Chapter 3

Kinematics I: Rectilinear Motion

Displacement

Displacement is the net change in position:

$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$

$\mathbf{r}_2$ is the position at t_2 and $\mathbf{r}_1$ is the position at t_1 with t_2 occurring after t_1. Displacement can have a positive or negative sign.

In one-dimension (rectilinear motion):

$\Delta\mathbf{r} = (x_2 - x_1)\mathbf{i}$

Note that displacement is not the same as total distance traveled ($|d|$).

Average Velocity

$$\mathbf{v}_{\text{av}} = \frac{\mathbf{r}_2 - \mathbf{r}_1}{t_2 - t_1} = \frac{\Delta \mathbf{r}}{\Delta t}$$

In one dimension: $\mathbf{v}_{\text{av}} = \frac{x_2 - x_1}{t_2 - t_1}\mathbf{i} = \frac{\Delta x}{\Delta t}\mathbf{i}$

Writing only the *x* component: $v_{\text{av}} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$

Velocity is a vector and has a magnitude and direction. In one dimension, the only "direction" is the positive or negative direction. But in more than one dimension, the direction is more complicated. This direction doesn't really affect the problem in one dimension, but will be very important when we work in two or more dimensions.

Average speed

$$\text{Average Speed} = \frac{d}{\Delta t}$$

For an object traveling in one direction along a straight line, the average speed is the magnitude of the average velocity.

Interactive Question

You jog around a 400 m track in 100 seconds, returning to the place where you started. Which of the following statements is true?

A) Your average speed and average velocity are the same, and neither is zero.
B) Your average speed and average velocity are the same, and both are zero.
C) Your average velocity is zero, and your average speed is 4 m/s.
D) Your average speed is zero, and your average velocity is 4 m/s.

Instantaneous Velocity and Speed

Instantaneous velocity is defined as:

$$\mathbf{v} = \lim_{\Delta t \to 0} \frac{\mathbf{r}(t + \Delta t) - \mathbf{r}(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{dr}{dt}$$

In one-dimension: $\mathbf{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \mathbf{i} = \frac{dx}{dt} \mathbf{i}$

The absolute value of the magnitude of the instantaneous velocity is the instantaneous speed.
For example, the speedometer in your car gives your instantaneous speed, but not instantaneous velocity.

When discussing velocity and speed, we will always mean instantaneous velocity or speed, unless explicitly stated otherwise.

Interactive Question

Which physical quantity is not correctly paired with its SI unit and dimension?

Quantity	Unit	Dimension
A) velocity	m/s	[L]/[T]
B) path length	m	[L]
C) speed	m/s	[L]/[T]
D) displacement	m/s^2	$[L]/[T]^2$
E) speed $\times$ time	m	[L]

Relative Velocity

Velocity is always measured relative to some fixed axis. Often, in everyday life, that axis is the earth. Relative velocity usually refers to the velocity of two objects relative to each other when they are both moving relative to a third object, say, the earth.

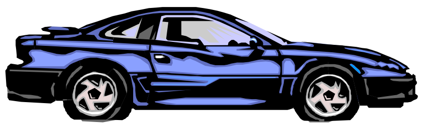
Problem Solving Steps

1. Think about the problem
   A. Read the problem twice carefully.
   B. Draw a detailed picture of the situation.
   C. Write down what the problem is asking for.
   D. Think about the physics principles and determine the approach to use.
2. Draw a “physics diagram” and define variables.
   A. Write down what is given in the problem.
   B. Determine which equations can be used.
3. Do the calculation.
   A. It is a good idea to start with the “target” variable
   B. Do algebra before using numbers
   C. Check the units.
4. Think about the answer.
   A. Is it reasonable? (Order of magnitude)

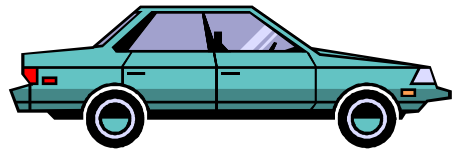
Problem: You are going to meet some friends at the Arbuckle mountains, about 110 km south of Norman. Your friends merge onto I-35 exactly three minutes before you. They are traveling 105 km/hr. If you are traveling 115 km/hr, will you catch up to them before they reach the Arbuckles? If so, where will you catch them? If not, how much later than them will you arrive?

Focus the Problem:

My car



Friend's car



3 minutes ahead of me

Speed = 115 km/hr

Speed = 105 km/hr

110 km

distance

What is the problem asking for:

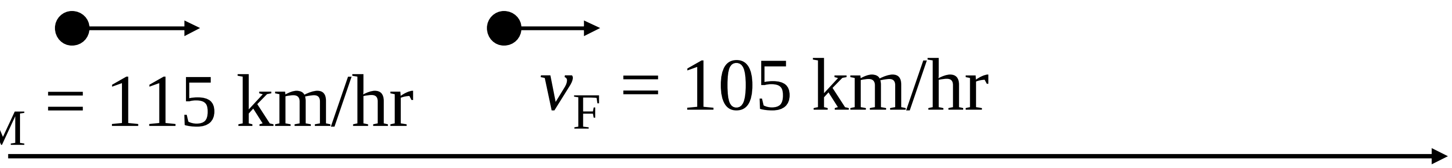
Will I reach my friend's car before 110 km. If not, when will I arrive compared with them?

Outline the Approach: Determine what point the two cars will meet, and compare it to 110 km. If it is after 110 km, then compare the location of cars when my friend arrives and calculate the time it takes for me to go the last few miles.

Describe the Physics:

Draw physics diagrams and define all quantities uniquely

My car　　　　Friend's Car



$v_M = 115$ km/hr　　$v_F = 105$ km/hr　　$+x$

$t_1 = 0$ s　　$t_1 = 0$ s　　$t_2 = ?$ s

$x_{M1} = 0$ m　　$x_{F1} = ?$　　$x_{M2} = x_{F2} = x_2 = ?$

$t_0 = -3 \text{ min} = -.050 \text{ hr}$

$x_{F0} = 0$ m

We have 3 unknown quantities

Which defined quantity is your target variable? x_2

Write equations you will use to solve this problem.

$v = (x_2 - x_1)/(t_2 - t_1)$

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

		Unknowns
$x_2 - x_{M1} = v_M(t_2 - t_1)$		x_2, t_2
$x_2 = v_M t_2$	(1)	
$x_2 - x_{F1} = v_F(t_2 - t_1)$		x_{F1}
$x_2 - x_{F1} = v_F t_2$	(2)	
$x_{F1} - x_{F0} = v_F(t_1 - t_0)$		
$x_{F1} = -v_F t_0$	(3)	

We have three equations and three unknowns. The problem is now just algebra:

$x_2 = v_M t_2$ (1)

$x_2 - x_{F1} = v_F t_2$ (2)

$x_{F1} = -v_F\, t_0$ (3)

From (1): $t_2 = x_2/v_M$ (4)

Plug (4) and (3) into (2)

$x_2 = v_F t_2 + x_{F1}$

$x_2 = v_F\, x_2/v_M - v_F\, t_0$

$x_2 - v_F x_2/v_M = -v_F\, t_0$

$x_2\,(1 - v_F/v_M) = -v_F\, t_0$

$x_2 = -v_F\, t_0 \div (1 - v_F/v_M)$

Check Units:

[L] ={[L]/[T]}[T]) ÷ 1 = [L]

Units are good!

EXECUTE the PLAN

$x_2 = -v_F\, t_0 \div (1 - v_F/v_M)$

$= -(105\text{ km/hr})(-0.050\text{ hr}) \div \{1-(105\text{ km/hr})/(115\text{ km/hr})\}$

$= 60.4\text{ km}$

EVALUATE the ANSWER

Is Answer Properly Stated?

No. The answer is that I will catch my friend 60.4 km from Norman

Is Answer Unreasonable?

We can check that this works.

$t_2 = x_2/v_M = 60.4\text{ km} \div 115\text{ km/hr} = 0.525\text{ hr}$

$t_2 - t_0 = (x_2 - x_0)/v_F$

$t_2 = x_2/v_F + t_0 = (60.4\text{ km} \div 105\text{ km/hr}) - 0.050\text{ hr} = 0.525\text{ hr}$

Is Answer Complete?

Yes, we would meet 60.4 km from Norman

Let’s solve this problem using the concept of relative velocity. “Focusing the Problem” and “Describing the Physics” are the same. There are extra equations to use.

PLAN the SOLUTION

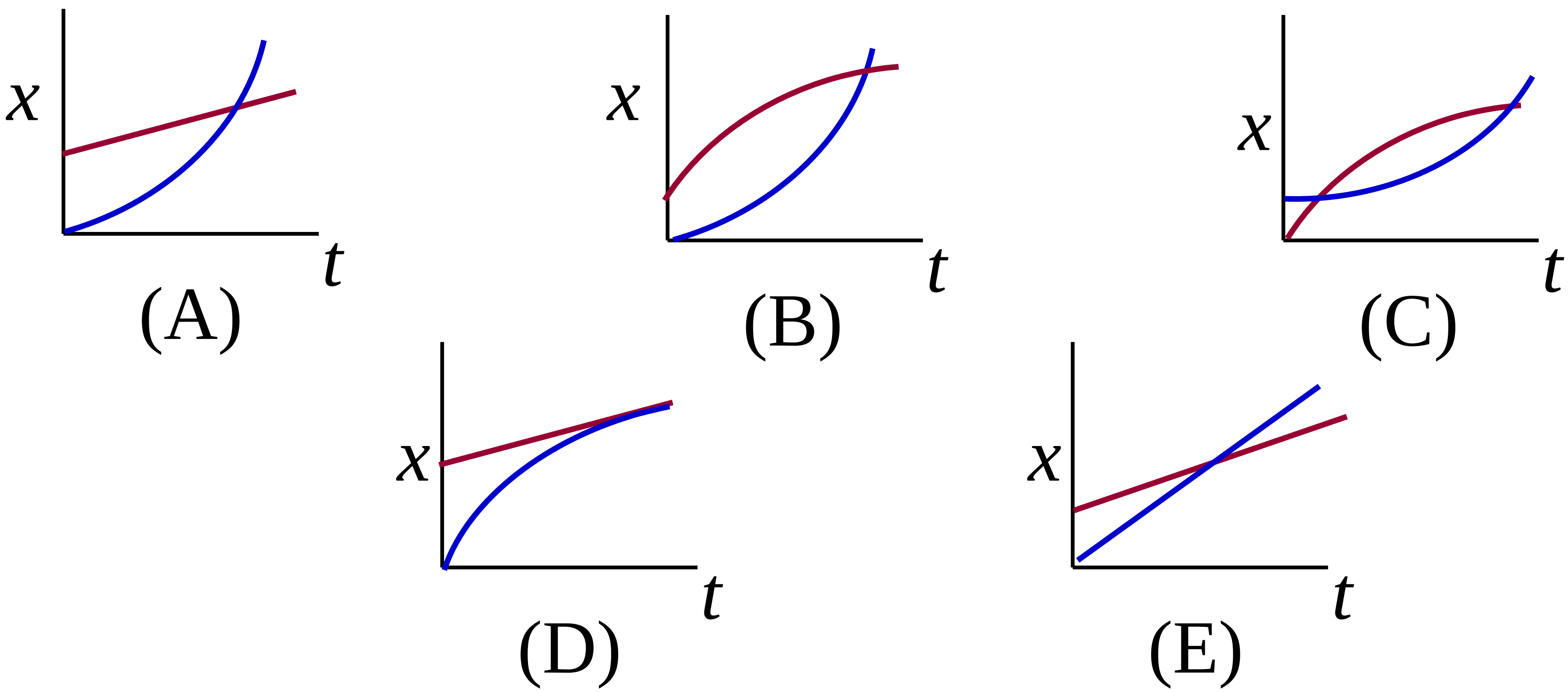
Construct Specific Equations (Same Number as Unknowns)

Unknowns

We have three equations and three unknowns.

