1 J

If we write the integral in components, we get

$$W = \int \mathbf{F} \cdot d\mathbf{r} = \int F_x dx + \int F_y dy + \int F_z dz$$

(with $\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$, $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$)

If there is only motion in one dimension (x),

$$W = \int F_x dx$$

If there is only motion in one dimension and the force constant, then

$$W = \int F_x dx = F_x \Delta x = F \cos\theta \, \Delta x$$

where θ is the angle between the force, F, and the displacement, Δx.

CAUTION

The work-energy theorem only works when considering the net work done on an object!

$$\Delta K = W_{\text{net}}$$

The definition of work gives the work done on an object by a force acting on an object. This will be the opposite sign to the work done the force. Be careful to differentiate between work done on an object and by an object, which usually means by a force.

$$W_i = \int \mathbf{F}_i \cdot d\mathbf{s}$$

Problem: (a) You pull a crate with a force of 98 N at an angle of 25° above the horizontal for a distance of 62 m. What is the total work done by you on the crate? (b) You now pull the crate with a force of 98 N at an angle of 25° above the horizontal for a distance of 62 m in the other direction. What is the total work you did on the crate in both directions?

Problem: What work is done by a force $\mathbf{F} = (2.0x \text{ N})\mathbf{i} + (3.0 \text{ N})\mathbf{j}$, with x in meters, that moves a particle from a position $\mathbf{r}_i = (2.0 \text{ m})\mathbf{i} + (3.0 \text{ m})\mathbf{j}$ to a position $\mathbf{r}_f = -(4.0 \text{ m})\mathbf{i} - (3.0 \text{ m})\mathbf{j}$?

Interactive Question

You lift a barbell with a mass of 50 kg up a distance of 0.70 m. Then you let the barbell come back down to where you started. How much net work did you do on the barbell?

A) – 340 J

B) 0 J

C) + 35 J

D) + 340 J

E) + 690 J

Interactive Question

You lift a 10 N physics book up in the air a distance of 1 meter at a constant velocity of 0.5 m/s. The work done by gravity is

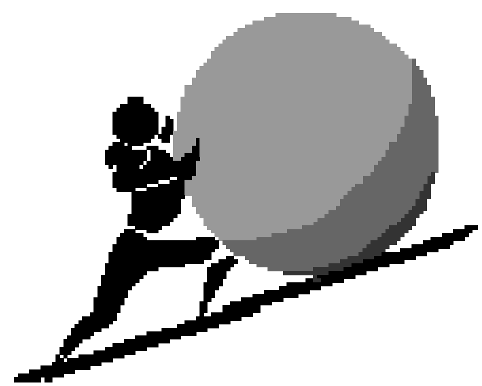
A) +10 J
B) – 10 J
C) +5 J
D) –5 J
E) zero

How much work is done by …

…the team on the right if the rope doesn’t move?

…the groom as he carries his bride over the threshold?

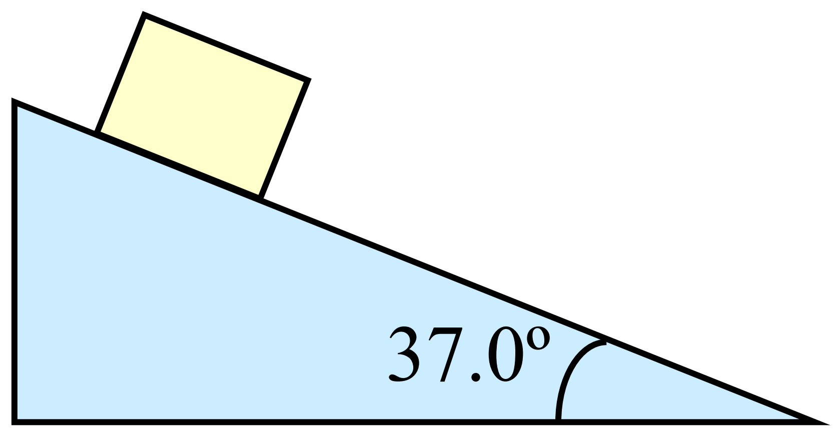
…the person as he pushes the rock but it doesn’t move.



Problem: A box weighing 23.0 kg slides down an inclined plane at a constant velocity as shown. The box slides 1.50 m.

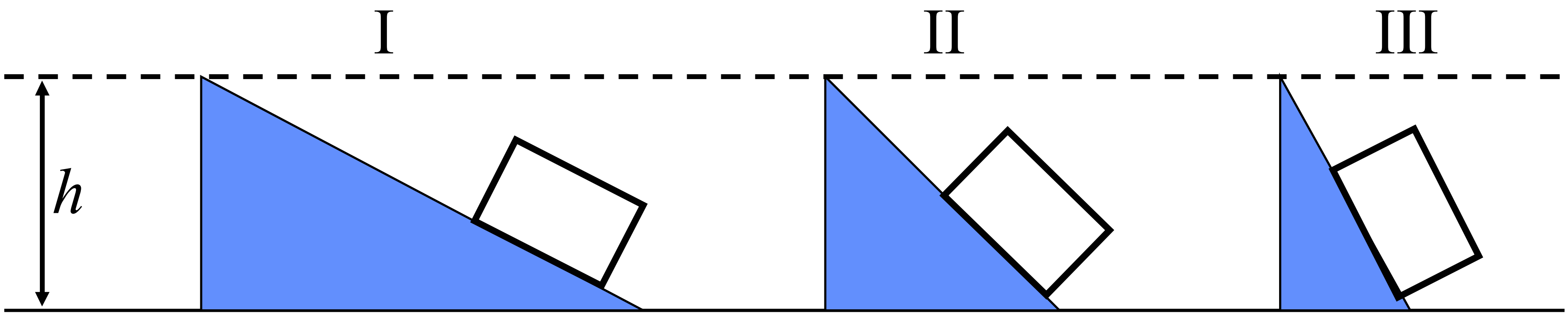
(a) What is the work done by the normal force, by gravity, and by friction?

(b) What is the total work done on the box?



Interactive Question

Consider the three different frictionless ramps shown below. If you push an identical box up each ramp at constant speed, rank the ramps in order from the one that would take the least work to the one that would take the most work.



A) I, II, III
B) III, II, I
C) II, I, III
D) I, III, II
E) None of the above

Interactive Question

Suppose you wanted to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. Compared to the work you would do if you took the short path, the work you do along the longer path is

A) four times as small.
B) three times as small.
C) half as small.
D) the same.
E) it depends on the time taken.

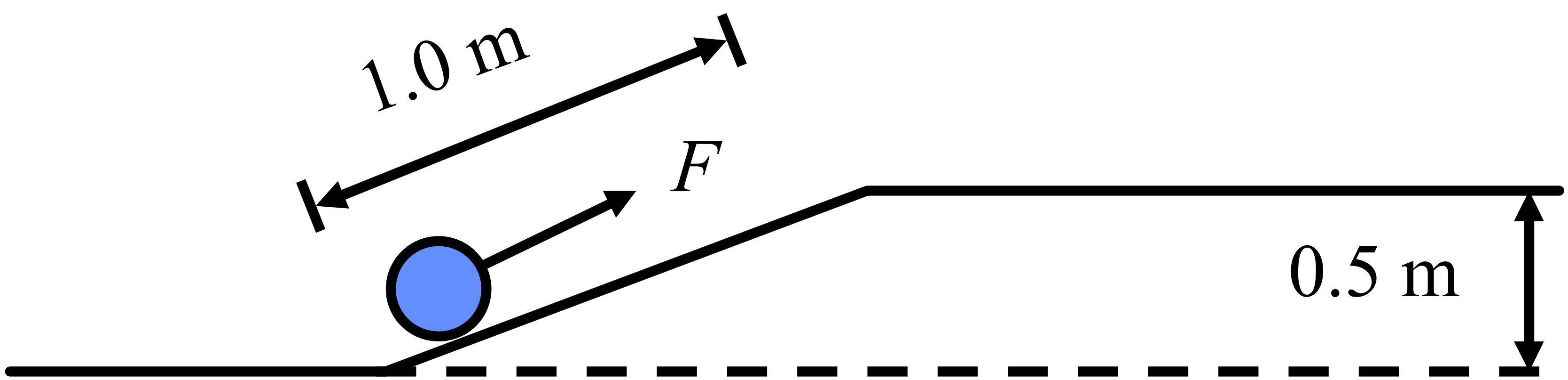
Interactive Question

Suppose you wanted to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

A) four times as small.
B) three times as small.
C) half as small.
D) the same.
E) it depends on the time taken.

