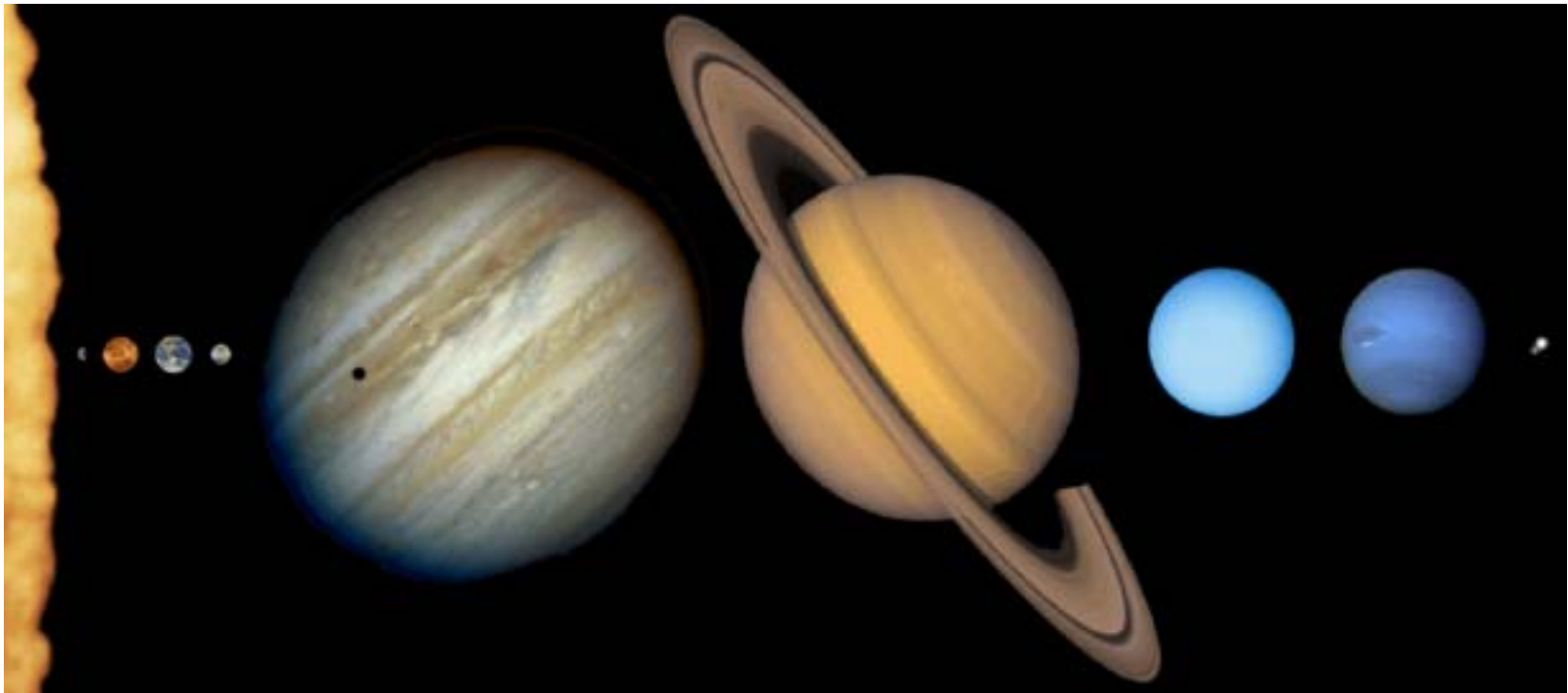


Chapter 6

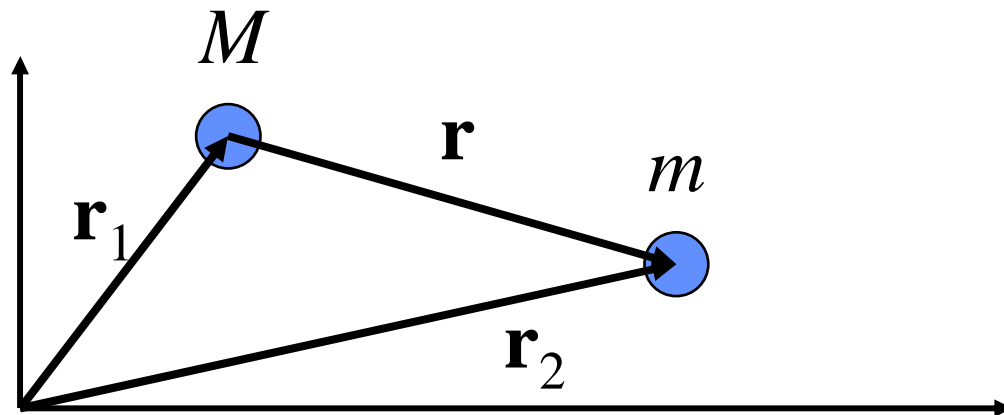
The Gravitational Force and the Gravitational Field