Interactive Question

You are going to meet some friends at the Arbuckle mountains, about 110 km south of Norman. Your friends merge onto I-35 exactly two minutes before you. They are traveling 105 km/hr. You are traveling 115 km/hr. Which graph below describes this situation?



Average Acceleration

$$\mathbf{a}_{\mathrm{av}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t}$$

In one dimension: $$\mathbf{a}_{\mathrm{av}} = \frac{v_{x2} - v_{x1}}{t_2 - t_1}\mathbf{i} = \frac{\Delta v_x}{\Delta t}\mathbf{i}$$

Writing only the *x* component: $$a_{x\,\mathrm{av}} = \frac{v_{x2} - x_{x1}}{t_2 - t_1} = \frac{\Delta v_x}{\Delta t}$$

Since acceleration is a vector with a magnitude and direction, then an object is accelerating if it changes speed and/or direction.

Acceleration is the rate at which velocity changes, while velocity is the rate at which position changes.

Problem: You are driving 35 mi/hr when a dog runs across your path. You slam on your brakes and in 2.0 seconds slow to 10 mi/hr. What was your average acceleration in m/s^2?

Instantaneous acceleration

$$\mathbf{a} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}$$

When discussing acceleration, we will always mean instantaneous acceleration, unless explicitly stated otherwise.

Interactive Question

A car travels in a straight line covering a total distance of 90.0 miles in 60.0 minutes. Which statement concerning this situation is true?

A) The velocity of the car is constant.
B) The acceleration of the car must be non-zero.
C) The first 45 miles must have been covered in 30 minutes.
D) The speed of the car must be 90 miles per hour throughout the entire trip.
E) The average velocity of the car is 90 miles per hour in the direction of motion.

Interactive Question

Suppose that an object is moving with constant acceleration. Which of the following is an accurate statement concerning its motion?

A) In equal times its speed increases by equal amounts

B) In equal times its velocity changes by equal amounts.

C) In equal times it moves equal distances.

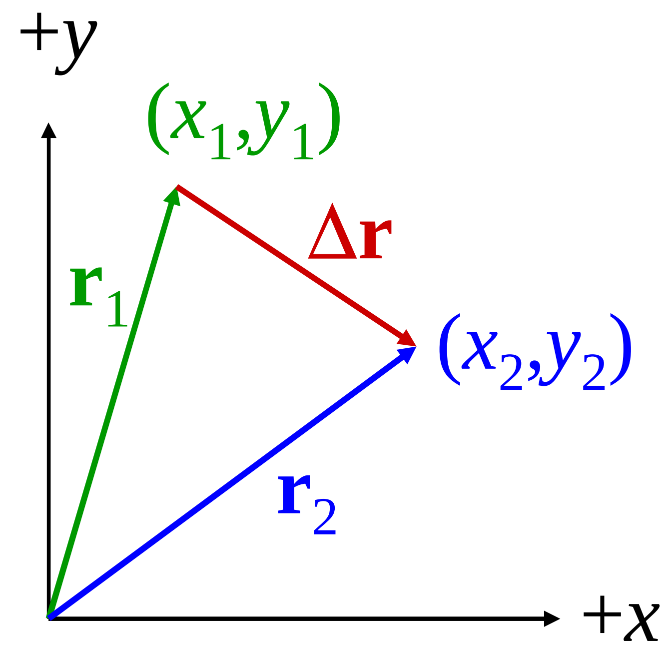
D) All of the above are true.

E) None of the above are true.

Multi-Dimensional Variables and Differences

$$\mathbf{r}_1 = x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}.$$

$$\mathbf{r}_2 = x_2\mathbf{i} + y_2\mathbf{j} + z_2\mathbf{k},$$



$$\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1 = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}.$$

If this change in position happens during a time interval Δt, then the average velocity is given by .

$$\mathbf{v}_{\mathrm{av}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

The instantaneous velocity is given by,

$$\mathbf{v} = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

If the velocity changes from $\mathbf{v}_1$ to $\mathbf{v}_2$ in a time interval Δt, then the average acceleration is given by .

$$\mathbf{a}_{\mathrm{av}} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1} = \frac{\Delta \mathbf{v}}{\Delta t}$$

The instantaneous acceleration is given by,

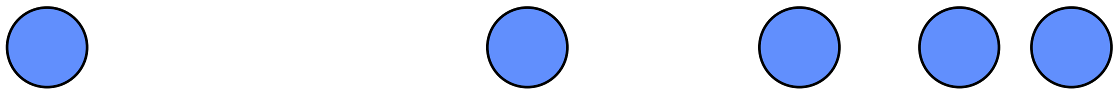
$$\mathbf{a} = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j} + \frac{dv_z}{dt}\mathbf{k} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

Motion Diagrams

A snapshot of an object at different times. A motion diagram will usually include quantitative information about the object’s position, velocity, and acceleration.

Interactive Question

The picture below shows snapshots of an object taken at equal time intervals. Which statement is true?

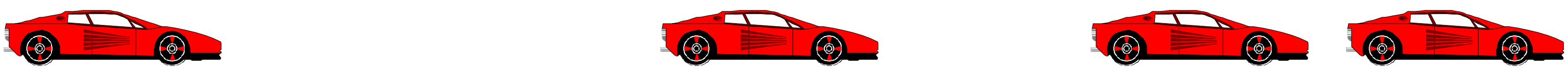


A) The object is definitely moving to the right
B) The object is definitely moving to the left
C) The object is definitely speeding up
D) The object is moving at a constant speed
E) None of the above is necessarily true

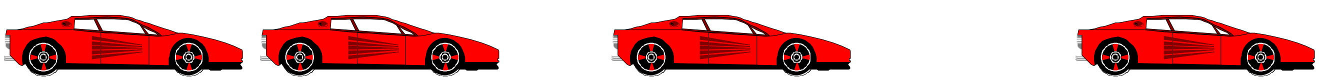
Interactive Question

The picture below shows snapshots of four cars taken at equal time intervals. If the cars are moving forward, which car has the greatest magnitude of acceleration?

1) 

2) 

3) 

4) 

A) Car 1

B) Car 2

B) Car 3

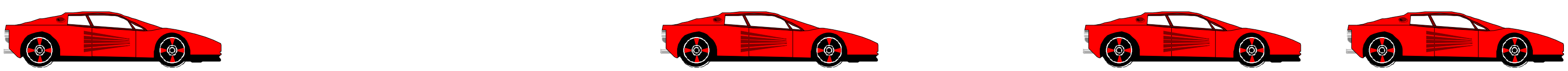
D) Car 4

E) Car 1 and 3 tie

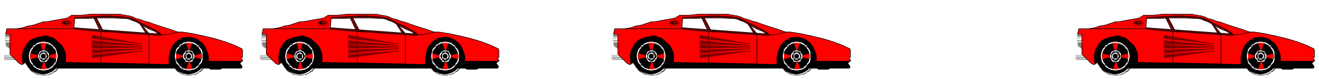
Interactive Question

The picture below shows snapshots of four cars taken at equal time intervals. If the cars are moving forward, which car has the smallest magnitude of acceleration?

1) 

2) 

3) 

4) 

A) Car 1

B) Car 2

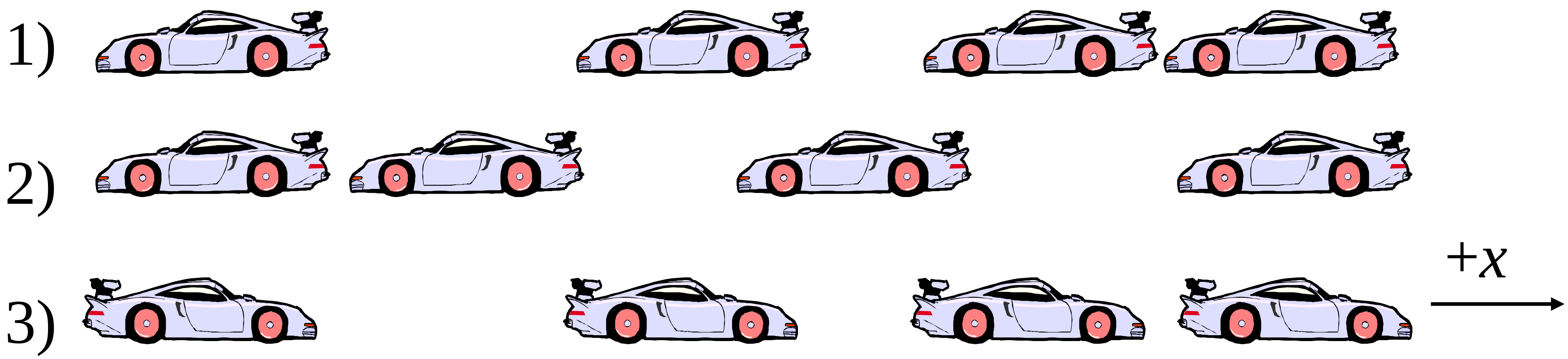
B) Car 3

D) Car 4

E) Car 1 and 3 tie

Interactive Question

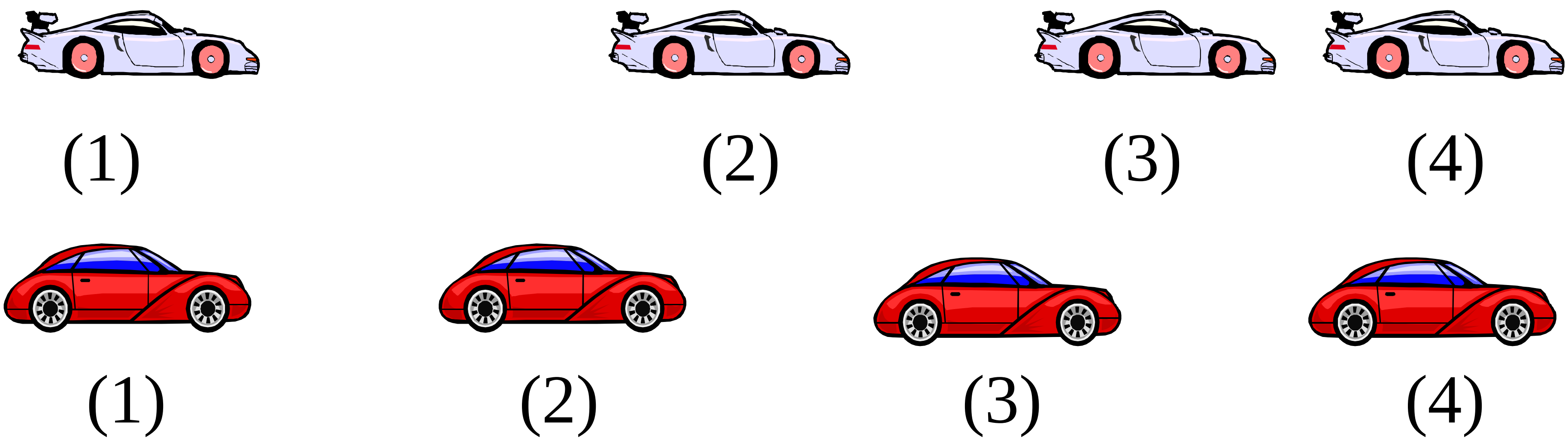
The picture below shows snapshots of four cars taken at equal time intervals. If the cars are moving forward, which car has a negative acceleration and which car is slowing down?