Interactive Question

At a bowling alley, the ball feeder must exert a force to push a 5.0 kg bowling ball up a 1.0 m long ramp. The ramp raises the ball 0.5 m above the base of the ramp. Approximately how much force must be exerted on the bowling ball.



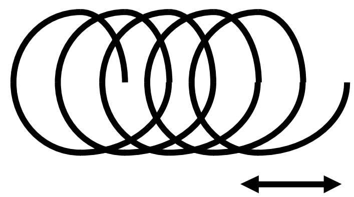
A) 200 N
B) 50 N
C) 25 N
D) 5.0 N
E) Not enough info to tell

Variable Forces

Consider the force exerted by an ideal spring

$$\mathbf{F} = -k(\mathbf{x} - \mathbf{x}_0)$$

Hooke's Law



$(x - x_0)$

$\longrightarrow +x$

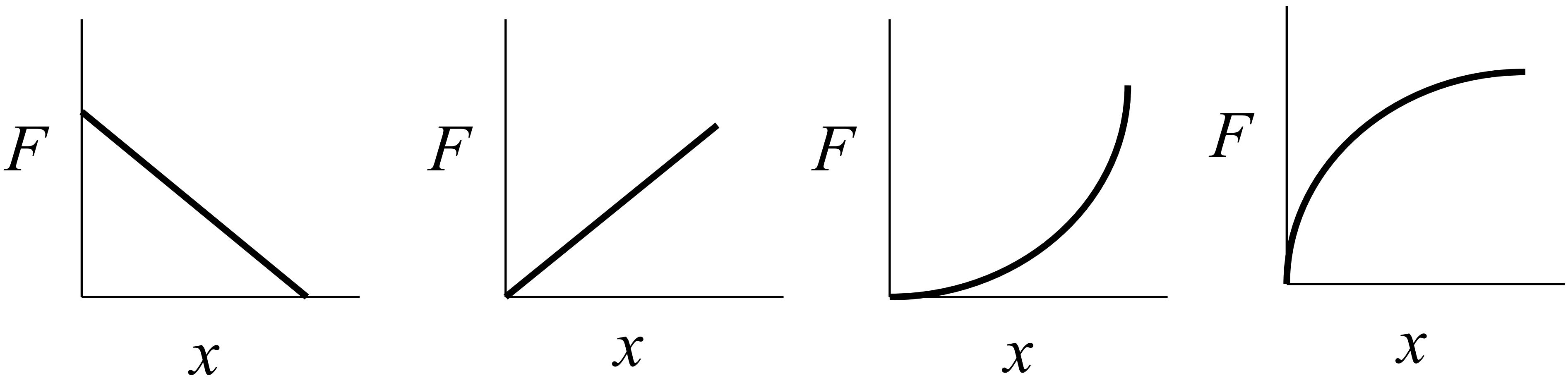
$$W = -\int_{x_0}^{x} F\, dx' = -\int_{x_0}^{x} kx'dx'$$

$$= \frac{1}{2}kx'^2\Big|_{x_0}^{x} = -\frac{1}{2}(kx^2 - kx_0^2)$$

$$= -\frac{1}{2}k(x^2 - x_0^2)$$

Interactive Question

The following plots show the value of a force directed along the x axis. Which plot is correct for an ideal spring?

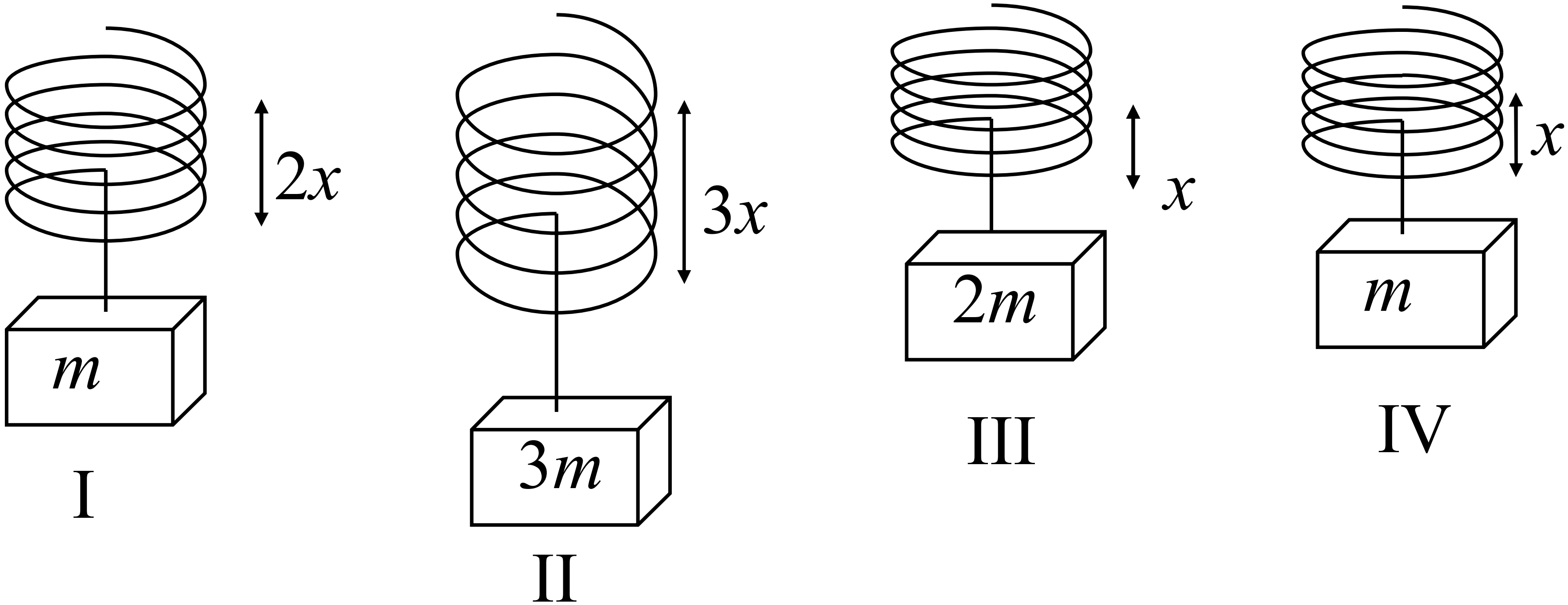


(A) (B) (C) (D)

(E) More than one of the above is correct.

Interactive Question

The following diagrams show the distance that a spring has been stretched from its equilibrium position when an object is hung from it. Which pair of springs have had the same work done on them?



