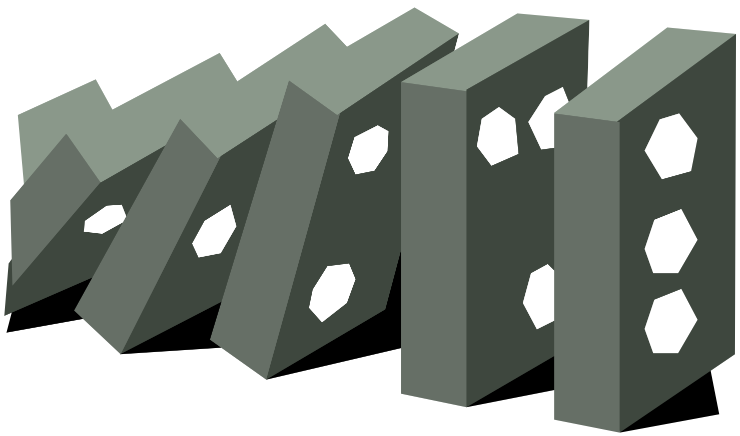
Chapter 12

Waves

Wave Motion



Falling dominoes are a good example of waves. Energy is transported from one place to another, but matter is not.

Waves can travel in:

1 dimension: (dominoes, string, laser)

2 dimensions: (water)

3 dimensions: (sound, light bulb)

Types of Waves

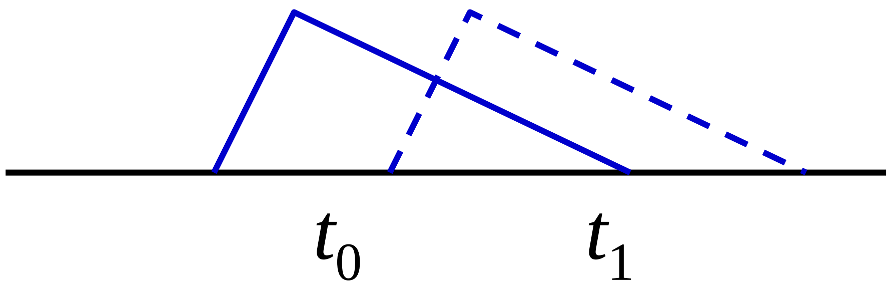
Longitudinal: The elements of the wave are in the same direction as the motion of the wave.

e.g. compressed slinky, sound waves

Transverse: The elements of the wave are in a perpendicular direction to the motion of the wave.

e.g. string, light waves

Consider a pulse traveling to the right

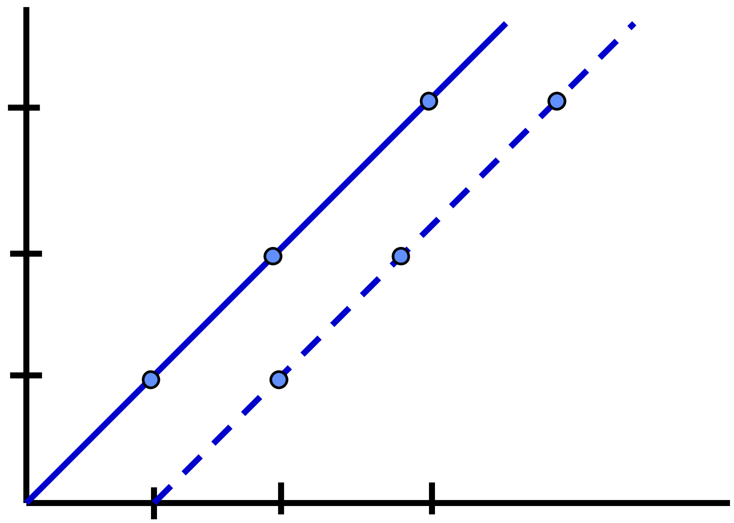


$y = f(x - vt)$

As t increases the value of y that was at a lower x, is now at a higher x.