	Negative Acceleration	Slowing Down
A)	Car 3 only	Car 3 only
B)	Car 1 only	Car 2 only
C)	Car 2 only	Car 1 only
D)	Car 1 and 3	Car 2 and 3
E)	Car 2 and 3	Car 1 and 3

Interactive Question

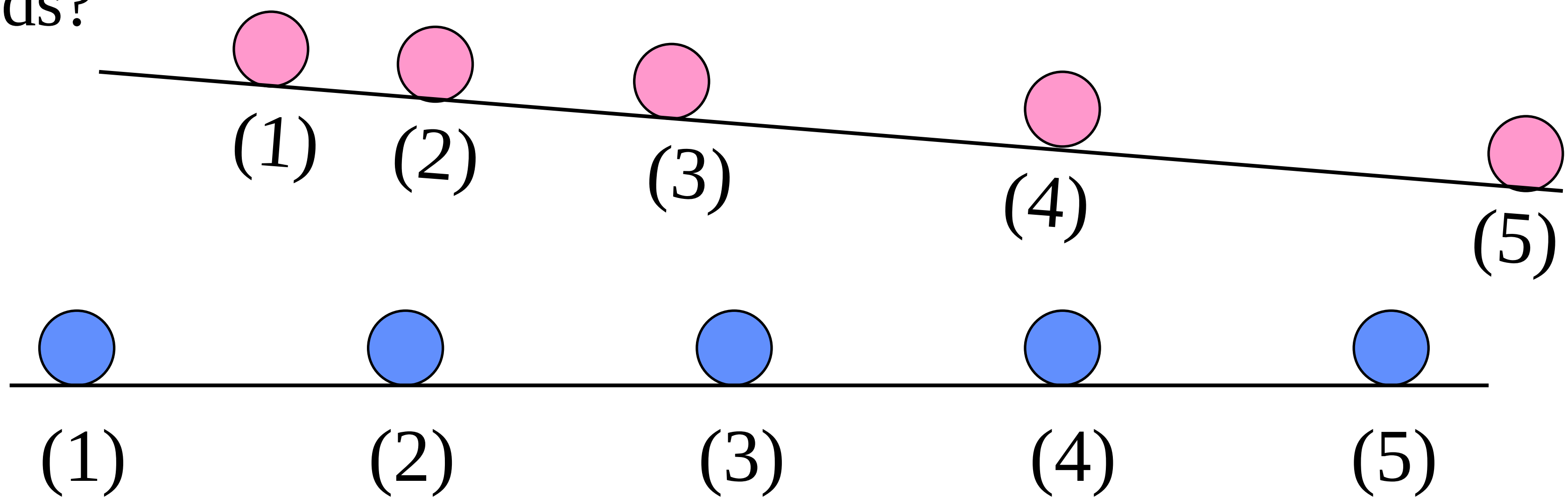
Consider the two cars shown with four pictures taken at the same equal time intervals for each car. At which point(s) do the two cars have equal speeds?



A) Point 1 only
B) Point 1 and 4
C) Point 2
D) Point 3
E) Somewhere between point 2 and 3

Interactive Question

Consider two balls, one rolling down a ramp and one rolling along the floor. At which point(s) do the two balls have equal speeds?



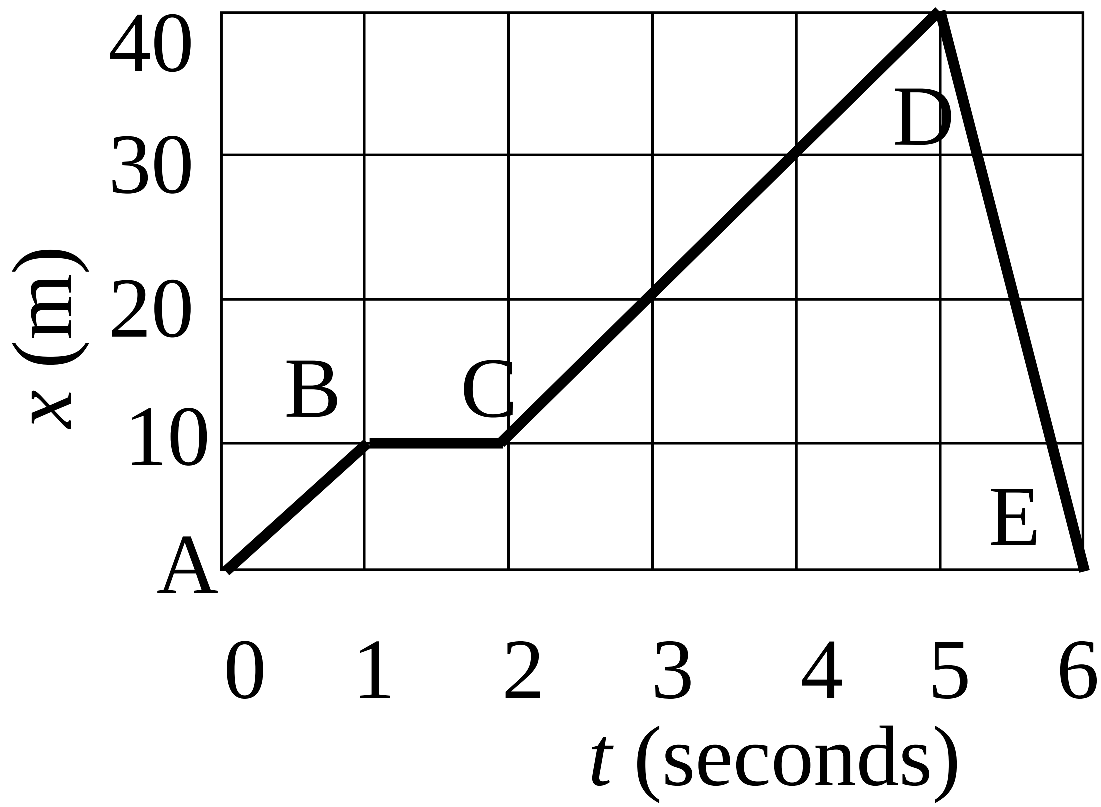
A) Point 2 only
B) Points 2 and 5
C) Somewhere between points 2 and 3
D) Somewhere between points 3 and 4
E) Either C or D

Analyzing Motion on a Graph

- Look carefully at what the axes on the graph represent.
- Look carefully at what is constant, and what is changing linearly (at a constant rate).
- Determine what the slope represents.
  - Example: If the vertical axis is displacement x and the horizontal axis is the time t, then the slope is $\Delta x/\Delta t = v$, or $dx/dt = v$ (the slope is the velocity).
  - If the vertical axis is velocity v, and the horizontal axis is time, t, then the slope is given by $\Delta v/\Delta t = a$, or $dv/dt = a$ ((the slope is the acceleration).

Interactive Question

An object is moving along a straight line. The graph at the right shows its position from the starting point as a function of time.

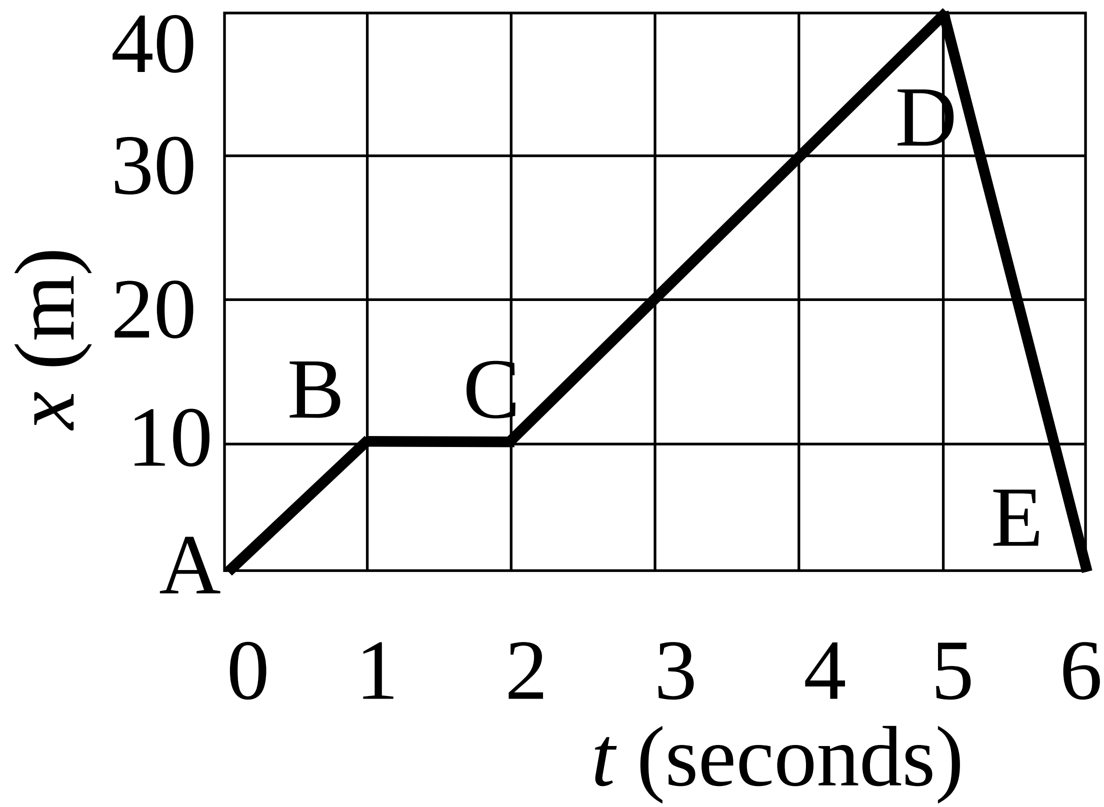


In what section of the graph does the object have the fastest speed?