A) I & II B) III & IV C) II & IV
D) I & III E) None of the above

Kinetic Energy and the Work-Energy Theorem

For simplicity, let's consider motion along a straight line, say the x axis.

$$v_f^2 = v_i^2 + 2a_x\Delta x$$
$$a_x = (v_f^2 - v_i^2)/2\Delta x$$

$$W_{\text{net}} = \sum \mathbf{F}\cdot\mathbf{d} = (\sum F_x)\ \Delta x = ma_x\Delta x$$
$$= m\{(v_f^2 - v_i^2)/2\Delta x\}\ \Delta x$$
$$W_{\text{net}} = (1/2)mv_f^2 - (1/2)mv_i^2 = K_f - K_i = \Delta K$$

where $K \equiv (1/2)mv^2$ is the kinetic energy.

Interactive Question

Two marbles, one twice as heavy as the other, are dropped to the ground from the roof of a building. Just before hitting the ground, the heavier marble has

A) as much kinetic energy as the lighter one.
B) twice as much kinetic energy as the lighter one.
C) half as much kinetic energy as the lighter one.
D) four times as much kinetic energy as the lighter one.
E) impossible to tell.

Problem: A 58-kg skier is coasting down a slope inclined at 35° above the horizontal. A kinetic frictional force of 90 N opposes her motion. Near the top of the slope the skier's speed is 3.6 m/s. Without using Newton's second law or kinematic equations, find her speed at a point which is 57 m downhill?

Interactive Question

A cart on an air track is moving at 0.5 m/s when the air is suddenly turned off. The cart comes to rest in 1 m. The experiment is repeated with the cart moving at 1 m/s when the air is turned off. How far does the cart travel before coming to rest?

A) 1 m
B) 2 m
C) 3 m
D) 4 m
E) impossible to determine

Interactive Question

A 4 kg mass with a speed of 2 m/s, and a 2 kg mass with a speed of 4 m/s, are gliding over a horizontal frictionless surface. Both objects encounter the same horizontal force, which opposes their motion, and are brought to rest. Which statement best describes their stopping distances?

A) The 4 kg mass travels twice as far as the 2 kg mass before stopping.

B) The 2 kg mass travels twice as far as the 4 kg mass before stopping.

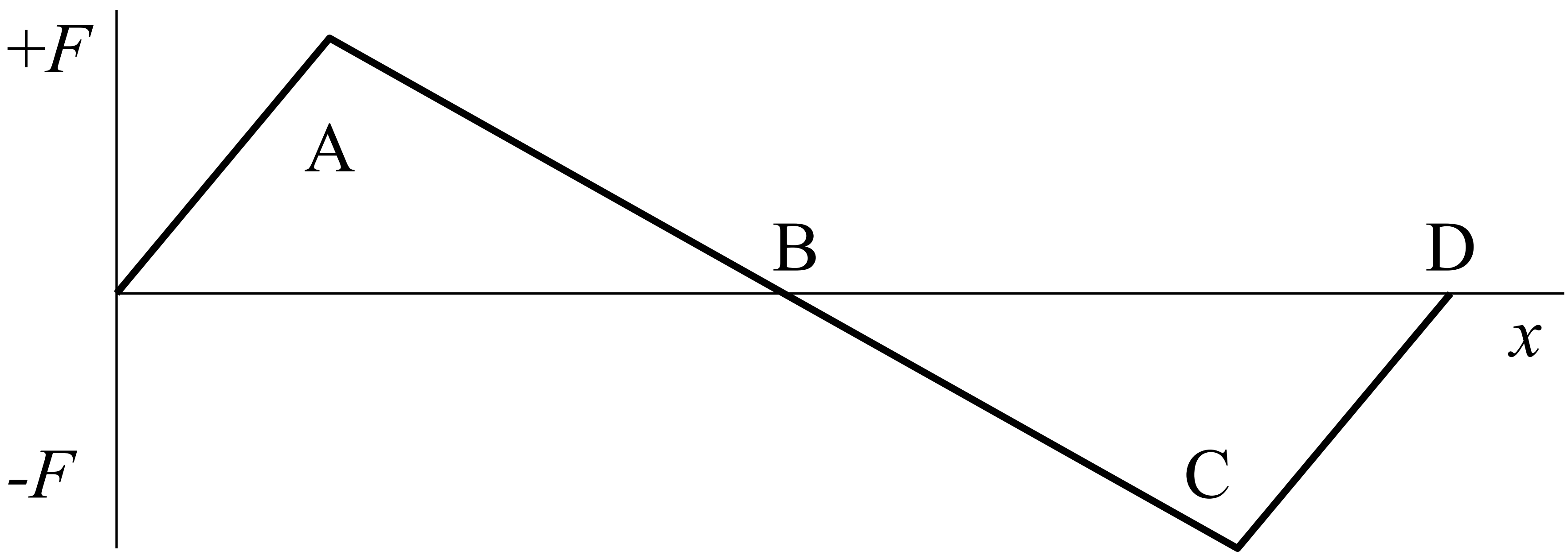
C) Both mass have the same stopping distance.

D) the 2 kg mass travels farther, but not necessarily twice as far.

Problem: A 5 mph (2.2 m/s) bumper on a car with a mass of 1100 kg compresses 15 cm and stops the car without any damage. What is the effective spring constant of the bumper?

Interactive Question

The graph below shows a plot of force as a function of distance for a force acting on an object directed along the x axis. If the velocity of the particle is zero at x=0, at which point is the speed the greatest?



E) None of the above

Conservative and Nonconservative Forces

If the work done by a force in moving an object between two positions is independent of the path of motion, the force is called a conservative force. The work done by a conservative force to move an object in a closed path is zero.

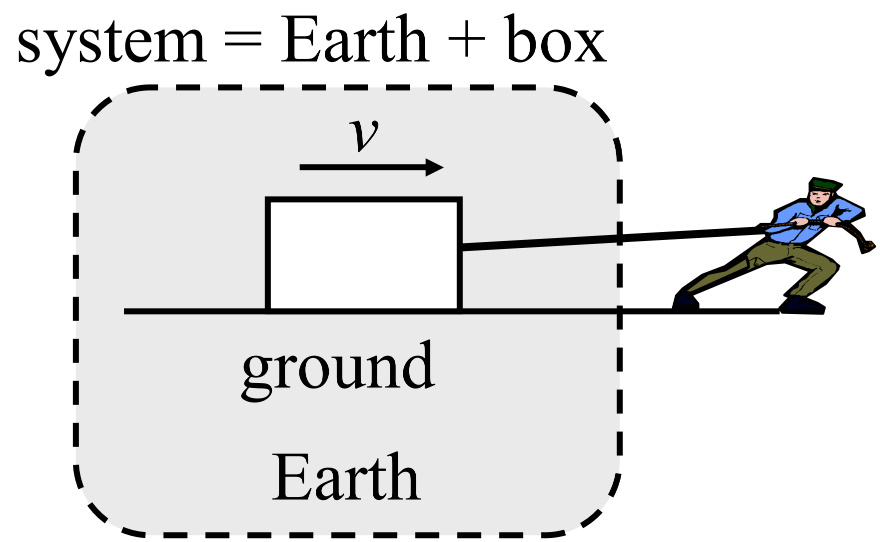
The only conservative forces we will encounter this semester are gravity and the elastic (spring) force.

$$\Delta K = W_{\text{net}}$$
$$\Delta K = W_{\text{int}} + W_{\text{ext}}$$
$$\Delta K = W_{\text{C}} + W_{\text{NC}} + W_{\text{ext}}$$

All of these are work done <u>on</u> the object in question.