Newton's Law of Universal Gravitation

$$\mathbf{F} = -\frac{GMm}{r^2} \hat{\mathbf{r}}$$

- \mathbf{F} is the force of an object with mass M on an object with mass m
- $\hat{\mathbf{r}}$ is a unit vector pointing in the direction from the object with mass M to the object with m .
- G is Newton's universal gravitational constant with a value is $G = 6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$



Interactive Question

A spaceship is traveling to the moon. At what point is it beyond the pull of the earth's gravity? The mass of the moon is $1/80$ the mass of the earth, and the surface gravity of the moon is $1/6$ that of the earth.

- A) When it gets out of the atmosphere.
- B) When it is half-way there.
- C) When it is $5/6$ of the way there.
- D) When it is $79/80$ of the way there.
- E) It is never beyond the pull of earth's gravity.

Observations Regarding Universal Gravitation

1. The force acts over all space. Every object in the universe feels gravity from every other object in the universe.
2. Force is proportional to mass.
3. The force that one object feels from another is equal and opposite to the force the other object feel on the first one. (This is the same as Newton's third law). The acceleration of the two objects would be different if their mass is different.

When the objects are spherical, the distance “between” the objects is measured from their centers. One of the reasons Newton developed calculus to demonstrate that this was true.

Problem: (a) What is the magnitude of the gravitational force on the moon from the earth? (b) What is the magnitude of the gravitational force on the earth from the moon?

Newton's Reasoning

A. $a_c = v^2/r = (2\pi r/T)^2/r = 4\pi^2 r/T^2$

- $a_{\text{near Earth}} / a_{\text{at Moon}} = 9.81/0.00272 = 3600$

- $r_{\text{at Moon}} / r_{\text{near Earth}} = 3.84 \times 10^8 / 6.37 \times 10^6 = 60$

- $F = ma \Rightarrow F \propto 1/r^2$

B. a_g is independent of mass of object

$$F = ma_g$$

$$K(Mm)^n/r^2 = ma_g$$

$$a_g = K(Mm)^n/mr^2 = (K M^n m^{n-1})/r^2$$

$$\Rightarrow n = 1$$

The Universal Gravitational force is a vector, as are all forces, and must be added like vectors, by using components. When there are more than two objects involved, we determine each of the forces independently, and then add them together as vectors. This is the **principle of superposition.**

Problem: Find the total gravitational force on the moon from both the earth and sun, assuming that they are at right angles to each other as shown.



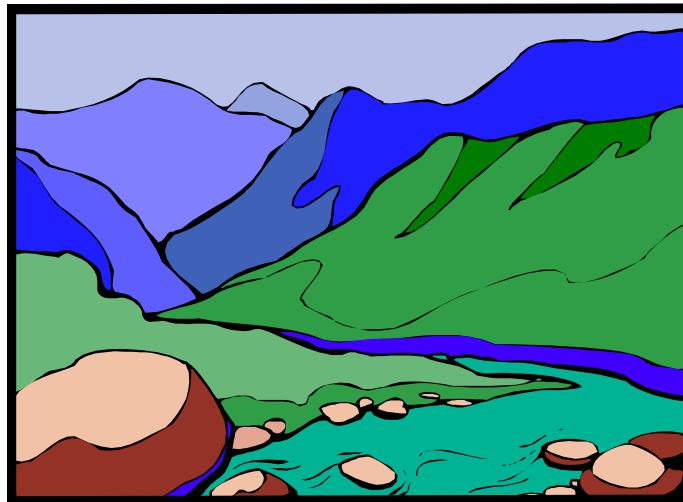
Gravity at the surface of the Earth

$$F_g = mg = Gmm_E/r_E^2$$

$$g = Gm_E/r_E^2$$

$$= (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg}) \div 6.38 \times 10^6 \text{ m}^2$$