A) AB
B) BC
C) CD
D) DE
E) AB and CD

Interactive Question

An object is moving along a straight line. The graph at the right shows its position from the starting point as a function of time.



What was the instantaneous velocity of the object at t = 4 seconds?

A) +6.0 m/s

B) +8.0 m/s

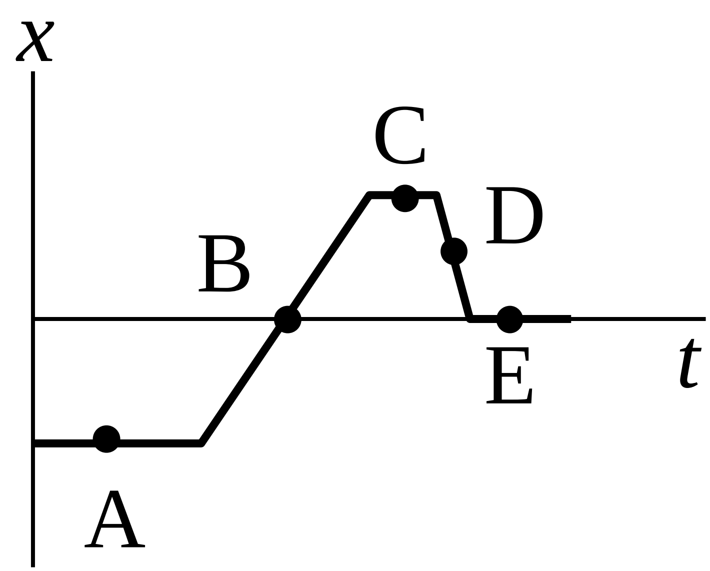
C) +10.0 m/s

D) + 13.3 m/s

E) +40 m/s

Interactive Question

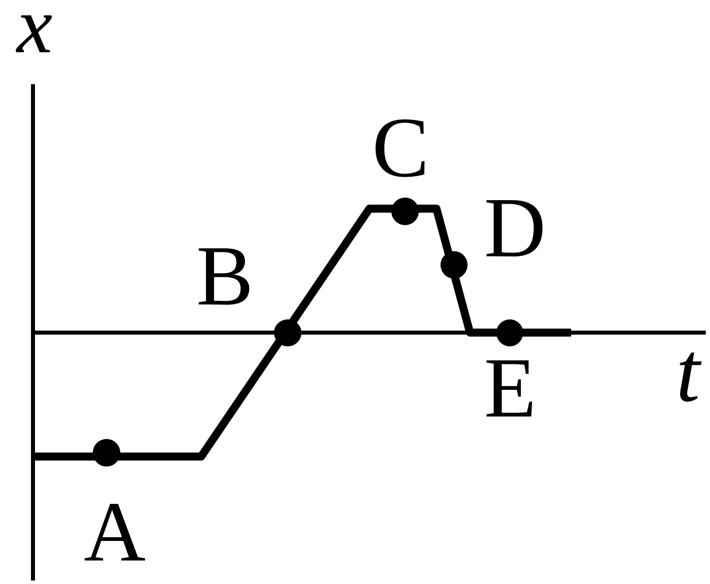
Consider the plot of x vs t at the right, at which point(s) is the motion slowest?



A) A
B) B
C) D
D) E
E) More than one of the above answers

Interactive Question

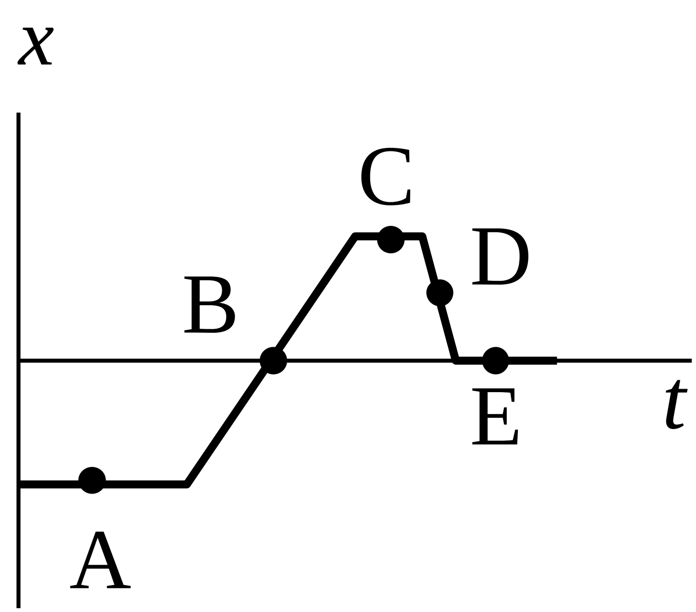
Consider the plot of *x* vs *t* at the right, at which point(s) is the object moving at a constant non-zero velocity?



A) A and C
B) A, C and D
C) C only
D) D only
E) B and D

Interactive Question

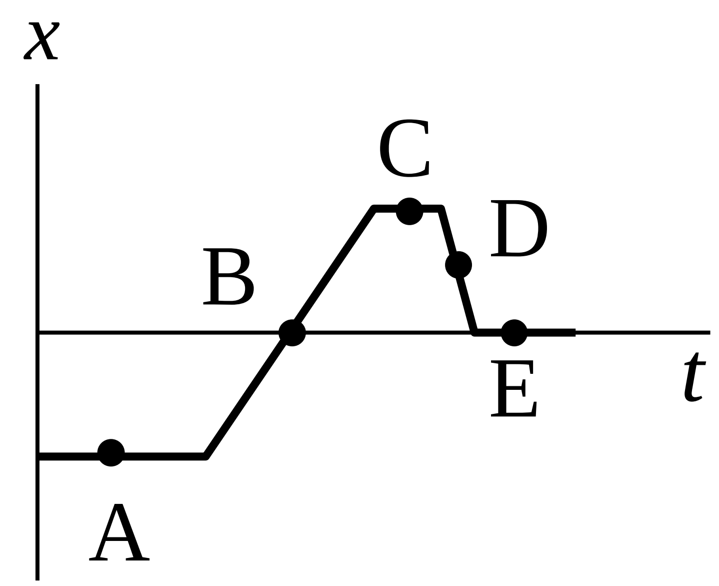
Consider the plot of x vs t at the right, at which point(s) is the object moving in the negative x direction?



A) A
B) B
C) C
D) D
E) E

Interactive Question

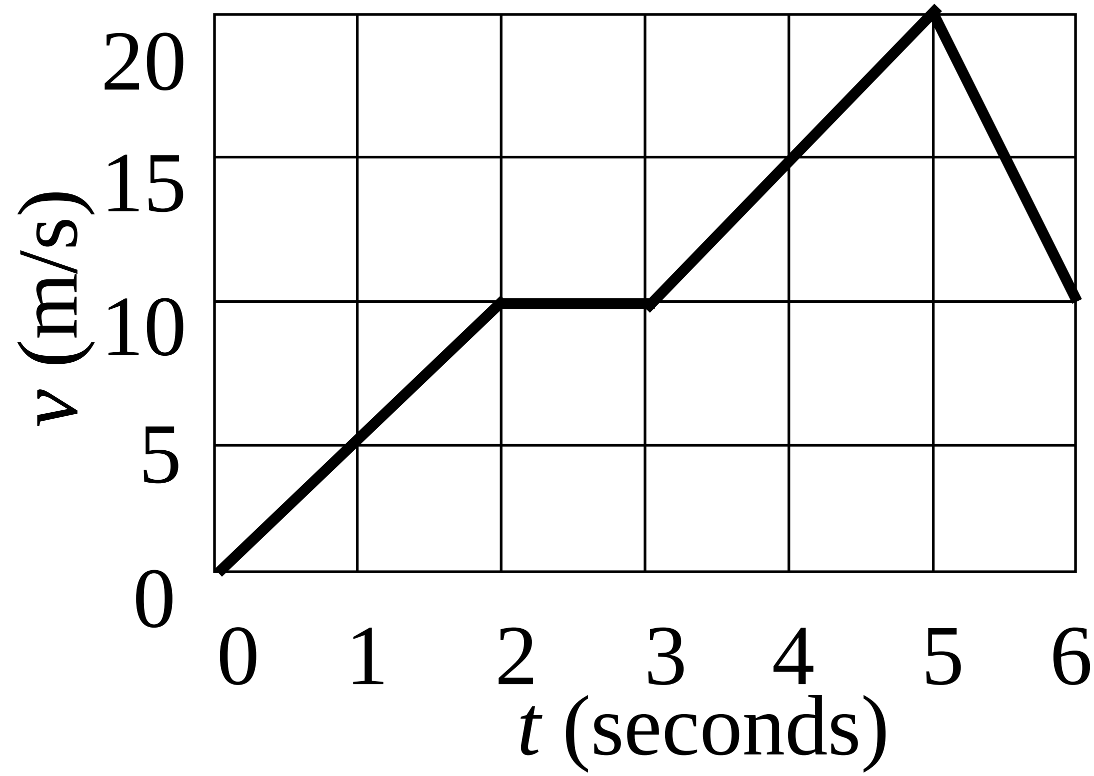
Consider the plot of *x* vs *t* at the right, at which point(s) is the object turning around?



A) A
B) B
C) C
D) D
E) E

Interactive Question

An object is moving along a straight line. The graph at the right shows its velocity as a function of time.



During which interval(s) of the graph does the object travel ***equal distances in equal times***?

A) 0 to 2 s

B) 2 s to 3 s

C) 3s to 5 s

D) 0 to 2 s and 3 s to 5 s

E) 0 to 2 s, 3 s to 5 s, and 5 s to 6 s

Interactive Question

The graph shows position as a function of time for two trains running on parallel tracks. Which is true:

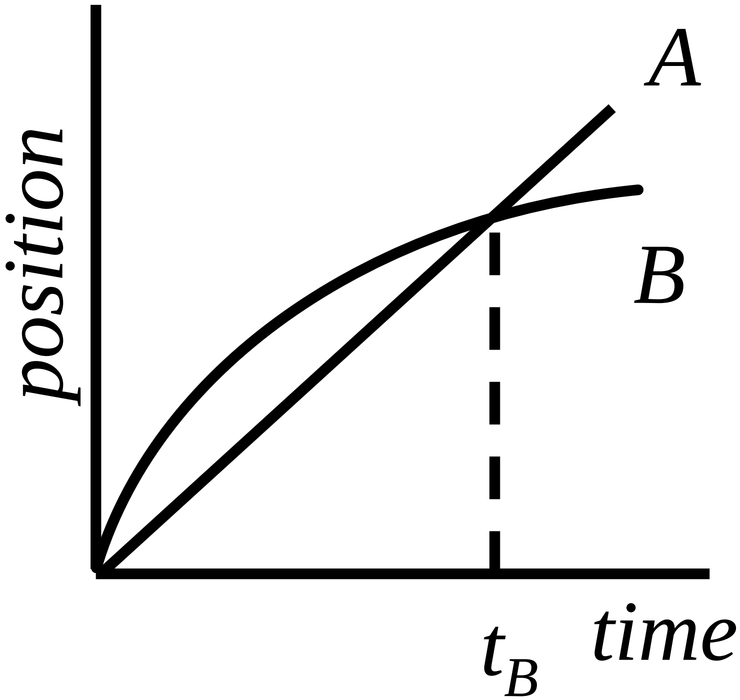
A) At time t_B both trains have the same velocity.