Interactive Question

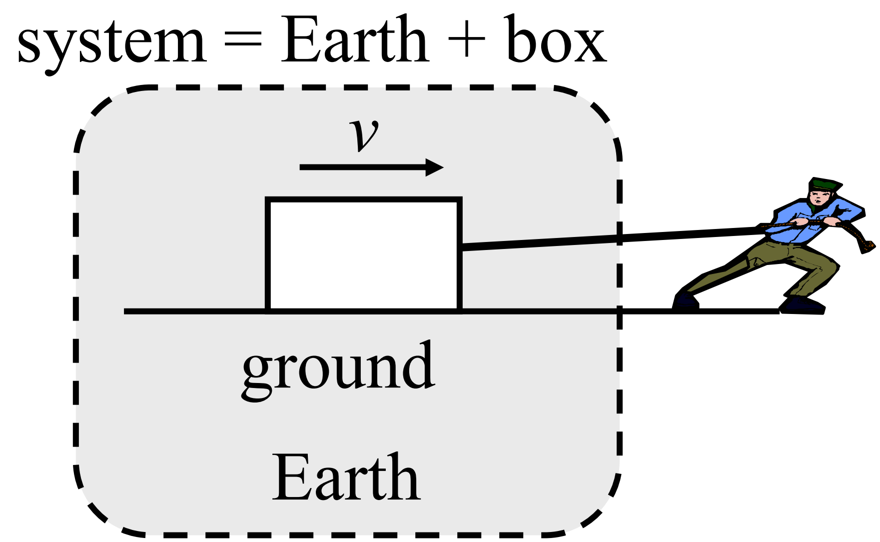
A person pulls a box along the ground at a constant speed. If we consider Earth and the box as our system, what can we say about the net external force on the system?



A) It is zero because the system is isolated.
B) It is nonzero because the system is not isolated.
C) It is zero even though the system is not isolated.
D) It is nonzero even though the system is isolated.
E) none of the above

Interactive Question

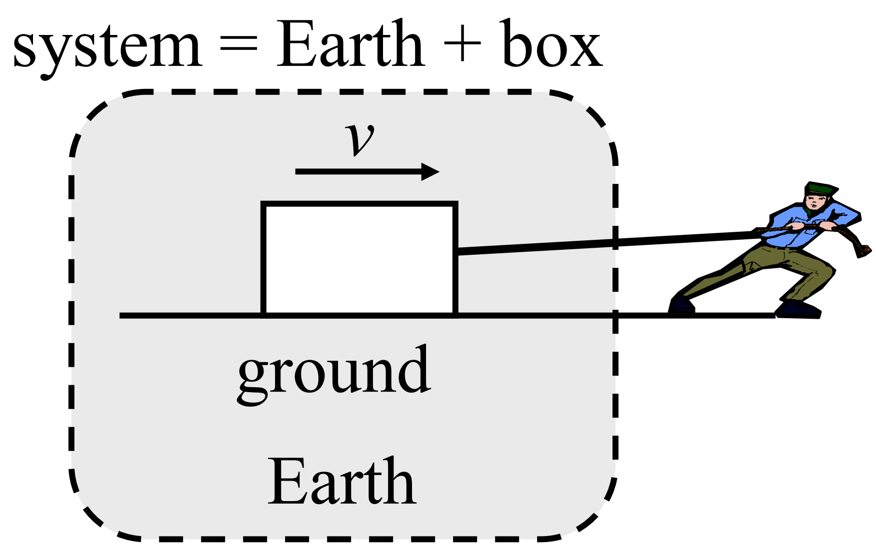
A person pulls a box along the ground at a constant speed. If we consider Earth and the box as our system, the net force exerted by the person on the system is



A) Zero
B) Not zero

Interactive Question

A person pulls a box along the ground at a constant speed. If we consider Earth and the box as our system, work done by the person on the system is



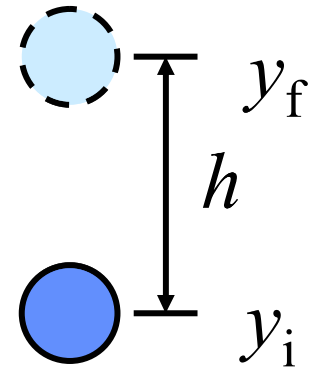
A) Zero
B) Not zero

Potential Energy

1. Potential energy is only defined for conservative forces.
2. Potential energy requires that at least two objects be a part of the system.
3. Potential energy is only dependent on the relative position of the two objects.

Gravitational Potential Energy

Consider the act of raising a ball to some height h at a constant velocity.



$$W_{you} = \int \mathbf{F} \cdot d\mathbf{r} = mg\,(y_f - y_i) = mgh$$

$$\Delta K = 0 = W_{you} + W_g$$

$$W_g = -W_{you} = mg\,(y_i - y_f) = mgy_i - mgy_f = U_i - U_f$$

$$W_g = -\Delta U$$

where $U = mgy$ is called the gravitational potential energy.

We can only measure changes in the gravitational potential energy, so we can set $U=0$ at any arbitrary point.

Why can a potential energy be defined only for a conservative force?

In general, $W_C = -\Delta U_C$

Force and Potential Energy for Gravity

For a conservative force: $W_C = -\int \mathbf{F}_C \cdot d\mathbf{r} = -\Delta U_C$

$dW = \mathbf{F} \cdot d\mathbf{r} = -dU$

For gravity, then

$$U = -\int \mathbf{F}_G \cdot d\mathbf{r} = \int (Gm_1m_2/r^2)\,\hat{\mathbf{r}} \cdot d\mathbf{r} = \int (Gm_1m_2/r^2)\,dr$$
$$= -Gm_1m_2/r + U_0$$

Setting the potential energy equal to zero for two objects that are an infinite distance apart gives:

$$U = -Gm_1m_2/r$$

$$E = K + U = (1/2)mv^2 - GMm/r$$

Elastic (Spring) Potential Energy

$$W_E = -\Delta U_E = (1/2)k\,\Delta x^2$$

Interactive Question

In which system is there a decrease in potential energy?

A) A boy stretches a spring.
B) A child slides down a sliding board.
C) A crate rests at the bottom of an inclined plane.
D) A car ascends up a steep hill.
E) More than one of the above

Problem: A block with a mass of 0.65 kg is hung on the end of a vertical spring with a spring constant of 22 N/m. What is the change in potential energy of this system?

Force and Potential Energy

Recall for a conservative force: $W_C = -\Delta U_C$

For simplicity, only consider motion in one dimension, along the x axis.