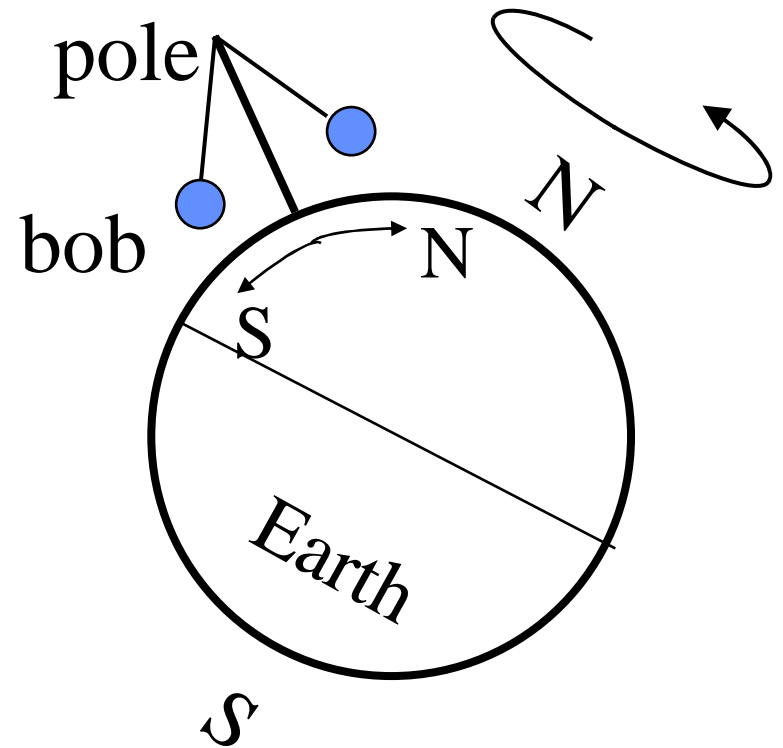
$$= 9.78 \text{ m/s}^2$$



Problem: How far above the earth must you go for the force you feel to be $1/2$ of what it is on the surface of the earth?

Interactive Question

A pendulum bob is suspended from a long pole in Oklahoma. When the pendulum is at rest, the combined action of gravitation and Earth's rotation makes the bob:



- A) deviate toward the east.
- B) deviate toward the west.
- C) deviate toward the north.
- D) deviate toward the south.
- E) point straight down toward the center of the Earth.

Gravitational and Inertial Mass

Let's look at Newton's second law, when there is only the force of gravity acting.

$$\Sigma \mathbf{F} = m\mathbf{a}$$

$$F_g = ma$$

$$Gmm_E/r_E^2 = ma$$

On the left hand side, m is the mass that interacts by the force of gravity, or the *gravitational mass*. On the right hand side m is the object's resistance to acceleration, or the *inertial mass*. Are these the same? Why should they be?

Near the surface of the earth: $mg = ma$

So $a = g$ for all objects only if these are the same.

Satellites and “Weightlessness”

Examples of objects having only the force of gravity acting on them.

