Chapter 4

Kinematics II: Motion in Two and Three Dimensions



Demonstrations:

- 1) Ball falls down and another falls out
- 2) Parabolic and straight line motion from two different frames. The truck with a dropping ball.
- 3) Monkey Gun

Multi-Dimensional Variables



A plot of y vs x is shown for a car driving through the Lloyd Noble parking lot. If the car starts at x=0 (point A) and drives to point C without stopping, which direction is the velocity at point B?

В

A



E

A plot of y vs x is shown for a car driving through the Lloyd Noble parking lot. If the car starts at x=0 and drives to point D without stopping, at which point is the velocity the greatest?



A ball hits a brick wall and bounces off as shown. Which arrow best represents the change in velocity Δv ?





B

E

A ball is thrown up in the air. The velocity vector is shown at four different points.

B)

A)

What is the direction of the average acceleration between points B and C?

B

E

A ball is thrown up in the air. The velocity vector is shown at four different points.

A

What is the direction of the average acceleration between points A and D?

<u>Problem:</u> A toy car rolls along a track from a position of (-2.0, 3.0, 0.0) meters to (5.0, 7.0, 3.0) meters in 3.5 seconds. What is the average velocity of the car?

<u>Problem:</u> A toy train moves along a track with a position described by the equation

 $\mathbf{r} = (.50 \text{ m/s})t \mathbf{i} + (.20 \text{ m/s}^2 t^2 - .50 \text{ m/s} t + .20 \text{ m})\mathbf{j}$ + (.25 m) \mathbf{k}

where *t* is in seconds.

- A) Find the train's average velocity between the 2nd and 4th second.
- B) Find equations for the instantaneous velocity and acceleration at all times.
- C) What is the velocity and acceleration after 4 seconds.

Projectile Motion

Projectile motion is the name we give to the motion of an object with a constant acceleration in one direction, and no acceleration in the other directions, like an object moving near the surface of the earth, if we neglect air resistance.

There is no acceleration in the horizontal direction, and the acceleration in the vertical direction is the same for all objects. The vertical and horizontal motions can be considered to act independently of each other.

A tennis ball is thrown upward at an angle from point A and follows a parabolic path as shown. (The motion is shown from the time the ball leaves the person's hand until just before it hits the ground.)



At what point is the vertical velocity equal to zero?

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At what point is the velocity equal to zero?

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At what point is the vertical acceleration equal to zero?

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At what point is the horizontal velocity equal to the horizontal velocity at A?

A) B
B) C
D
D) All of the above
E) None of the above

A tennis ball is thrown upward at an angle from point A and follows a parabolic path as shown. (The motion is shown from the time the ball leaves the person's hand until just before it hits the ground.)



At what point is the horizontal acceleration equal to zero?

For Projectile Motion

In this case, if we define +y as being vertical,

$$v_{y2} = v_{y1} - g(t_2 - t_1)$$

$$v_{y2}^2 = v_{y1}^2 - 2g(y_2 - y_1)$$

$$y_2 - y_1 = (1/2)(v_{y2} + v_{y1})(t_2 - t_1)$$

$$y_2 - y_1 = v_{y1}(t_2 - t_1) - (1/2)g(t_2 - t_1)^2$$

$$y_2 - y_1 = v_{y2}(t_2 - t_1) + (1/2)g(t_2 - t_1)^2$$

In the horizontal direction, there is no acceleration, $x_2 - x_1 = v_x(t_2 - t_1)$

A pilot drops a bomb from a plane flying horizontally. When the bomb hits the ground, the horizontal location of the plane will

- A) be behind the bomb.
- B) be over the bomb.
- C) be in front of the bomb.
- D) depend on the speed of the plane when the bomb was released.

<u>Problem:</u> You are working as a consultant on a video game designing a bomb site for a World War I airplane. In this game, the plane you are flying is traveling horizontally at 30.0 m/s at an altitude of 150 m when it drops a bomb. You need to determine how far from the target you should drop the bomb and at what angle the bomb site should be set to accurately hit a target.