B) Both trains speed up all the time.

C) Both trains have the same velocity at some time before t_B.

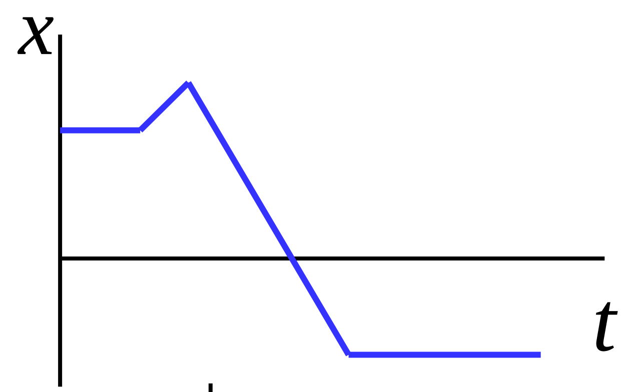
D) Somewhere on the graph, both trains have the same acceleration.

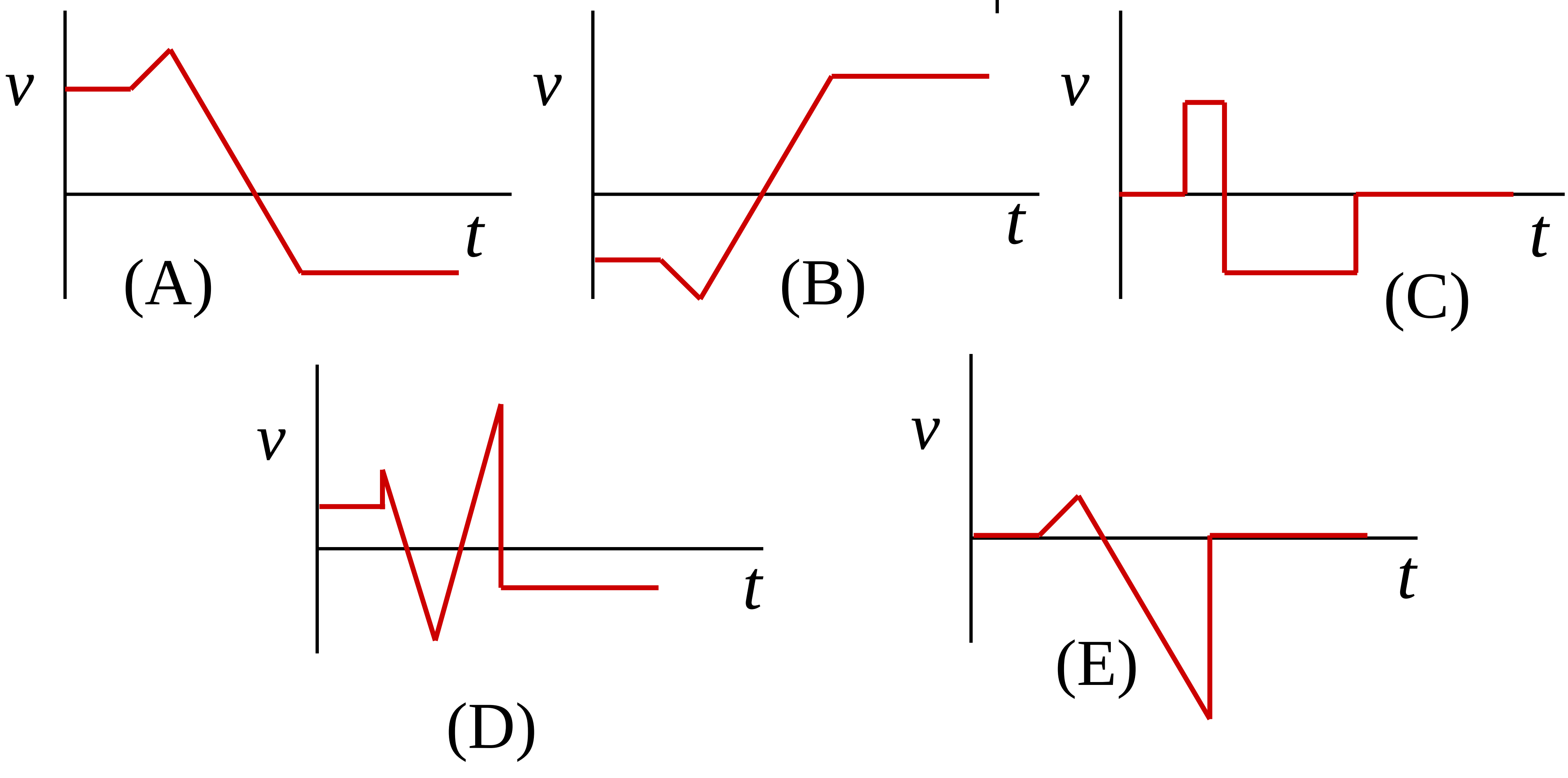
E) More than one of the above is true.



Interactive Question

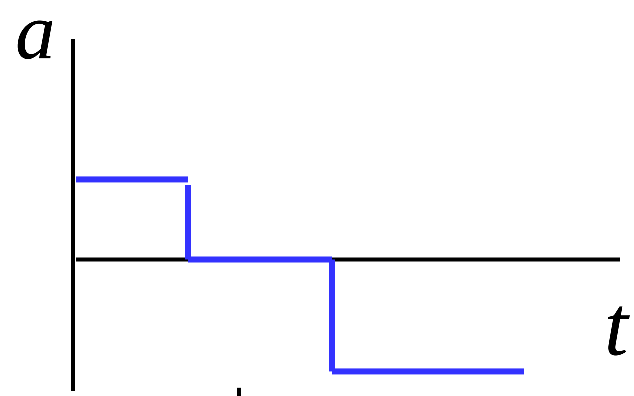
Consider the graph to the right. Which graph below represents the same motion?

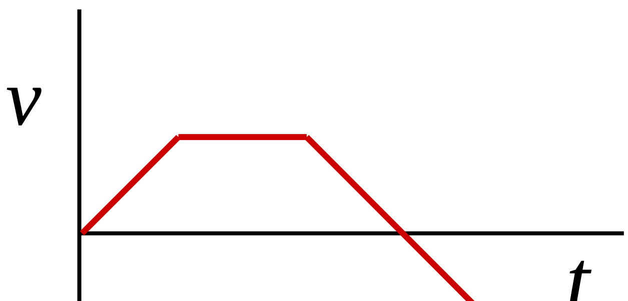




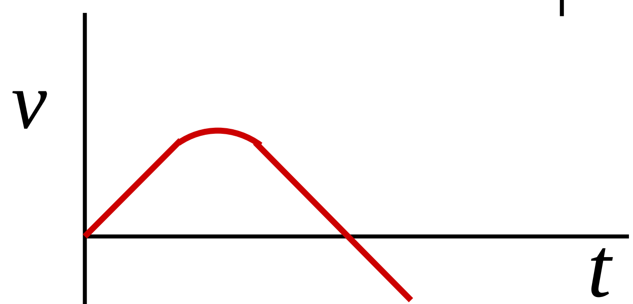
Interactive Question

Consider the graph to the right. Which graph below represents the same motion?

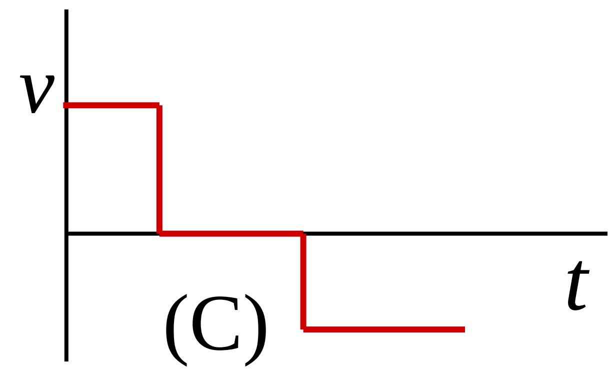




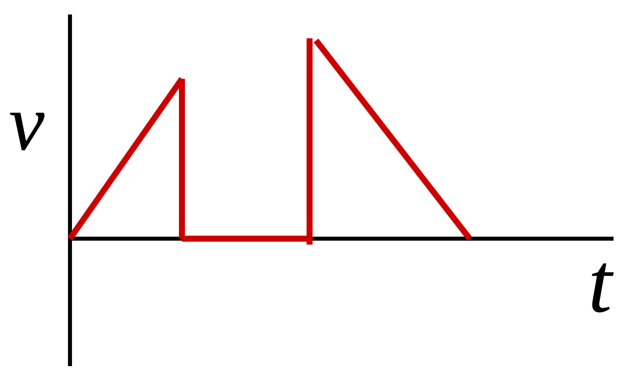
(A)



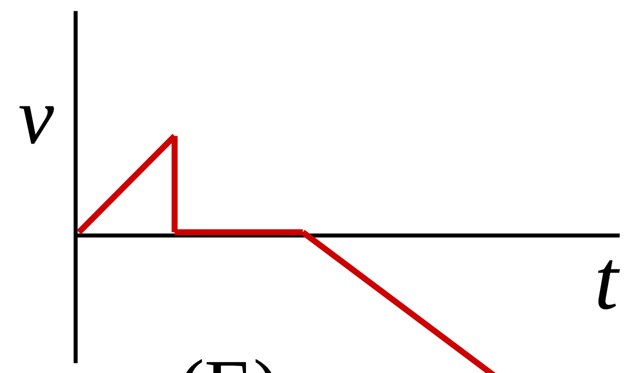
(B)



(C)



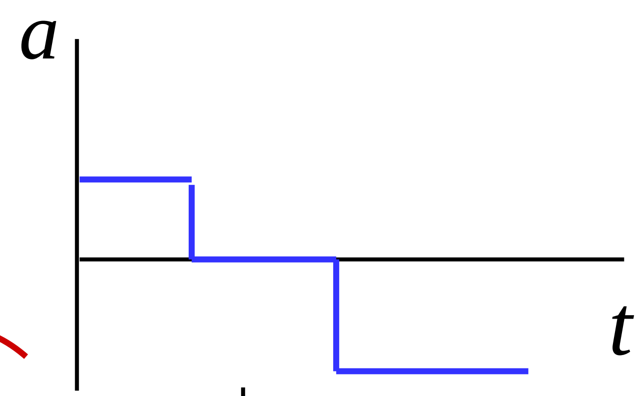
(D)

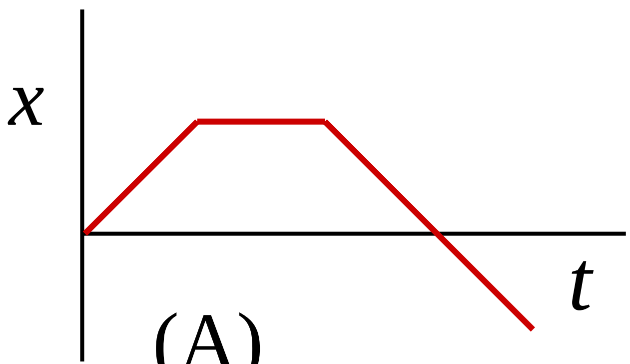


(E)

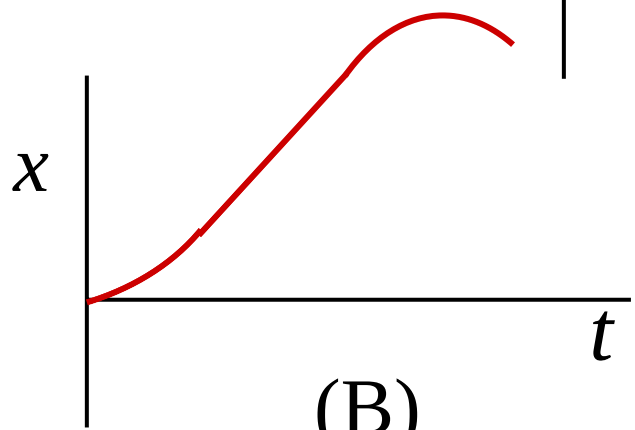
Interactive Question

Consider the graph to the right. Which graph below represents the same motion?