$dW = F\,dx = -dU$

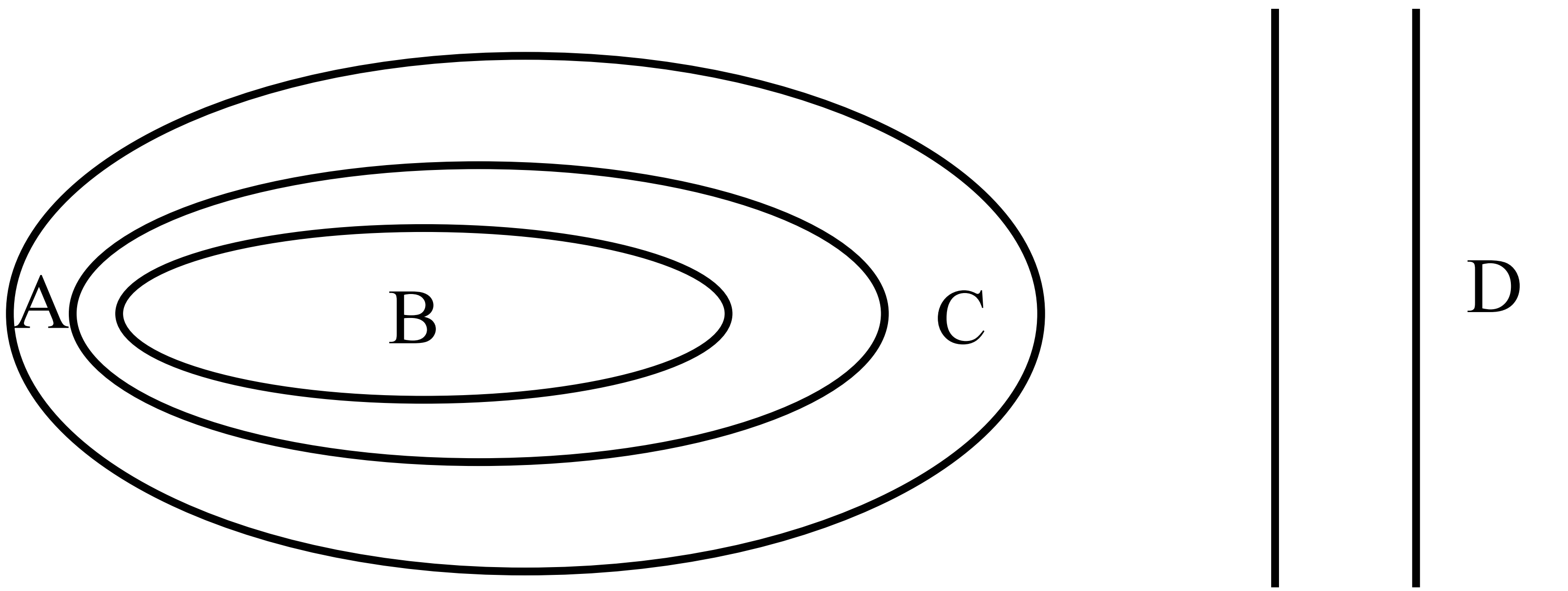
$F = -dU/dx$ This is a general result.

If the force is in more than one direction, we can write this as a partial differential equation:

$$\mathbf{F} = -\frac{\partial U}{\partial x}\hat{\mathbf{i}} - \frac{\partial U}{\partial y}\hat{\mathbf{j}} - \frac{\partial U}{\partial z}\hat{\mathbf{k}}$$

Interactive Question

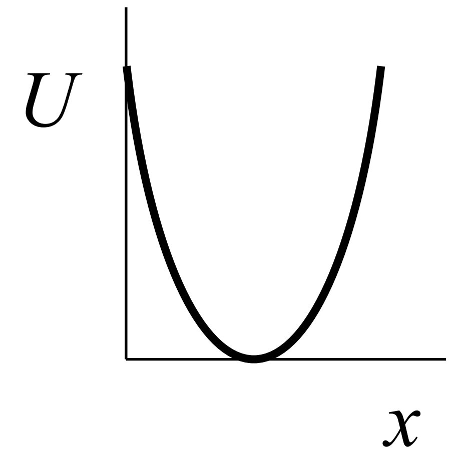
You are using the following topographical map while hiking. Each line is drawn along a constant elevation. At which location would it require the most force to walk up hill?

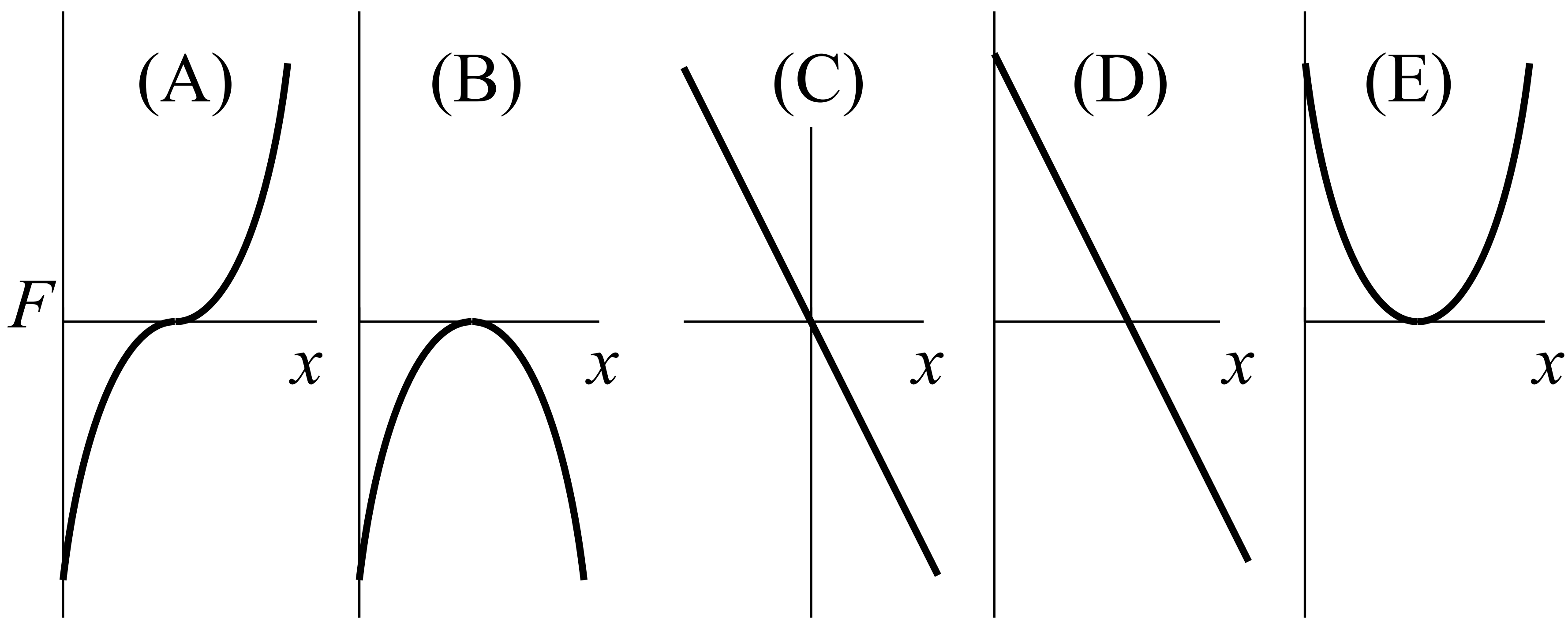


E) It is impossible to tell with the information given.