Focus the Problem:

Speed = 30.0 m/sAltitude = 150 m

What is the problem asking for:



- (1) How far (horizontally) will the bomb travel?
- (2) What will be the angle between a horizontal line from where the plane is when the bomb is released to where the bomb hits?

<u>Outline the Approach</u>: Use the kinematic equations for constant acceleration in the vertical direction and no acceleration in the horizontal direction. The only important object is the bomb. The two important times are when the bomb is released and when it hits. The horizontal distance and altitude give the correct angle.

Describe the Physics:

Draw physics diagrams and define all quantities uniquely



Which defined quantity is your Target variable?

Quantitative Relationships:

$$v_{2} = v_{1} + a(t_{2} - t_{1}) \qquad v_{2}^{2} = v_{1}^{2} + 2a(x_{2} - x_{1})$$

$$x_{2} - x_{1} = (1/2)(v_{2} + v_{1})(t_{2} - t_{1})$$

$$x_{2} - x_{1} = v_{1}(t_{2} - t_{1}) + (1/2)a(t_{2} - t_{1})^{2}$$

$$x_{2} - x_{1} = v_{2}(t_{2} - t_{1}) - (1/2)a(t_{2} - t_{1})^{2}$$

<u>PLAN the SOLUTION</u> Construct Specific Equations

<u>Unknowns</u>



EXECUTE the PLAN

EVALUATE the ANSWER Is Answer Properly Stated?

Is Answer Unreasonable?

Answer Complete?

<u>Problem</u>: For the previous problem, calculatea) where the plane is when the bomb hits.b) the velocity of the bomb when it hits the ground.

<u>Problem</u>: You are watching a baseball game on television, when the batter hits a home run. The ball sails over the 390 foot sign on the center-left fence. It eventually lands about 50 feet beyond the fence and about 50 feet high into the stands. On the replay you notice that the ball left the bat at an angle of about 35°. You begin to wonder how fast the ball left the bat. Since you are a bright physics student, you decide that you will just sit down and figure it out.

<u>Problem</u>: Neglecting air resistance, at what angle should an object be launched in order for it to go the maximum distance on level ground?

If an object is thrown into the air near the earth and eventually lands on the ground at the same level that it started, what variable(s) are required to determine how long it is in the air?

A) x velocity (v_x)
B) x displacement (x-x₀)
C) initial y velocity (v_{y0})
D) y displacement (y-y₀)
E) More than one of the above variables

If an object is thrown into the air near the earth and eventually lands on the ground at a different level that it started, what variable(s) are required to determine how long it is in the air?

x velocity (v_x)
 initial y velocity (v_{y0})

2) x displacement (x-x₀)
4) y displacement (y-y₀)

A) 1 and 2
B) 1 and 3
C) 2 and 4
D) 3 and 4
E) either (A) or (D)

A battleship simultaneously fires two shells toward two enemy ships, one close by (A), and one for away (B). The shells leave the battleship at different angles and travel along the parabolic trajectories indicated. Which of the two enemy ships gets hit first?



A) A B) B
C) They both get hit at the same time.
D) It is impossible to tell from the information given

<u>Problem</u>: A hunter aims his gun at a monkey hanging on a branch. Just as the hunter shoots, the monkey drops from the branch to avoid the bullet. What happens?



t=0 when the hunter shoots and the monkey drops

$$\frac{v_{0b}}{v_{0by}} = v_{0b} \sin\theta$$
$$v_{0bx} = v_{0b} \cos\theta$$

Projectile Motion is Parabolic

For simplicity, set $x_0 = y_0 = 0$ and $t_0 = 0$

$$x = v_x t$$

$$y = v_{y0} t - (1/2)gt^2$$

$$t = x/v_x$$

$$y = v_{y0} x/v_x - (1/2)g (x/v_x)^2$$

$$y = (v_{y0}/v_x) x - (g/2v_x^2) x^2$$

This is the equation of a parabola: $y = Ax + Bx^2$

Relative Velocities

We can use vectors to solve problems involving two objects different velocities. The use of proper subscripts can be very helpful for sorting out which velocities are in which reference frame.