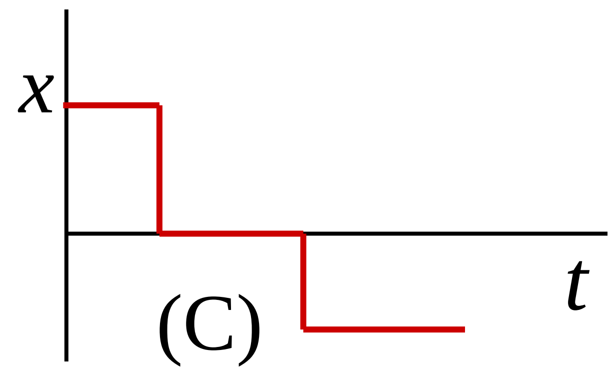




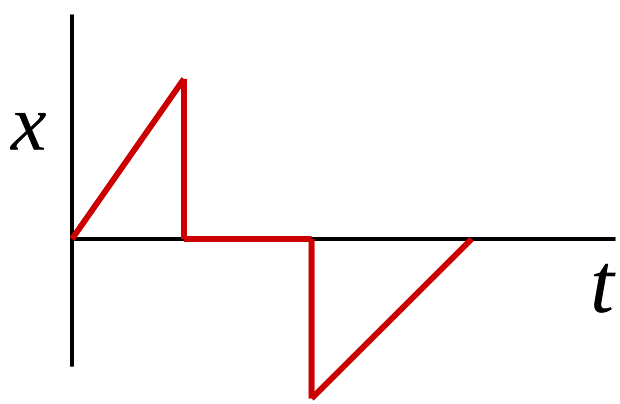
(A)



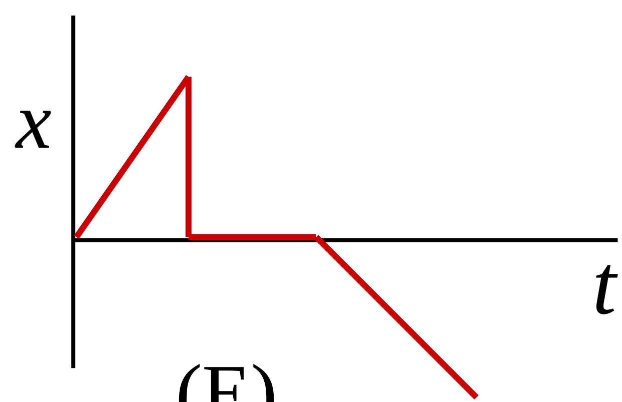
(B)



(C)



(D)



(E)

Equations for Constant Acceleration

$a = (v_2 - v_1)/(t_2 - t_1)$

$v_2 = v_1 + a(t_2 - t_1) = v_1 + a\Delta t$ (1)

When the acceleration is a constant, the average velocity is simply midway between the initial and final velocities. (On a *v* vs. *t* graph, the acceleration is just the slope of a line. For constant acceleration, that slope never changes).

$v_{av} = (v_2 + v_1)/2$ (A)

$v_{av} = (x_2 - x_1)/(t_2 - t_1) = (v_2 + v_1)/2$

$x_2 - x_1 = (1/2)(v_2 + v_1)(t_2 - t_1) = (1/2)(v_2 + v_1)\Delta t$ (2)

From equation (1), $v_2 = v_1 + a\Delta t$

$x_2 - x_1 = (1/2)(v_1 + a\Delta t + v_1)\Delta t$

$x_2 - x_1 = v_1\Delta t + (1/2)a\,\Delta t^2$ (3)

Now start with (2), and substitute Δt from (1)

$x_2 - x_1 = (1/2)(v_2 + v_1)\Delta t$

$x_2 - x_1 = (1/2)(v_2 + v_1)(v_2 - v_1)/a$

$x_2 - x_1 = (1/2)(v_2^2 - v_1^2)/a$

$v_2^2 = v_1^2 + 2a(x_2 - x_1)$ (4)

Substituting $v_1 = v_2 - a(t_2 - t_1)$ from (1) into (2),

$x_2 - x_1 = v_2\Delta t - (1/2)a\,\Delta t^2$ (5)

	$x_2 - x_1$	v_1	v_2	a	$t_2 - t_1$
$v_2 = v_1 + a\Delta t$		✓	✓	✓	✓
$v_2^2 = v_1^2 + 2a\,\Delta x$	✓	✓	✓	✓	
$\Delta x = (1/2)(v_2 + v_1)\Delta t$	✓	✓	✓		✓
$\Delta x = v_1\Delta t + (1/2)a\Delta t^2$	✓	✓		✓	✓
$\Delta x = v_2\Delta t - (1/2)a\Delta t^2$	✓		✓	✓	✓

Also $v_{av} = (v_1 + v_2)/2$

These equations only work when acceleration is constant!

If a=0, then there is only one equation,

$\Delta x = (1/2)(v_2 + v_1)\Delta t$, with $v_2 = v_1$

$\Delta x = v\,\Delta t$

distance equals velocity times time

Often these are written by setting the subscript on the initial point to 0, having no subscript on the final point, and setting t_0=0 and x_0=0.

	x	v_0	v	a	t
$v = v_0 + at$		✓	✓	✓	✓
$v^2 = v_0^2 + 2ax$	✓	✓		✓	✓
$x = (1/2)(v + v_0)t$	✓	✓	✓		✓
$x = v_0t + (1/2)at^2$	✓	✓		✓	✓
$x = vt - (1/2)at^2$	✓		✓	✓	✓

This is simpler to write, but can create confusion if there are more than two important points in a problem.

One advantage of writing the equations this way is you can remember one equation and derive others readily

Start with this equation:

$x = v_0 t + (1/2)at^2$ (1)

Take the derivative

$dx/dt = v_0\, dt/dt + (1/2)a\, d(t^2)/dt$

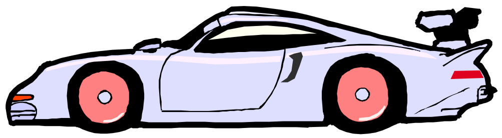
$v = v_0 + at$ (2)

Square the result

$v^2 = v_0^2 + 2v_0 at + a^2 t^2 = v_0^2 + 2a(v_0 t + (1/2)at^2)$

$v^2 = v_0^2 + 2ax$ (3)

Problem: A sports car can accelerate at $4.5\ m/s^2$. It starts at rest and accelerates in the negative x direction. After 8.0 seconds what is its speed and how far has it gone?



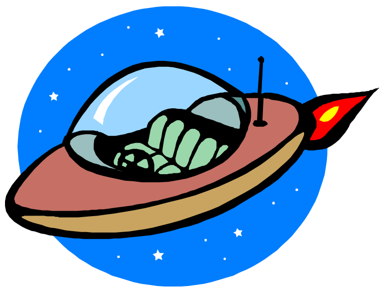
$+x$ →

$a = -4.5\ m/s^2$

Initial velocity is 0

Time = 8.0 s

Problem: A spacecraft is traveling with a speed of 3250 m/s, and it slows down by firing its retro rockets, so that it decelerates at a rate of 10 m/s^2. What is the velocity of the spacecraft after it has traveled 215 km?