Interactive Question

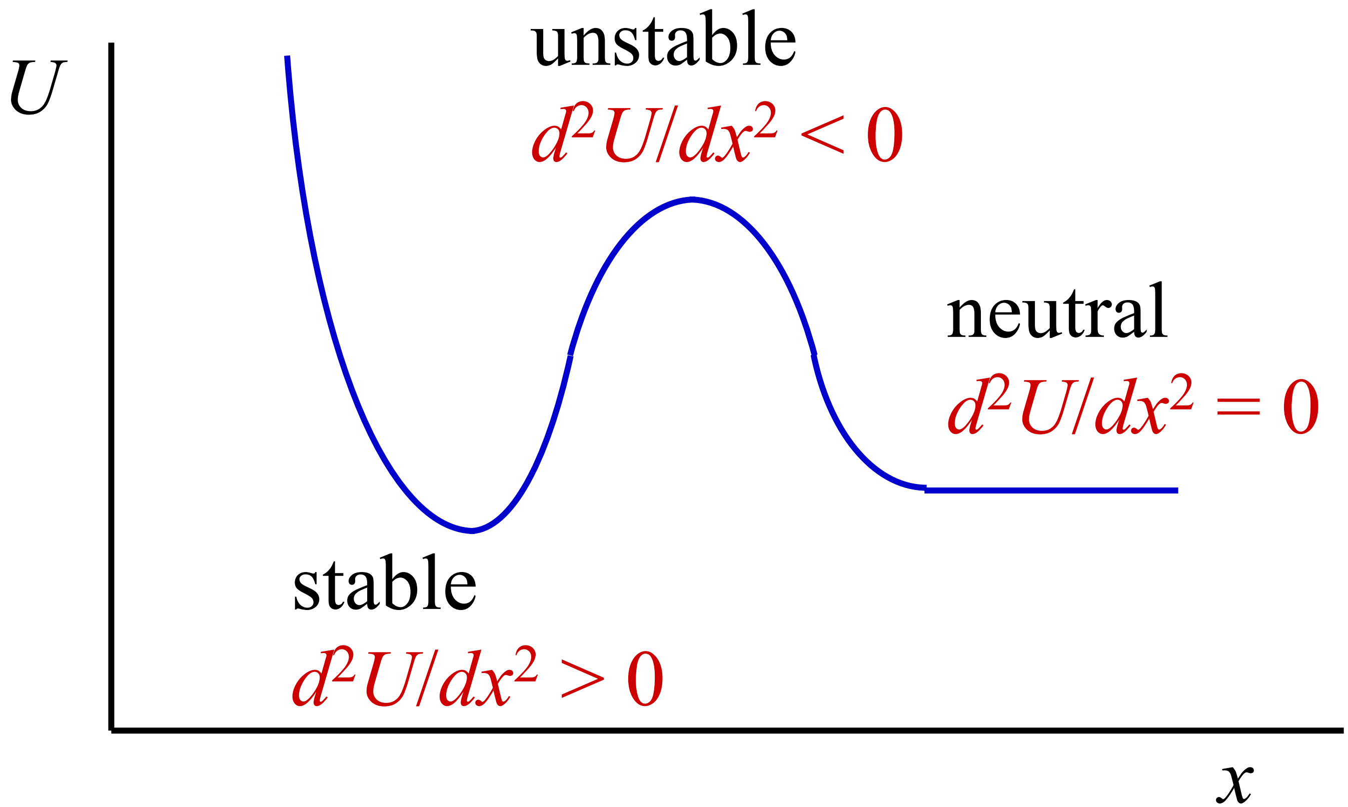
The graph to the right shows the potential energy for an object moving along the x axis. Which of the following graphs correctly gives the force F exerted on the object?





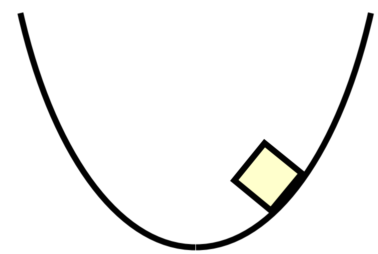
$$F = -dU/dx$$

The slope of a plot of the potential energy as a function of x is the force. If the slope is zero, we are at a point of equilibrium.



Interactive Question

A small block is placed on a frictionless ramp that is in the shape of a parabola. (The equation for a parabola centered at $x = 0$ is given by $y = Cx^2$ where C is a constant.) When released, will this block exhibit simple harmonic motion?



A) No
B) Yes, but only for small oscillations
C) Yes, for all oscillations

We have already studied everything we need to develop the conservation of energy. Recall the idea of a system and energy transfer.

Energy transfer

Environment Work = W

System

- Motional energy
  = kinetic energy = K
- Stored energy
  = potential energy = U
- Internal energy (not discussed too much this semester)

Mechanical energy = $E = U + K$

Also recall: $W_C = -\Delta U_C$

And the Work-Energy Theorem: $\Delta K = W_{net}$

Let's just write the Work-Energy theorem as

$$\Delta K = W_{\text{net}} = W_{\text{int}} + W_{\text{ext}}$$
$$\Delta K = W_{\text{C}} + W_{\text{NC}} + W_{\text{ext}}$$
$$\Delta K = -\Delta U - \Delta E_{\text{int}} + W_{\text{ext}}$$

$$E_{\text{int}} = E_{\text{ther}} + E_{\text{chem}} + E_{\text{nucl}} + E_{\text{sound}} + E_{\text{light}} + \ldots = 0 \text{ (for now)}$$

$$\Delta K + \Delta U = W_{\text{ext}}$$

This is the conservation of energy equation for this semester. It is the most important equation regarding energy, that we will use, and the starting point for most problems dealing with energy. (It is the same as the Work-Energy theorem.) We will expand this equation with other terms next semester when we deal with thermodynamics.

$$\Delta K + \Delta U = W_{\text{ext}}$$

We define the "Mechanical Energy" as $E = K + U$

The conservation of energy equation is sometimes written

$$\Delta E = W_{\text{ext}}$$

Conservation of Mechanical Energy

$$\Delta E = W_{\text{ext}}$$

In many cases, it is approximately true that $W_{\text{ext}} = 0$. In such cases, $\Delta E = 0$ and mechanical energy is conserved. This happens when only conservative forces are acting within the system, and no external forces do work on the system. Then,

$$\Delta E = 0$$
$$\Delta K + \Delta U = 0$$
$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$
$$K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$$
$$K_{\text{i}} + U_{\text{gi}} + U_{\text{Ei}} = K_{\text{f}} + U_{\text{gf}} + U_{\text{Ef}}$$

Problem: A child and sled with mass of 50 kg slide down a frictionless hill. If the sled starts from rest and has a speed of 6.00 m/s at the bottom, how high is the hill?

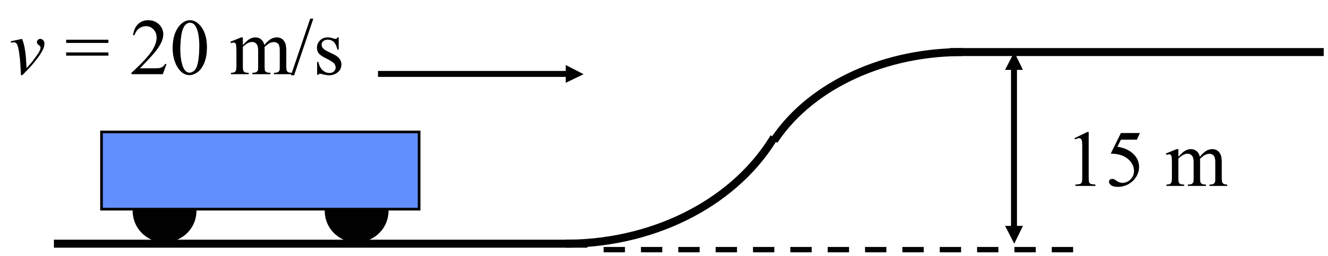
Interactive Question

A woman stands on the edge of a cliff. She throws a stone *vertically downward* with an initial speed of 10 m/s. The instant before the stone hits the ground below, it has 450 J of kinetic energy. If she were to throw the stone *horizontally outward* from the cliff with the same initial speed of 10 m/s, how much kinetic energy would it have just before it hits the ground?

A) 50 J
B) 100 J
C) 450 J
D) 800 J
E) Not enough information is given to answer the question

Interactive Question

A roller coaster travels along a straight path at a speed of 20 m/s. What would its speed be after climbing a 15 m hill if friction is ignored?