For a wave traveling to the left: $y = f(x + vt)$

e.g. $y = x - vt$

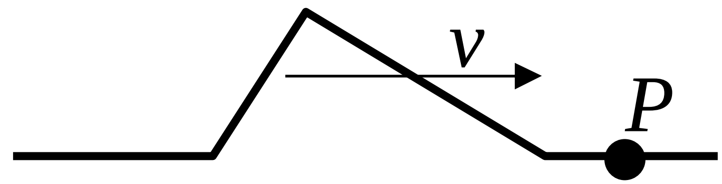


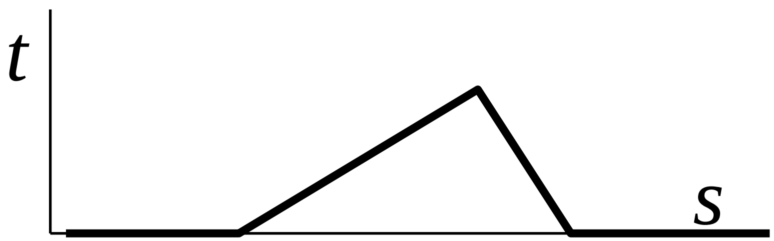
At $t = 0$, three points on the pulse are (1,1), (2,2), (3,3)

At a later time when $vt = 1$, these points become (1,0), (2,1), (3,2)

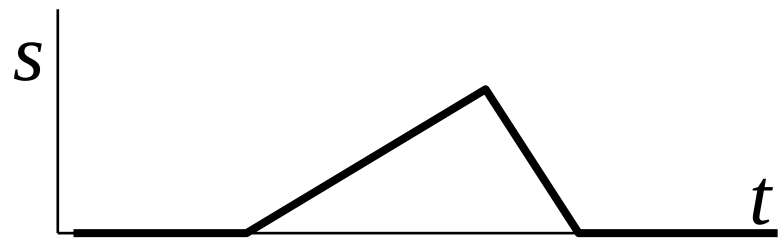
Interactive Question

A wave pulse is moving as illustrated, with uniform speed *v* along a rope. Which of the graphs below correctly shows the relation between the displacement *s* of point *P* and time *t*.





(A)



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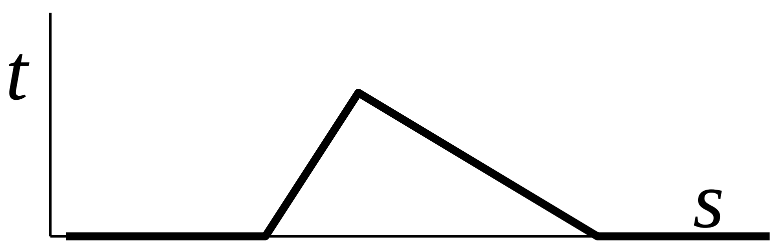

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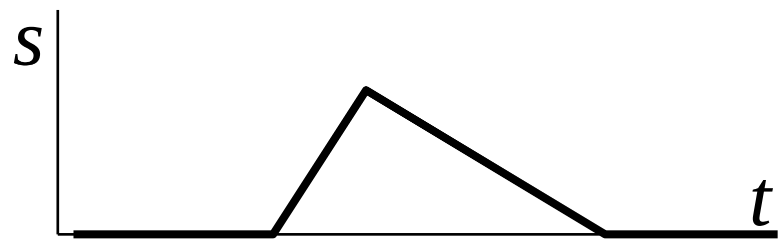
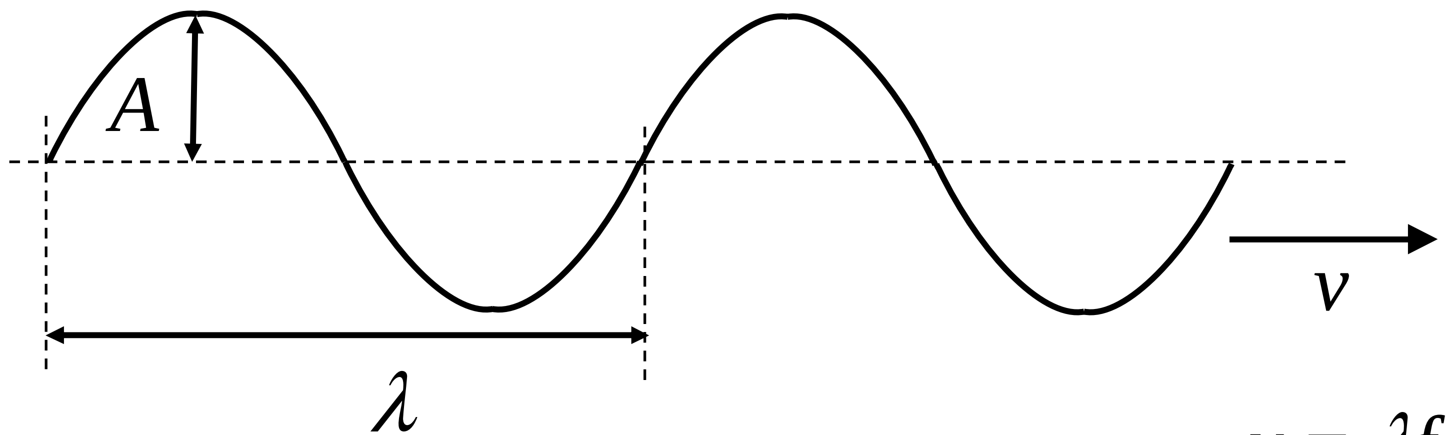