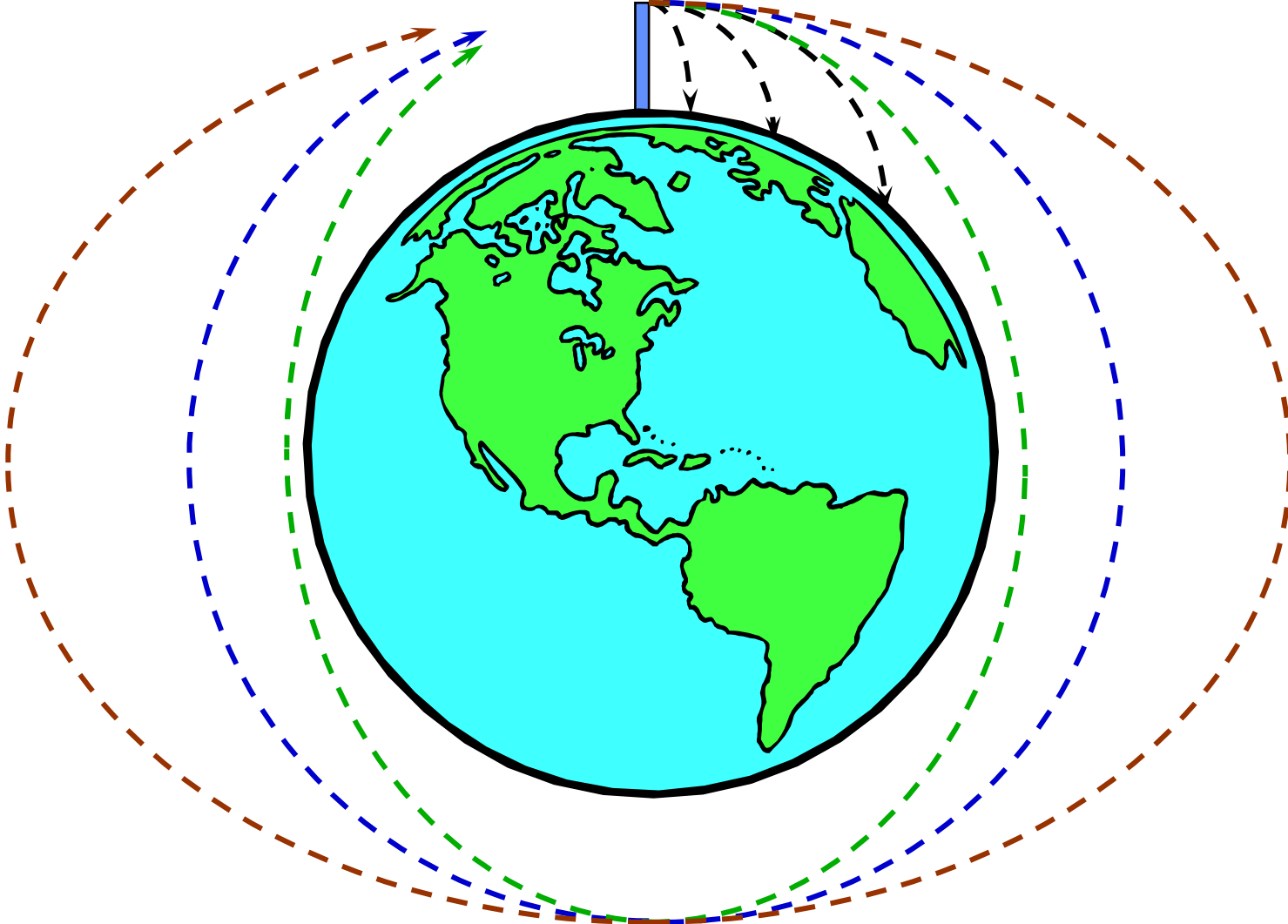


When an object is in freefall, we call it “weightless” because a scale would not read any weight. But the object still has weight (the force of gravity acting on it).

Interactive Question

The Moon does not fall to Earth because

- A) It is in Earth's gravitational field
- B) The net force on it is zero.
- C) It is beyond the main pull of Earth's gravity.
- D) It is being pulled by the Sun and planets as well as by Earth.
- E) None of the above.



Interactive Question

An astronaut is inside a space shuttle in orbit around the earth. She is able to float inside the shuttle because

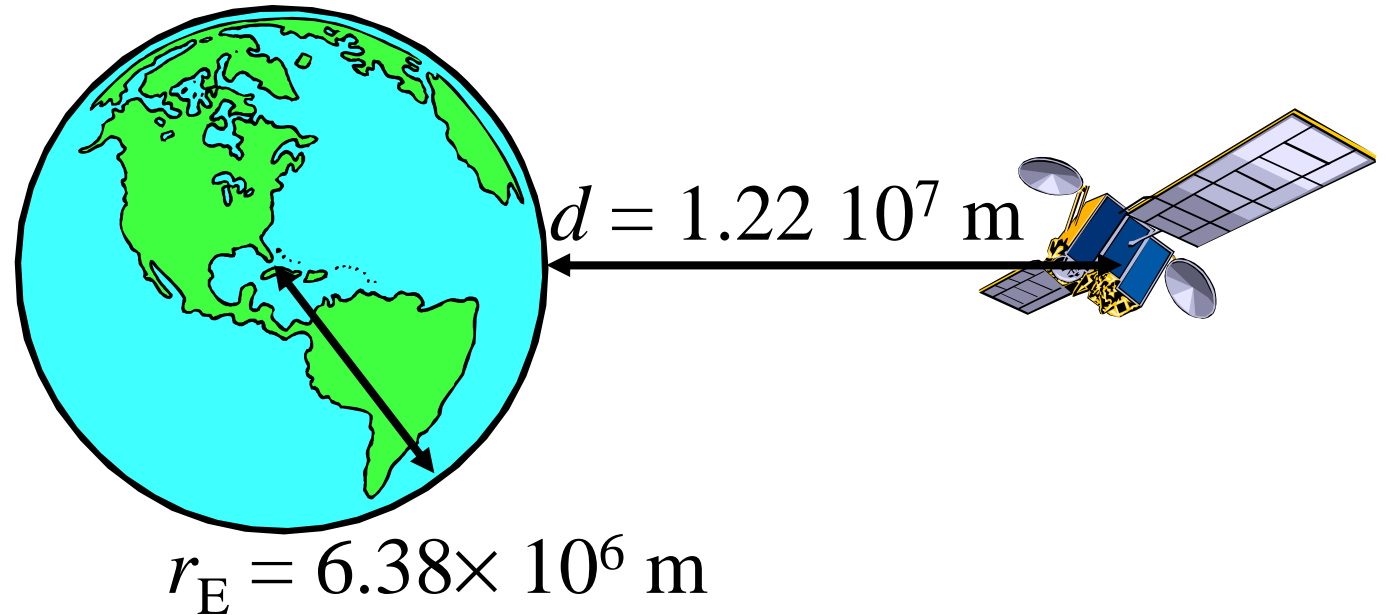
- A) her weight is zero, and her shuttle's weight is zero.
- B) the force of gravity is very small on the astronaut.
- C) she and her shuttle move with the same constant velocity.
- D) she and her shuttle move with the same centripetal acceleration.
- E) The force of the earth on the spaceship and the force of the spaceship on the earth cancel because they are equal in magnitude and opposite in direction.