For instance if a plane is flying through the wind, we have the velocity of the plane relative to the ground v_{pg} , the velocity of the plane relative to the wind v_{pw} , and the velocity of the wind relative to the ground v_{wg} . When adding the relative velocities, always put the subscripts that are the same next to each other in the addition:

$$\mathbf{v}_{\mathrm{pg}} = \mathbf{v}_{\mathrm{pw}} + \mathbf{v}_{\mathrm{wg}}$$

<u>Problem:</u> A person looking out of a window of a stationary train notices that rain drops are falling at a rate of 5.0 m/s. When the train is moving, the drops are make an angle of 25° with the vertical. How fast is the train moving?



$$\mathbf{v}_{\mathrm{RT}} = ? \operatorname{m/s}_{0} / [\theta = 25^{\circ}]_{0} /$$

Using the subscript notation, we know that $\mathbf{v}_{TG} = \mathbf{v}_{TR} + \mathbf{v}_{RG} = -\mathbf{v}_{RT} + \mathbf{v}_{RG}$ <u>Problem:</u> A bicyclist is traveling 10 m/s north and a jogger is running 3 m/s at an angle of 40° west of north.
a) What is their velocity with respect to each other?
b) Does their relative velocity depend on their relative location?



A boat that can travel at 4 km/hr in still water crosses a river with a current of 2 km/hr. At what angle relative to the shore must the boat be pointed to go straight across the river?

A) 27°
B) 30°
C) 60°
D) 63°
E) 90°

A boat that can travel at 4 km/hr in still water crosses a river with a current of 2 km/hr. At what angle relative to the shore must the boat be pointed to get across the river quickest?

A) 27°
B) 30°
C) 60°
D) 63°
E) 90°

- <u>Problem:</u> An ultra-lightweight plane can fly 19 m/s in still air. The plane is trying to fly east, but there is a wind blowing 20° east of north at 15 m/s.
- a) What direction must the plane fly in order to go east?
- b) How fast is the plane going relative to the ground?

A rock is twirled on a string at a constant speed in a clockwise direction, as shown. What is the direction of the average acceleration between points P_1 and P_2 ?





A rock is twirled on a string at a constant speed in a clockwise direction, as shown. What is the direction of the average acceleration between points P_1 and P_2 ?





A 1500 kg car travels at a constant speed of 22 m/s around a circular track which has a radius of 80 m.

Which statement is true concerning this car?

- A) The velocity of the car is changing.
- B) The car is characterized by constant velocity.
- C) The car is characterized by constant acceleration.
- D) The car has a velocity vector that points along the radius of the circle.
- E) More than one of the above is true.

Some Definitions used in Rotation

Rigid bodies: Objects that have a definite unchanging shape.
Fixed axis of rotation: A single-nonchanging axis around which the object rotates.
Translational motion: Motion with a fixed direction of net force.

Rotational motion: Motion around an axis of rotation

Angular Displacement

Consider an object rotating about a fixed axis from point P_1 at time t_1 to point P_2 at time t_2 . The points are both the same distance r from the center of the circle. The object rotates through an angle θ , and covers an arclength s.



The angular displacement, in radians is, $\theta = s/r$

Usually we set counterclockwise rotations to have a positive angular displacement and clockwise rotations to have a negative angular displacement.

Note that $360^\circ = 2\pi$ radians.

Steve (S) and his brother Mark (M) are riding on a merrygo-round as shown.