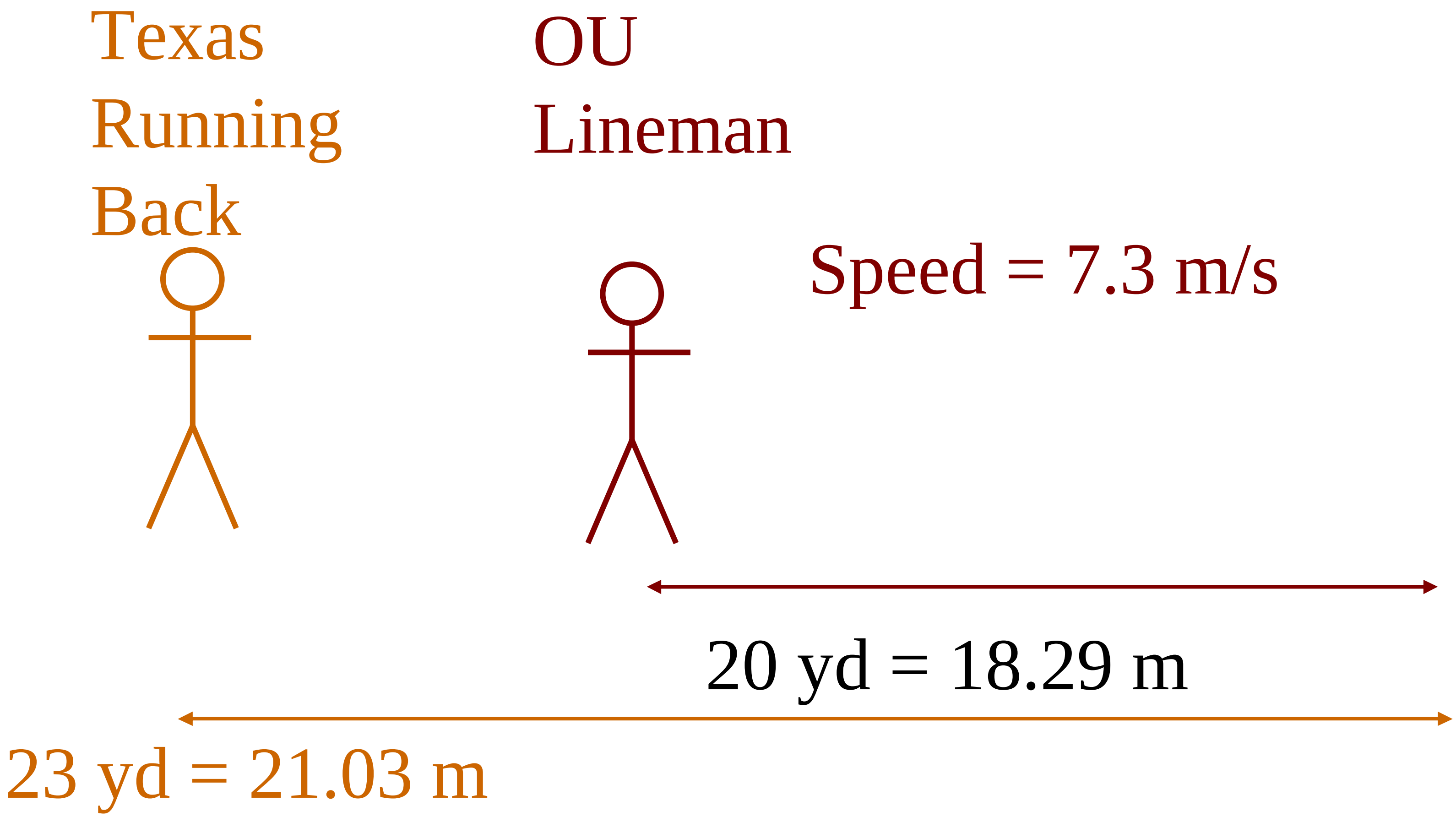
Speed = 3250 m/s

Deceleration = 10 m/s^2

Distance = 325 km

Problem: You are attending the OU-Texas football game. During the game, the Texas quarterback is tackled behind the line of scrimmage and fumbles the football. An OU defensive lineman picks up the fumbled football on the 20 yard line and runs toward the end zone at a speed of 7.3 m/s. A Texas running back is standing on the 23 yard line and wants to catch up to the lineman before he scores a touchdown. If the running back can accelerate at a constant rate, what must be his minimum acceleration to catch the lineman before OU scores a touchdown. What will be the result of the play?

<u>Focus the Problem:</u>



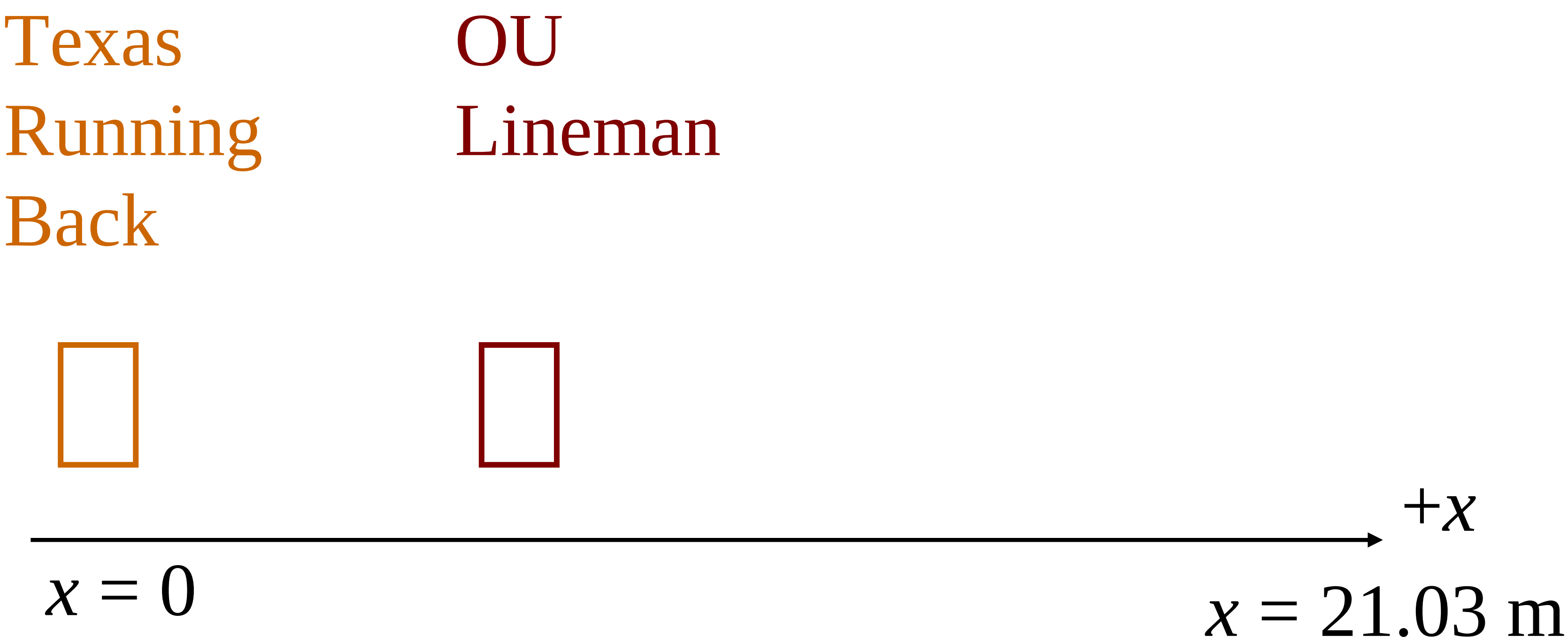
What is the problem asking for:

(1) What acceleration is necessary for the running back to just catch the lineman when the lineman reaches the goal line?
(2) What will be the result of this play?

Outline the Approach: I will use the kinematic equations for constant acceleration. The lineman will have a constant velocity, so no acceleration. I will determine the time it takes for the lineman to reach the goal line and from that determine the running back's acceleration. Then I will evaluate my answer and try to determine the result of the play.

Describe the Physics:

Draw physics diagrams and define all quantities uniquely



Which of your defined quantities is your Target variable(s)?

Quantitative Relationships: Write equations you will use to solve this problem.

$$v_2 = v_1 + a(t_2 - t_1)$$
$$v_2^2 = v_1^2 + 2a(x_2 - x_1)$$
$$x_2 - x_1 = (1/2)(v_2 + v_1)(t_2 - t_1)$$
$$x_2 - x_1 = v_1(t_2 - t_1) + (1/2)a(t_2 - t_1)^2$$
$$x_2 - x_1 = v_2(t_2 - t_1) - (1/2)a(t_2 - t_1)^2$$

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

Unknowns

We have two equations and two unknowns. The problem is now just algebra:

Check Units:

EXECUTE the PLAN

EVALUATE the ANSWER

Is Answer Properly Stated?

Is Answer Unreasonable?

Is Answer Complete?

Objects in a Uniform Gravitational Field

One of the most important cases of uniform accelerated motion is the case of objects that are near to earth which undergo free falling motion. In the late 1500's Galileo showed that even objects of different weights all fell at the same rate if air resistance is neglected.

All objects near the surface of the earth fall with a constant acceleration of about 9.80 m/s^2 which we call g, the acceleration due to gravity. **Therefore, all of the equations we have derived for constant acceleration apply to an object in free fall, neglecting air resistance.**

The reason that some things actually fall slower **in the air** is because air resistance pushes against the moving object. If there were no air resistance then even a piece of paper would drop at the same rate as a heavy ball. Astronauts on the moon, where there is no air, dropped a feather and a hammer and they actually fell at the same rate. When an object is dropped, its velocity increases by 9.80 m/s every second. It continues to go faster. If there were no air resistance, this would continue every second. Because there is air resistance, the object eventually reaches a terminal velocity and doesn't go any faster. But for many applications, we can neglect air resistance. Then, everything near the surface of the earth accelerates toward the center of the earth with a constant acceleration.

Problem: A stone is thrown upward with a speed of 10.0 m/s from the top of a building 40.0 m high. How long will it take for the stone to reach the ground?

Interactive Question

Ball **A** is dropped from a window. At the same instant, ball **B** is thrown downward and ball **C** is thrown upward from the same window. Which statement concerning the balls is necessarily true if air resistance is neglected?

A) At one instant, the acceleration of ball **C** is zero.
B) All three balls strike the ground at the same time.
C) All three balls have the same velocity at any instant.
D) All three balls have the same acceleration at any instant.
E) All three balls reach the ground with the same velocity.

Interactive Question

If you drop a brick from a building in the absence of air resistance, it accelerates downward at 9.8 m/s^2. If instead you throw it downward, its downward acceleration after release is

A) less than 9.8 m/s^2
B) 9.8 m/s^2
C) more than 9.8 m/s^2
D) impossible to determine with the information given

Interactive Question

A person standing at the edge of a cliff throws one ball straight up and another ball straight down at the same initial speed. Neglecting air resistance, the ball that hits the ground below the cliff with the greater speed is the one initially thrown

A) upward
B) downward
C) neither, they both hit at the same speed.
D) It is impossible to tell with the information given.

Interactive Question

Two balls are thrown straight up. The first is thrown with twice the initial speed of the second. Ignore air resistance. How much higher will the first ball rise?

A) $\sqrt{2}$ times as high.
B) Twice as high.
C) Three times as high.
D) Four times as high.
E) Eight times as high

Interactive Question

Two balls are thrown straight up. The first is thrown with twice the initial speed of the second. Ignore air resistance. How much longer will it take for the first ball to return to earth?

A) $\sqrt{2}$ times as high.
B) Twice as high.
C) Three times as high.
D) Four times as high.
E) Eight times as high

Interactive Question

Two balls are thrown straight up. The first one takes twice as long to return to earth as the second one. Ignore air resistance. How much faster was the first ball thrown?

A) $\sqrt{2}$ times as fast.
B) Twice as fast.
C) Three times as fast.
D) Four times as fast.
E) Impossible to tell without knowing the exact times.

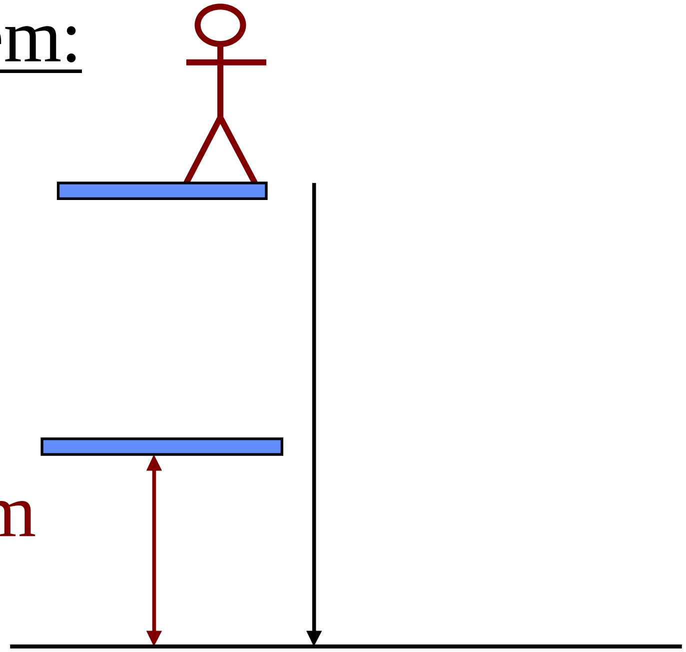
Interactive Question

Two rocks are dropped into two different deep wells. The first one takes three times as long to hit bottom as the second one. Ignore air resistance. How much deeper is the first well than the second?