Interactive Question

The earth exerts the necessary centripetal force on an orbiting satellite to keep it moving in a circle at constant speed. Which statement best explains why the speed of the satellite does not change even though there is a net force exerted on it?

- A) The acceleration of the satellite is zero.
- B) The centripetal force has a magnitude of mv^2/r .
- C) The centripetal force is canceled out by the reaction force.
- D) The centripetal force is always perpendicular to the velocity.
- E) The satellite is in equilibrium, which means there is no net force acting on it.

Problem: What is the speed of a satellite in orbit around the earth at a distance of 12,200 km above the surface of the earth.



- Note that all satellites at the same distance from the earth have the same speed, regardless of their mass.
- We often write the speed as a function of the period (T), the time it takes to make one orbit.

$$v = 2\pi r/T$$

$$T = 2\pi r/v$$

Problem: How long does it take the satellite in the previous problem to circle the Earth?

Interactive Question

Two satellites A and B of the same mass are going around the earth in concentric orbits. The distance of satellite B from center of the earth is twice that of satellite A . What is the ratio of the centripetal force acting on B to that acting on A ?

- A) $1/8$
- B) $1/4$
- C) $1/2$
- D) $1/\sqrt{2}$
- E) 1

Interactive Question

Two satellites A and B of the same mass are going around the earth in concentric orbits. The distance of satellite B from the center of the earth is twice that of satellite A .

What is the ratio of the tangential speed of B to that of A ?

A) $1/2$

B) $1/\sqrt{2}$

C) 1

D) $\sqrt{2}$

E) 2

Interactive Question

A satellite encircles Mars at a distance above its surface equal to 3 times the radius of Mars. The acceleration of gravity of the satellite, as compared to the acceleration of gravity on the surface of Mars, is