- A) They have the same speed, but different angular velocities.
- B) They have the same speed, and the same angular velocities
- C) They have different speeds and different angular velocities.
- D) They have different speeds and the same angular velocity



Polar Coordinates and Unit Vectors $y + \frac{1}{r} = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}(y/x)$ $x = r \cos \theta$ $s = r\theta$ $y = r \sin \theta$

$$\hat{\mathbf{r}} = \frac{x}{r}\hat{\mathbf{i}} + \frac{y}{r}\hat{\mathbf{j}} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}}$$

$$\widehat{\mathbf{t}} = -\frac{y}{r}\widehat{\mathbf{i}} + \frac{x}{r}\widehat{\mathbf{j}} = -\sin\theta \,\widehat{\mathbf{i}} + \cos\theta \,\widehat{\mathbf{j}}$$

 $\widehat{\mathbf{r}} \cdot \widehat{\mathbf{t}} = -\cos\theta \sin\theta + \sin\theta \cos\theta = 0$ $\widehat{\mathbf{r}} \cdot \widehat{\mathbf{r}} = \widehat{\mathbf{t}} \cdot \widehat{\mathbf{t}} = 1$

Angular Displacement and Angular Velocity

Angular Displacement: $\Delta \theta = \theta_2 - \theta_1$ Often (as in previous slide), just denoted as θ

Angular Velocity: $\omega = \Delta \theta / \Delta t$

The instantaneous angular velocity is given by

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \qquad \left\{ \frac{\operatorname{rad}}{\operatorname{sec}} = \mathrm{s}^{-1} \right\}$$

Relationship Between Angular and Linear Velocity

$$\mathbf{r} = r \cos\theta \mathbf{i} + r \sin\theta \mathbf{j}$$

$$\mathbf{v} = dr/dt = r d(\cos\theta)/dt \mathbf{i} + r d(\sin\theta)/dt \mathbf{j}$$

$$= -r \sin\theta d\theta/dt \mathbf{i} + r \cos\theta d\theta/dt \mathbf{j}$$

$$= r d\theta/dt (-\sin\theta \mathbf{i} + \cos\theta \mathbf{j})$$

$$= r\omega \hat{\mathbf{t}}$$

This is the velocity in the tangential direction with magnitude: $v = r\omega$

The time it takes to move around the circumference of a circle is called the period, T. At a constant speed, v

 $v = \Delta r / \Delta t = 2 \pi R / T$ where *R* is the radius of the circle.

 $T = 2\pi R/v$

The frequency f is defined as the rate of rotation, f = 1/T $\omega = 2\pi f$ {s⁻¹ = (2 π rad/cycle)(cycles/s)}

Angular Acceleration

Angular Acceleration: $\alpha = \Delta \omega / \Delta t$

The instantaneous angular acceleration is given by

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2 \theta}{dt^2} \qquad \left\{ \frac{\operatorname{rad}}{\operatorname{sec}^2} = \mathrm{s}^{-2} \right\}$$

Angular and Linear Acceleration

$$\mathbf{a} = \frac{d\mathbf{v}(t)}{dt} = \frac{d(v\hat{\mathbf{t}})}{dt} = \frac{dv}{dt}\hat{\mathbf{t}} + v\frac{d\hat{\mathbf{t}}}{dt}$$
$$= \left(\frac{d(r\omega)}{dt}\right)\hat{\mathbf{t}} + v\frac{d(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j})}{dt}$$
$$= \left(r\frac{d\omega}{dt} + \omega\frac{dr}{dt}\right)\hat{\mathbf{t}} + v\left(-\cos\theta\,\frac{d\theta}{dt}\mathbf{i} - \sin\theta\,\frac{d\theta}{dt}\,\mathbf{j}\right)$$
$$= r\alpha\,\hat{\mathbf{t}} + v\omega(-\cos\theta\,\mathbf{i} - \sin\theta\,\mathbf{j})$$
$$= r\alpha\,\hat{\mathbf{t}} - v\omega\,\hat{\mathbf{r}}$$
$$= r\alpha\,\hat{\mathbf{t}} - r\omega^{2}\,\hat{\mathbf{r}}$$
$$\mathbf{a} = \mathbf{a}_{\mathrm{T}} + \mathbf{a}_{\mathrm{C}}$$

Centripetal and Tangential Acceleration

 $a_C = \omega^2 r = v^2/r$ with a direction toward the center of the circle. This gives the object's change in direction. An object moving at a uniform (constant) speed, but turning in a circle has this acceleration directed toward the center of the circle. Centripetal means "center-seeking."