A) 3 m/s
B) 5 m/s
C) 10 m/s
D) 14 m/s
E) 17 m/s

Interactive Question

Suppose you wanted to ride your mountain bike down a steep hill. Two paths lead from the top to the base, one twice as long as the other. Neglect friction and air resistance. Compared to the maximum speed you would reach if you took the short path, the maximum speed you will reach along the longer path is

A) one quarter as fast.
B) twice as fast.
C) the same speed.
D) half as fast.
E) four times as fast.

You have volunteered to be a human cannonball at a charity circus. The "cannon" is a 1.5 m diameter tube with a spring inside which is attached to the bottom of the tube. A small seat is attached to the free end of the spring. The ringmaster tells you that before you enter the cannon, the spring will be compressed to 1/10 its normal length and held in that position. When the spring is released, the spring and the chair do not touch the sides of the 4.5 m long tube and you will be shot out of the cannon. You should fly over a wall that is 8 m higher than the uncompressed spring point. You should clear the wall at the highest point of your flight while moving at a speed of 4.3 m/s, then land in a net. Before the rehearsal, the cannon is taken apart for maintenance. You see the spring hanging straight down with one end attached to the ceiling. You determine that the spring is 3.0 meters long. When you hang on the free end of the spring without touching the ground, it stretches by about 0.50 m. Your mass is about 75 kg. Is it possible for you to make it over the wall?

Interactive Question

A spring loaded toy dart gun shoots a dart straight up in the air and the dart reaches a maximum height of 24 m. The same dart is shot straight up a second time but this time the spring is compressed only half as far. How far up does the dart go this time neglecting friction?

A) 96 m
B) 48 m
C) 24 m
D) 12 m
E) 6 m

General Gravitational Potential Energy (again)

Consider an object of mass m orbiting an object of mass M. The total energy of the object with mass m is the sum of its kinetic energy plus its potential energy.

$$E = K + U = (1/2)mv^2 - GMm/r$$

Let's look at two different applications of this idea.

Escape Speed

If an object is trying to escape a planet's gravity, it must have enough kinetic energy near the surface of the planet to reach an infinite distance away. If the object starts at the surface of the planet with radius R and mass M, then the minimum escape velocity is given by

$$K_\mathrm{i} + U_\mathrm{i} = K_\mathrm{f} + U_\mathrm{f}$$
$$(1/2)mv_\mathrm{e}^2 - GMm/R = 0 + 0$$
$$(1/2)mv_\mathrm{e}^2 = GMm/R$$
$$v_\mathrm{e} = \{2GM/R\}^{1/2}$$

If an object is in a circular orbit with radius r around a planet or around the sun with mass M, then we know that

$$F = ma$$

$$GMm/r^2 = mv^2/r$$

$$GM/r = v^2$$

So that the total mechanical energy is given by

$$E = K + U = (1/2)mv^2 - GMm/r = (1/2)GMm/r - GMm/r$$
$$= -(1/2)GMm/r$$

The distance r in these equations is the distance from the center of the earth (or the object being orbited), not the distance above the planet. The total mechanical energy is a negative number which leads to a stable orbit.

Problem: Two neutron stars, each with a mass of 10^{30} kg and a radius of 10^5 m, are separated by 10^{10} m. They are initially at rest with respect to each other.

a) What is their speed when their separation has decreased to one-half its initial value?

b) How fast are they moving just before they collide?

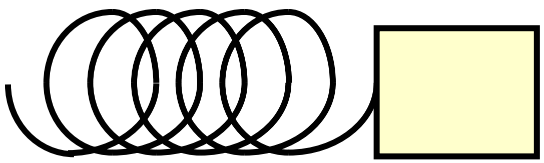
Interactive Question

A rock, initially at rest with respect to the Earth and located an infinite distance away is released and accelerates toward the Earth. An observation tower is built 3 Earth-radii high to observe the rock as it plummets to Earth. Neglecting friction, the rock's speed when it hits ground is

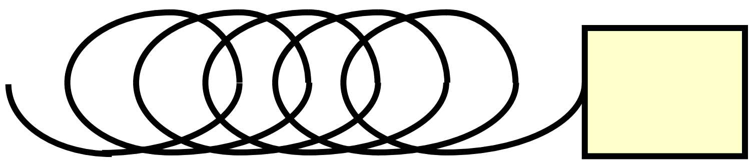
A) twice
B) three times
C) four times
D) nine times
E) sixteen times

its speed at the top of the tower

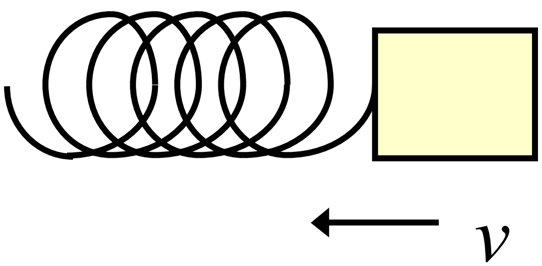
Energy of Simple Harmonic Motion



Equilibrium position



Spring stretched, no velocity
$K = 0$, $U = (1/2)k\,\Delta x^2 = (1/2)k\,A^2$



Spring unstretched, maximum velocity
$U = 0$, $K = (1/2)mv_{\text{max}}{}^2$

The total energy at any time is given by:
$E\ = (1/2)mv^2 + (1/2)kx^2$

Substituting for x and v:

$$
\begin{aligned}
E &= (1/2)mv^2 + (1/2)kx^2 \\
&= (1/2)mA^2\omega^2 \sin^2(\omega t + \delta) + (1/2)kA^2 \cos^2(\omega t + \delta) \\
&= (1/2)mA^2(k/m) \sin^2(\omega t + \delta) + (1/2)kA^2 \cos^2(\omega t + \delta) \\
&= (1/2)kA^2\{\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)\} \\
&= (1/2)kA^2
\end{aligned}
$$

or

$$E = (1/2)mv_{max}^2 = (1/2)m\omega^2A^2$$

Interactive Question

A mass vibrates back and forth from the free end of an ideal spring (k =20 N/m) with an amplitude of 0.30 m. What is the kinetic energy of this vibrating mass when it is 0.30 m from its equilibrium position?

A) Zero
B) 1.80 J
C) 0.90 J
D) 0.45 J
E) It is impossible to know without knowing the object's mass.

Interactive Question

A mass vibrates back and forth from the free end of an ideal spring (k = 20 N/m) with an amplitude of 0.30 m. What is the maximum kinetic energy of this vibrating mass?

A) Zero
B) 1.80 J
C) 0.90 J
D) 0.45 J
E) It is impossible to know without knowing the object's mass.

Interactive Question

Doubling only the amplitude of a vibrating mass and spring system produces what effect on the system's mechanical energy?

A) It decreases the energy by a factor of four.
B) It decreases the energy by a factor of two.
C) It increases the energy by a factor of two.
D) It increases the energy by a factor of four.
E) It produces no change.

Problem: A .24-kg mass is attached to a horizontal spring that has a spring constant of 86 N/m. The spring is initially stretched to 0.23 m. If there is no friction so that the spring oscillates with SHM how much energy is kinetic and potential when the spring is at

(a) $x = .10$ m

(b) $x = 0.0$ m

Interactive Question

The amplitude of any oscillator can be doubled by:

A) Doubling only the initial displacement
B) Doubling only the initial speed.
C) Doubling the initial displacement and halving the initial speed.
D) Doubling the initial speed and halving the initial displacement.
E) Doubling both the initial displacement and the initial speed.