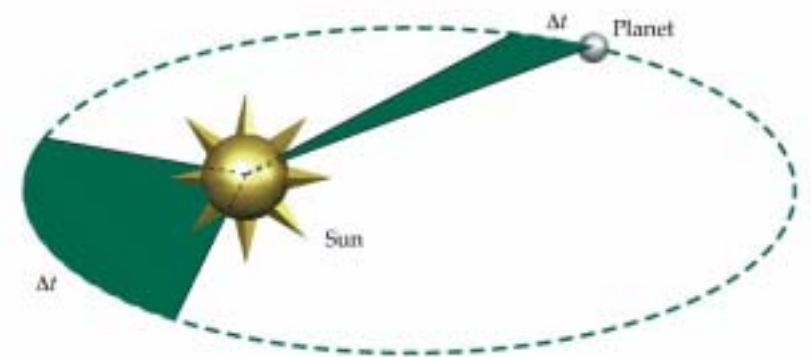
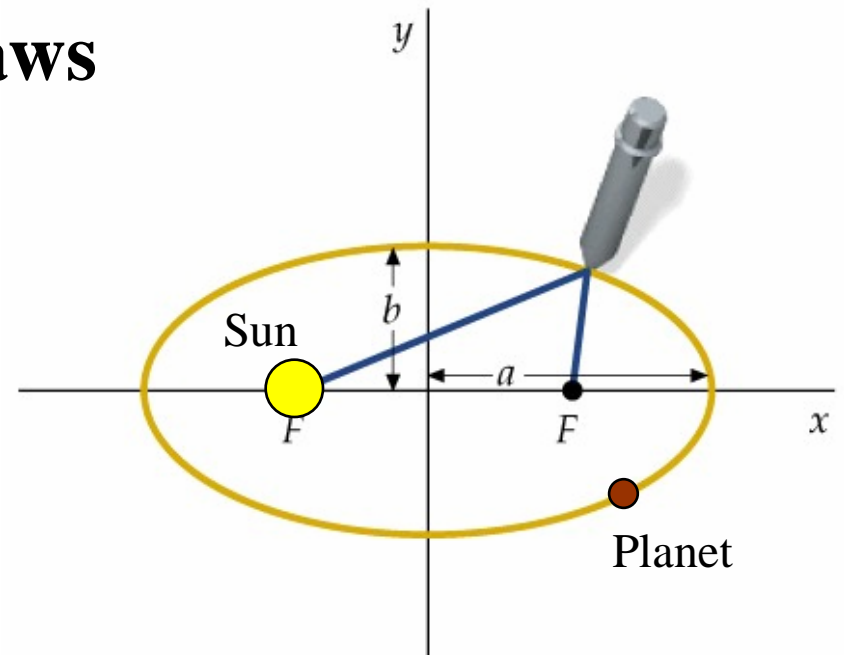
- A) zero.
- B) the same
- C) one-third as much.
- D) one-ninth as much.
- E) one-sixteenth as much.

Kepler's Laws

Long before Newton proposed his three laws of motion and his universal law of gravitation, Johannes Kepler had developed three laws that describe the motion of the planets. Newton showed that his law of universal gravitation **predicted** Kepler's three laws. It was an amazing prediction and helped convince people that Newton's law of universal gravitation was correct.

Kepler's Laws

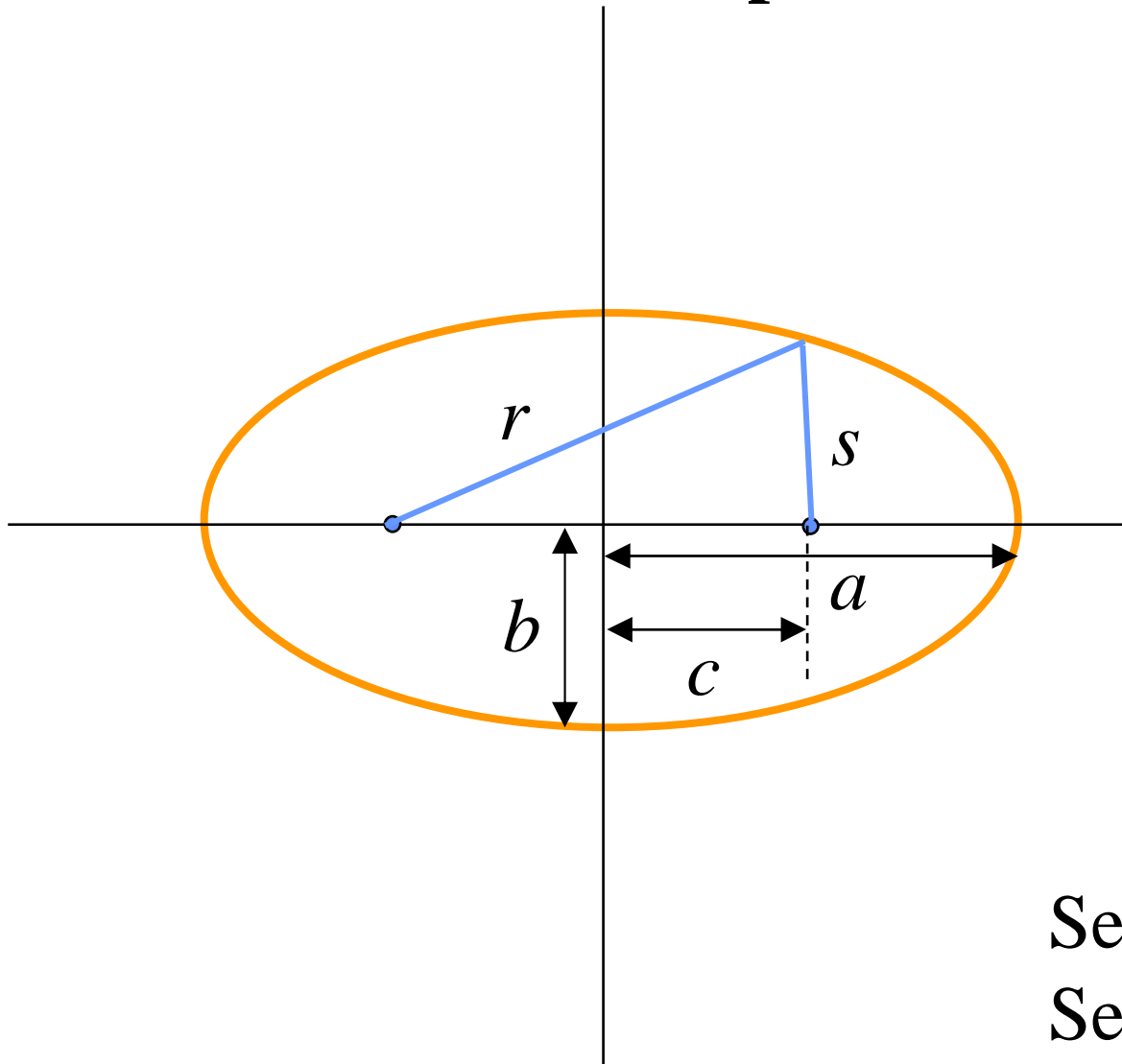
1. All planets move in elliptical orbits with the Sun at one focal point.
2. A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet is proportional to the cube of the average distance from the planet to the Sun.