 $a_T = r \alpha$

with a direction along the tangent of the circle. This gives the object's change in speed.

The total acceleration is the vector sum of the two accelerations: $\mathbf{a} = \mathbf{a}_c + \mathbf{a}_T$ which has a magnitude $a = (a_c^2 + a_T^2)^{1/2}$ A More Pictorial Look at Uniform Circular Motion Uniform circular motion is the motion of an object traveling at a constant (uniform) speed on a circular path.



The direction of this acceleration is toward the center of the circle. (Note the direction of the change of velocity).

A rock is twirled on a string at a constant speed. The direction of its acceleration at point P is





You and a friend are riding on a merry-go-round. You are sitting on a horse that is twice as far from the center of the merry-go-round as your friend. How does your centripetal acceleration relate to that of your friend's?

A) It is one-fourth as muchB) It is one-half as muchC) It is the sameD) It is twice as muchE) It is four times as much

Rotational Kinematics

The definitions of the angular quantities are analogous to the definitions of linear quantities with the following substitutions.

 $x \rightarrow \theta$

- $v \to \omega$
- $a \rightarrow \alpha$

Consequently: $\omega_{2} = \omega_{1} + \alpha \Delta t$ $\omega_{2}^{2} = \omega_{1}^{2} + 2 \alpha \Delta \theta$ $\Delta \theta = \omega_{1} \Delta t + (1/2) \alpha \Delta t^{2}$ $\Delta \theta = (1/2)(\omega_{1} + \omega_{2}) \Delta t$

- <u>Problem:</u> A compact disk with a radius of 6.0 cm starts from rest and accelerates to its final velocity of 3.50 rev/s in 1.50 s.
- a) What is the disk's average angular acceleration?
- b) How many rotations does the CD make while coming up to speed?
- c) What is the final speed of the outside of the disk?
- d) At this speed, what is the centripetal acceleration of the outside of the disk?

Vector Nature of Angular Quantities

The angular equivalent of all the linear quantities that are vectors are also vectors. (Actually, they are pseudovectors). The direction of the vector is chosen by the right hand rule.



- 1. Angular displacement is not really a vector because it does not act like a vector. In particular, two rotations performed one after another are not necessarily commutative when the rotations take place along different axes of rotation. We call angular displacement a pseudo-vector.
- 2. The relationship between angular quantities and linear quantities (like $v = \omega R$, $a_T = R\alpha$, and $a_C = \omega^2 R$), are not vector equations. The linear quantities do not point in the same direction as the angular quantities. For fixed-body rotation, the direction of ω and α will not change but the direction of v, a_T , and a_C will constantly be changing.

The record playing on the turntable is rotating clockwise as seen from above. After turning it off, the turntable is slowing down, but hasn't stopped yet. The direction of the acceleration at point P is





A car is speeding up as it enters the interstate on a circular entrance ramp as shown in the figure at right. What is the direction of the acceleration of the car when it is at the point indicated?



E)

Which of the following is true concerning the magnitude of non-zero centripetal and tangential accelerations for an object moving in a circle?

- A) It is possible to have a changing centripetal acceleration and a constant tangential acceleration.
- B) It is possible to have a constant centripetal acceleration and a constant tangential acceleration.
- C) It is possible to have a constant centripetal acceleration and a changing tangential acceleration.
- D) More than one of the above is true.
- E) None of the above.

<u>Problem:</u> While driving your car, you accelerate from 0 to 22 m/s along a road that turns exactly 90° in a circular path with a radius of 87 m.

- (a) Assuming constant magnitude tangential acceleration, what is a_{T} ?
- (b) What is a_c when your velocity is 15 m/s?
- (c) What is the magnitude and direction of the total acceleration at the point indicated in (c)?

A toy car rolls down a track and flies off the end. What direction is the instantaneous acceleration at points 1, 2 and 3?