A) $\sqrt{3}$ times as deep.
B) Three times as deep.
C) Four and a half times as deep.
D) Six times as deep.
E) Nine times as deep.

Problem: You are part of a citizen's group evaluating the safety of a high school athletic program. To help judge the diving program, you would like to know how fast a diver hits the water in the most complicated dive. The coach has his best diver perform for your group. The diver jumps from the high dive and performs her routine. During her dive, she passes near a lower diving board, which is 3.0 m above the water. With a stopwatch, you determine that it took 0.20 seconds to enter the water from the time the diver passed the lower board. From this you determine how fast she was going when she hit the water. After you have done this, another judge wonders out loud how high the high dive is. You quickly calculate the height and, to the judges surprise, announce it.

Focus the Problem:



Distance = 3 m

Time = 0.20 s

What is the problem asking for:

Outline the Approach:

Describe the Physics:

Draw physics diagrams and define all quantities uniquely

Target variable(s)?

Quantitative Relationships:

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

Unknowns

Check Units:

EXECUTE the PLAN: Calculate Target Quantity(ies)

EVALUATE the ANSWER

Is Answer Properly Stated?

Is Answer Unreasonable?

Answer Complete?

Interactive Question

Ball A is dropped from the top of a building. One second later, ball B is dropped from the same building. Neglecting air resistance, as time progresses the *difference* in their speeds

A) increases.
B) remains constant.
C) decreases.
D) depends on the size of the balls.

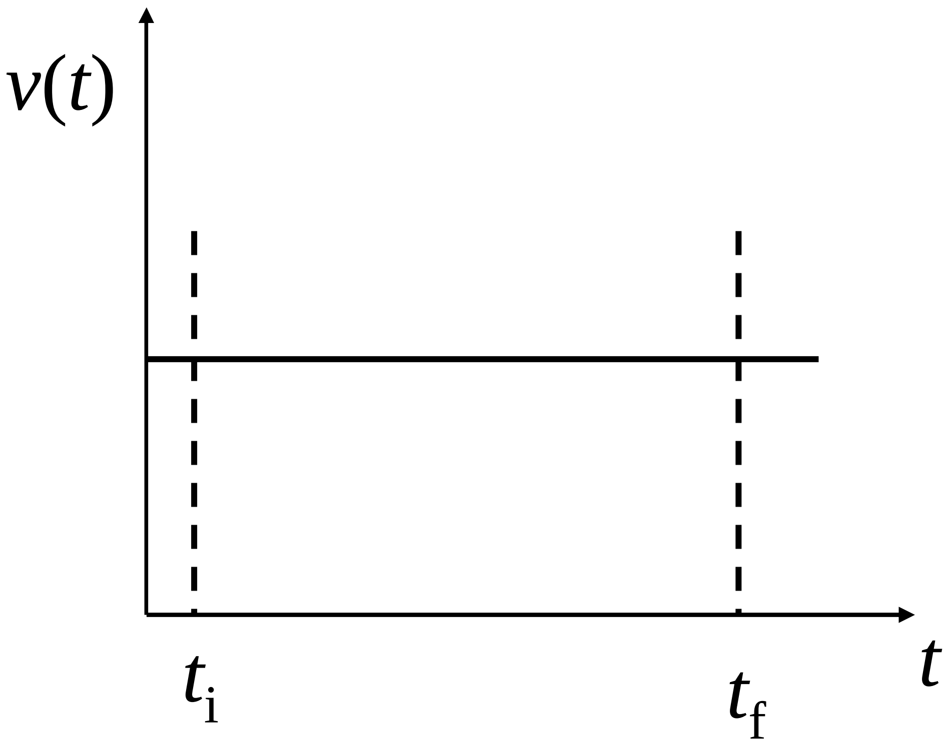
Interactive Question

Ball A is dropped from the top of a building. One second later, ball B is dropped from the same building. Neglecting air resistance, as time progresses the *distance* between them

A) increases.
B) remains constant.
C) decreases.
D) depends on the size of the balls.

Problem: The university skydiving club has asked you to help plan a stunt for an air show. In this stunt, two skydivers will step out of opposite sides of a stationary hot air balloon 500 m above the ground. The second skydiver will leave the balloon 20 seconds after the first skydiver, but you want them to land on the ground at the same time. To get a rough idea of the stunt, you decide to neglect air resistance while the skydivers are falling and assume a constant decent rate of 3 m/s with parachutes open. If the first skydiver waits 3 seconds after stepping out of the balloon before opening her parachute, how long must the second skydiver wait after leaving the balloon before opening his parachute?

Integration

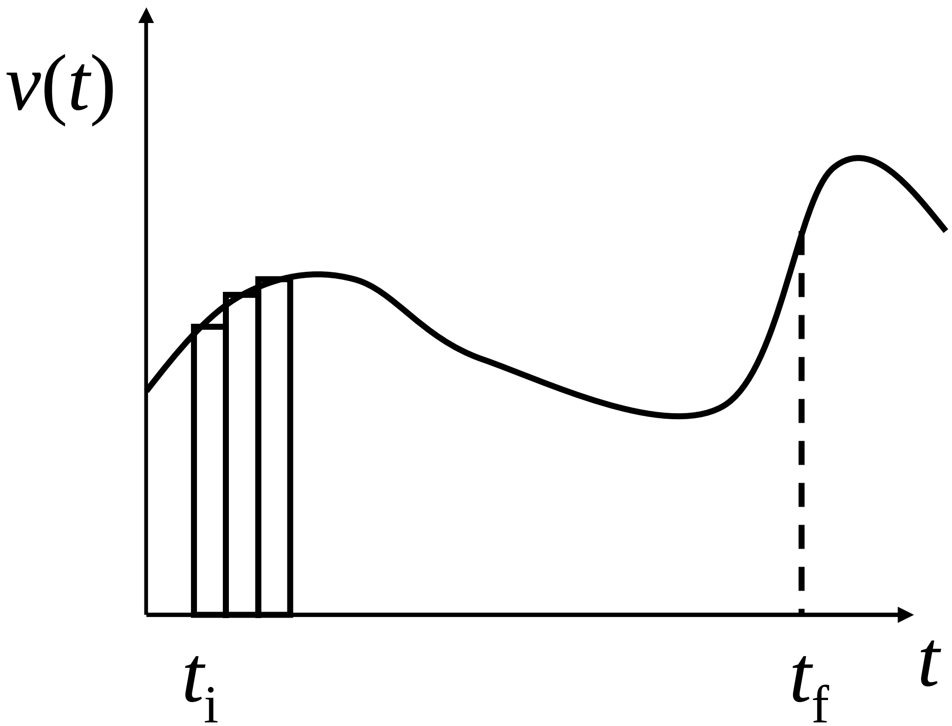


The area under the velocity curve is given by $v(t)\Delta t$.

$$\Delta x = v(t)\,\Delta t$$

So the area under the curve is Δx or vt

Integration



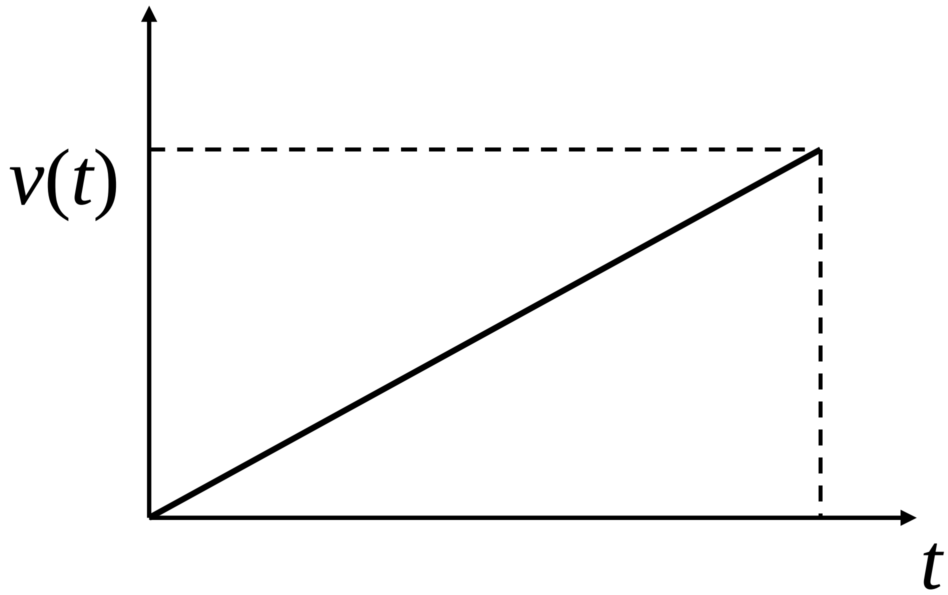
Each section has a width of Δt and an area of $v(t)\,\Delta t$ which is equal to Δx.

$$\Delta x = \sum_n v_n(t)\,\Delta t_n$$

$$\Delta x = \lim_{\Delta t_n \to 0} \sum_n v_n(t)\,\Delta t_n = \int_{t_i}^{t_f} v(t)\,dt$$

The definite integral between two points is the area under the curve.

Let’s look at an example: $v = at$



The area under this curve, the integral, is clearly
$x = (1/2)vt = (1/2)at^2$

Notice that the derivative of the area under the curve is
$dx/dt = d\{(1/2)at^2\}/dt$
$v = at$

Integration gives the area under a curve and is the opposite of differentiation. It is the “antiderivative.”

Indefinite Integrals

Consider the indefinite integral:

$$\int \left(3x^2 + 4x + 7\right)dx = x^3 + 2x^2 + 7x + C$$

We must add some arbitrary constant C, that depends on the initial conditions.

For example, using the definition of acceleration:

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$\int dv = \int a\,dt$$

$$v = at + C = at + v_0$$

Taking the integral again:

$$v_{\mathrm{f}} = \frac{dx}{dt} = at + v$$

$$dx = (at + v)\,dt$$

$$\int dx = \int (at + v)\,dt$$

$$x = \frac{1}{2}at^2 + vt + c = \frac{1}{2}at^2 + vt + x_0$$

Definite Integrals

A definite integral specifically indicates starting and ending values:

$$y = \int_1^5 \left(3x^2 + 4x + 7\right)dx$$

$$= \left(x^3 + 2x^2 + 7x\right)_1^5$$

$$= \left(5^3 + 2\times5^2 + 7\times5\right) - \left(1^3 + 2\times1^2 + 7\times1\right)$$

$$= 210 - 10 = 200$$