$$T^2 \propto r^3$$

$$T_1^2/T_2^2 = r_1^3/r_2^3$$

Kepler's 1st Law



$$r + s = 2a$$

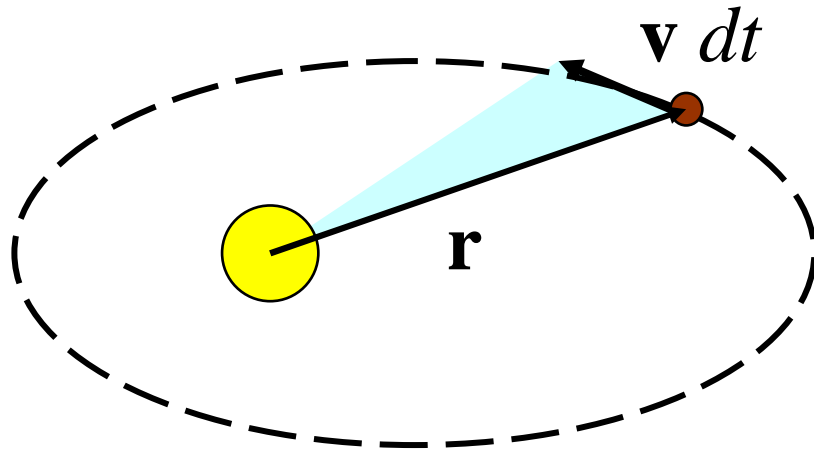
$$\varepsilon = c/a$$

Semi-major axis: a

Semi-minor axis: b

Eccentricity: ε

Let's derive **Kepler's 2nd Law**



$$\begin{aligned}dA &= (1/2)|\mathbf{r} \times \mathbf{v} dt| \\ &= (1/2m)|\mathbf{r} \times m\mathbf{v}| dt\end{aligned}$$

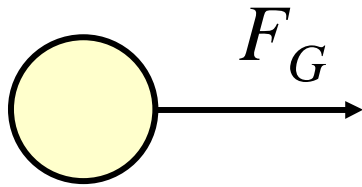
$$\begin{aligned}dA/dt &= (1/2m)|\mathbf{r} \times m\mathbf{v}| \\ dA/dt &= (1/2m) L\end{aligned}$$

Since there are no external torques, L cannot change, so the area swept out per unit time doesn't change. (We will see this in a later chapter).

This means that as a planet moves closer to the sun, its speed must increase, and vice versa. How can this be with a central force?

Let's derive **Kepler's 3rd Law**

for an object in a circular orbit using Newton's laws.