(D)

Description of Periodic Waves

Whether the wave is transverse or longitudinal, we usually draw it as a longitudinal wave to illustrate its properties.



$$v = \lambda f$$

A: Amplitude

λ: Wavelength

v: Wave velocity, or speed of the wave

T: Period, or time it takes one wavelength to pass

f: Frequency, or number of waves to pass per time, (1/*T*)

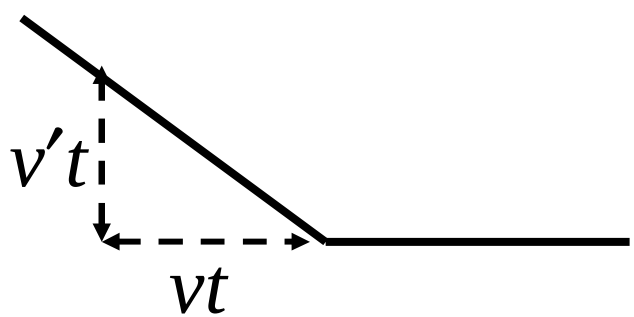
v´: Velocity of element on the wave

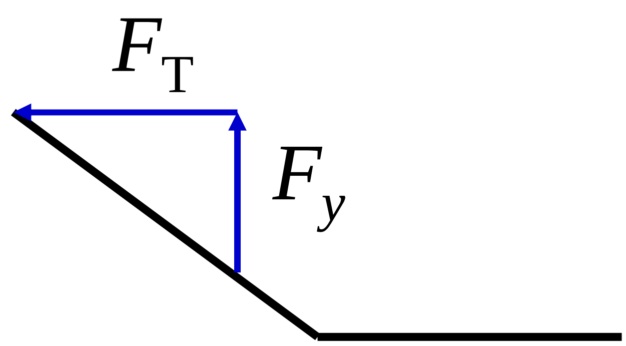
Wave speed for a stretched string

Mass density of string: $\mu = dm/dL$
Velocity of pulse moving to the right: v
Velocity of a point on the string moving up: v'

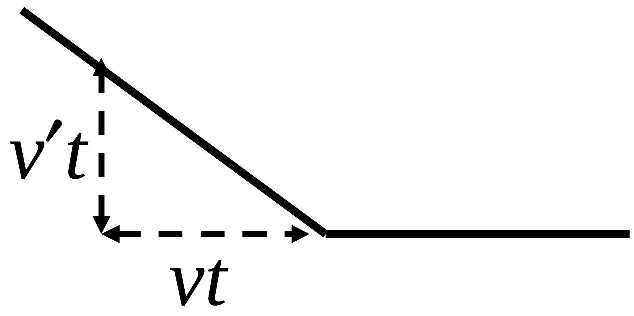
In a time t, the wave moves to the right a distance vt, and up a distance $v't$.

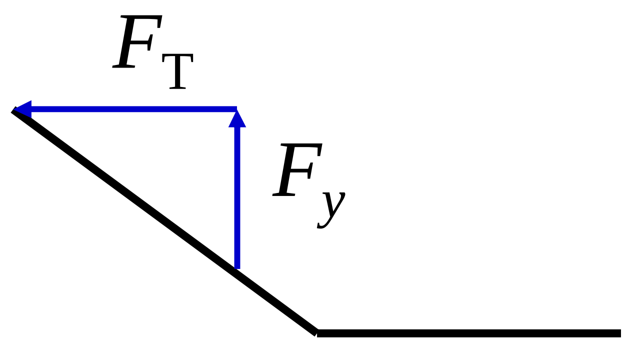
The string has a tension pulling on it of F_T, and a force of F_y causing it to move up.