Nonconservative Forces and Conservation of Energy

The total energy in any process is not increased or decreased. Energy can be transformed from one form to another and transferred from one body to another, but the total energy remains constant.

$$\Delta K + \Delta U = W_{\text{ext}}$$

Problem: A child and sled with mass of 50 kg slide down a hill with a height of 1.8 m.

a) If the sled starts from rest and has a speed of 4.6 m/s at the bottom, how much work is done by friction.
b) If the hill has an angle of 20° above the horizontal what was the frictional force.

Problem: An object is attached to an ideal spring with and stretched a distance of 0.64 m on a horizontal surface. When the object is released the spring moves a distance of 0.33 m past the equilibrium point. If the spring constant is 260 N/m, what is the average force of friction between the object and the surface?

Interactive Question

A ball drops some distance and gains 30 J of kinetic energy. Do not ignore air resistance. How much gravitational potential energy did the ball lose?

A) More than 30 J
B) Exactly 30 J
C) Less than 30 J
D) It depends on the mass of the ball.
E) More information is needed to determine the answer.

Interactive Question

A stone is thrown upward into the air. In addition to the force of gravity, the stone is subject to air resistance. The time the stone takes to reach the top of its flight is

A) larger than
B) equal to
C) smaller than

the time it takes to return from the top to its original position

Expansion of the CWE Theorem

$$\Delta K + \Delta U + \Delta E_{\text{int}} = Q + W_{\text{ext}}$$

$$E_{\text{int}} = E_{\text{ther}} + E_{\text{chem}} + E_{\text{nucl}} + E_{\text{sound}} + E_{\text{light}} + \ldots$$

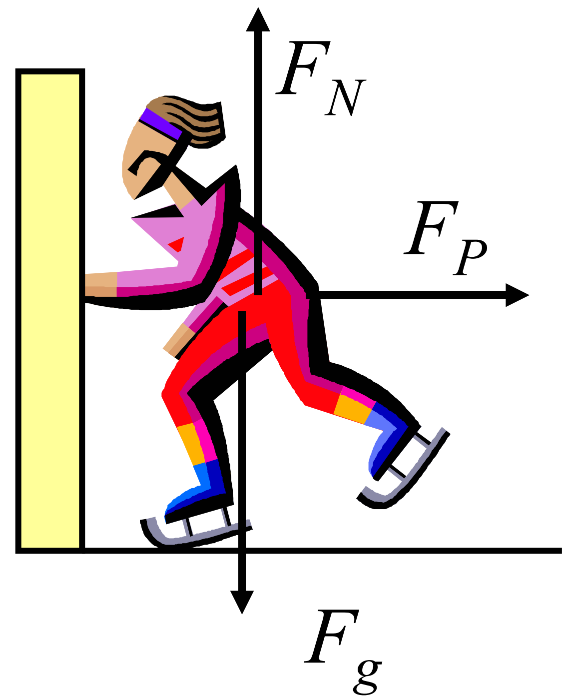
Depending on the situation, we often need to keep terms that we have neglected in order to really understand a situation. Let's look at some examples when

$$E_{\text{int}} = E_{\text{ther}} \neq 0$$

$$\Delta K + \Delta U + E_{\text{ther}} = W_{\text{ext}}$$

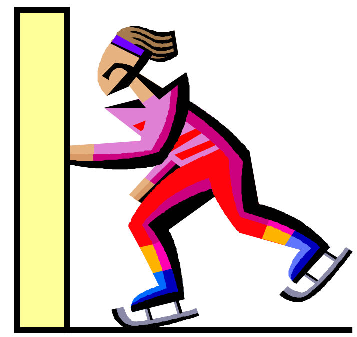
Interactive Question

An ice skater starting from rest, pushes off a wall and accelerates to a speed v, on a frictionless surface. The figure shows a correct free body diagram for the skater. Consider the earth, the wall and the skater as a system. Which force does work on the skater?



A) F_N
B) F_P
C) F_g
D) F_P and F_g
E) None of the above

Interactive Question



An ice skater starting from rest, pushes off a wall and accelerates to a speed v, on a frictionless surface. Consider the earth, the wall and the skater as a system.

Consider the relationships $\Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}}$ and $\Delta E_{\text{int}} = \Delta E_{\text{ther}} + \Delta E_{\text{chem}} + \Delta E_{\text{nucl}} + \Delta E_{\text{sound}} + \Delta E_{\text{light}} + \ldots$

Since ΔK is positive, what term must be negative, allowing the left hand side to add up to zero?

A) ΔE_{ther}

B) ΔE_{chem}

C) ΔU

D) A different internal energy

E) None of the above, the left hand side is not zero.

Interactive Question

A car starts from rest and accelerates to a speed v on level ground. Consider the car and the earth as the system. Which term(s) in the equation

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}}$$

are not equal to zero?

A) ΔK and ΔE_{int}
B) ΔK and W_{ext}
C) ΔK and ΔU
D) ΔK, ΔE_{int} and W_{ext}
E) only one term is not equal zero.

Interactive Question

A car starts from rest and accelerates to a speed v on level ground. Consider the relationships

$$\Delta K + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}}$$

and

$$\Delta E_{\text{int}} = \Delta E_{\text{ther}} + \Delta E_{\text{chem}} + \Delta E_{\text{nucl}} + \Delta E_{\text{sound}} + \Delta E_{\text{light}} + \dots$$

Which of the following is true?

A) $\Delta K \approx \Delta E_{\text{chem}}$ and everything else is nearly zero
B) $\Delta K \approx \Delta E_{\text{them}}$ and everything else is nearly zero
C) $\Delta K > \Delta E_{\text{ther}} > \Delta E_{\text{chem}}$ and everything else is nearly zero
D) $\Delta E_{\text{chem}} > \Delta K > \Delta E_{\text{ther}}$ and everything else is nearly zero
E) $\Delta E_{\text{chem}} > \Delta K$, we don't know how ΔK and ΔE_{ther} compare, and everything else is nearly zero

Power

Power is defined as the rate at which energy is transferred or transformed. Energy transfer into the system is done by external work, so Power is often defined as the rate that work is done.

$$P = dE_{tot}/dt$$

The SI unit of power is watts (W): 1 J/s = 1 W
Sometimes we measure power in horsepower (hp)
1 hp = 550 ft·lb/s = 746 W

$$W = \int \mathbf{F}\cdot d\mathbf{s}$$
$$dW = \mathbf{F}\cdot d\mathbf{s}$$
$$P = dW/dt = \mathbf{F}\cdot d\mathbf{s}/dt = \mathbf{F}\cdot\mathbf{v}$$

Problem: A deep sea observation chamber is raised from the bottom of the ocean 1700 m below the surface by means of a steel cable. The chamber moves at constant velocity and takes 5.00 minutes to reach the surface. The cable has a constant tension of 8900 N. How much power is required to pull the cable in? Give the answer in horsepower.

Problem: A 1000 kg elevator carries an 800 kg load. A constant frictional force of 4000 N retards its motion. What minimum power must the motor deliver to lift the elevator at a constant rate of 3.00 m/s?

Interactive Question

A 50 kg woman runs up a flight of stairs in 5 s. Her net upward displacement is 5 m. What power did the woman exert while she was running?

A) 250 W
B) 750 W
C) 0.5 kW
D) 1.0 kW
E) 5 kW

Interactive Question

A car accelerates from 0 to 30 mph in 1.5 s. How long does it take for it to accelerate from 0 to 60 mph assuming the power of the engine to be independent of velocity and neglecting friction?

A) 2.0 s
B) 3.0 s
C) 4.5 s
D) 6.0 s
E) 9.0 s