F_G is toward the
center of the sun

$$\Sigma F = ma$$

$$F_G = ma_c$$

$$GM_S m / r_1^2 = mv^2 / r = m(2\pi r / T)^2 / r$$

$$GM_S / r^3 = 4\pi^2 / T^2$$

$$T^2 = (4\pi^2 / GM_S) r^3$$

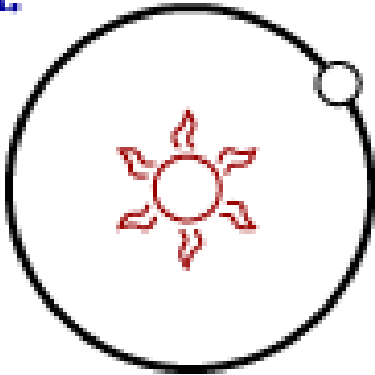
Problem: Saturn is about 9.7 times as far from the sun as the earth is. What is the length of Saturn's year?



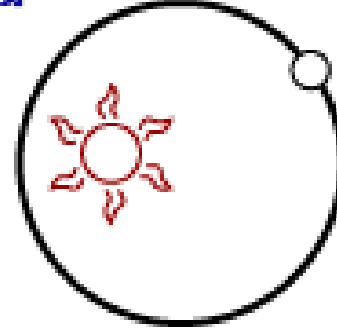
Interactive Question

Which of the following diagrams most accurately depicts the shape of Earth's orbit around the Sun?

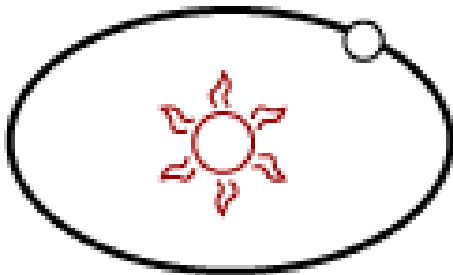
A.



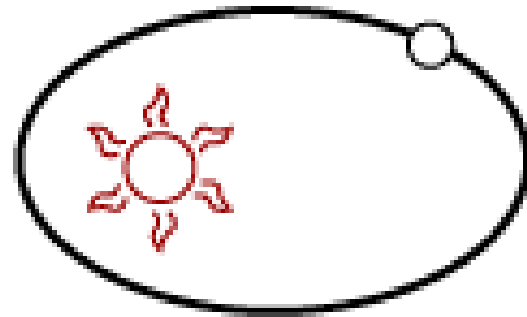
B.



C.



D.



The Gravitational Field

Many years after Newton lived, an idea was developed to explain how a force could act over a distance without two objects touching. The idea was that any single object can create a field that can be felt by another object. For the case of gravity, an object with mass can create a gravitational field that can be felt by other objects with mass. It is the interaction of the second object with the field that creates the force.

The relationship between the gravitational force (\mathbf{F}) and the gravitational field (\mathbf{g}) is given by

$$\mathbf{F}/m = \mathbf{g}$$

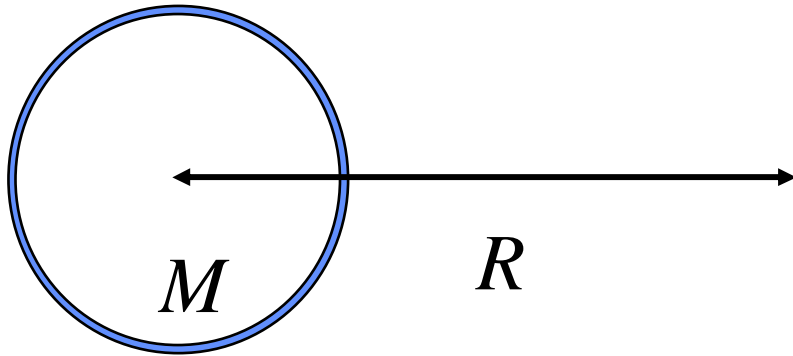
where \mathbf{g} is the gravitational field set up by the first object, (say with mass M), and m is the mass of the second object which feels the force \mathbf{F} . Using Newton's universal law of gravitation, we can determine what \mathbf{g} is for a single object.

$$\mathbf{g} = \frac{\mathbf{F}}{m} = -\frac{GMm}{mr^2} \hat{\mathbf{r}}$$

$$\mathbf{g} = -\frac{GM}{r^2} \hat{\mathbf{r}}$$

Problem: What is the gravitational field inside a thin spherical shell of uniform density?

What is the gravitational field outside any spherically symmetric object with total mass M ?



$$\mathbf{g} = -\frac{GM}{R^2} \hat{\mathbf{r}}$$

Of course the force on an object with mass m is found by multiplying the field by m , so

$$\mathbf{F} = -\frac{GMm}{R^2} \hat{\mathbf{r}}$$

Problem: What is the gravitational field inside a solid sphere with a total mass M and radius R of uniform mass density, at a distance r from the center?