(We are assuming that $v't << vt$)





These forces cause the string that is initially stationary to move.

$$v' = a't = F_y t/m \qquad v = at = F_T t/m = F_T t/(\mu vt)$$

$$t = v'm/F_y = vm/F_T$$

$$F_y/F_T = v'/v$$

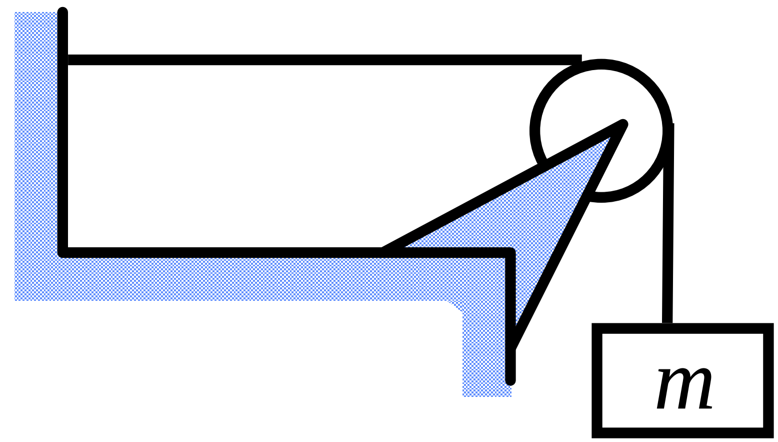
From the first equation, $F_y = mv'/t = \mu vtv'/t = \mu vv'$

Combining these two equations gives $\mu vv'/F_T = v'/v$

$$v = \sqrt{F_T/\mu}$$

Interactive Question

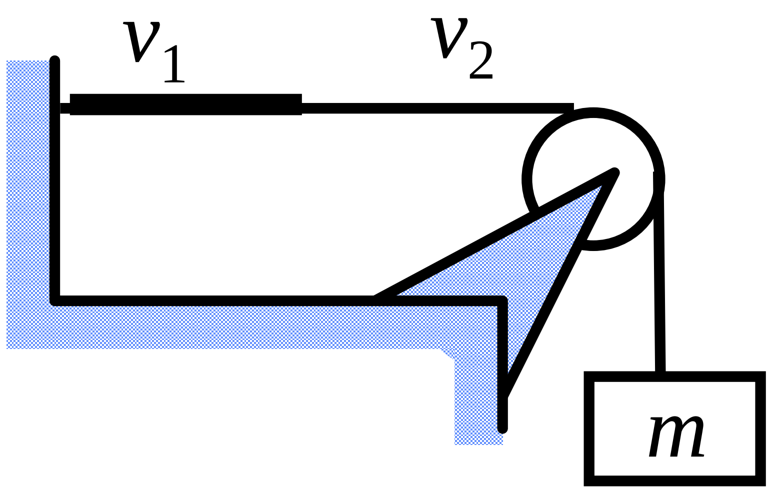
A mass *m* is hanging on the end of a pulley as shown. If the mass is doubled to 2*m*, what happens to the frequency of the wave produced when the string is plucked?



A) The frequency is increased by 4
B) The frequency is increased by 2
C) The frequency is increased by √2
D) The frequency is not changed
E) The frequency is deceased by 1/2

Interactive Question

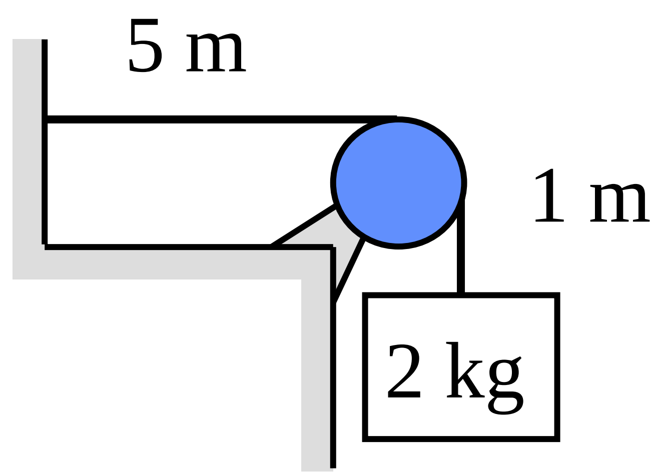
A weight is hung over a pulley as shown. The string is composed of two parts, each made of the same material. Part 1 has four times the diameter as part 2. What is the ratio of the velocity of the wave in part 1 (v_1) to that in part 2 (v_2)?



A) 4 B) 2 C) 1

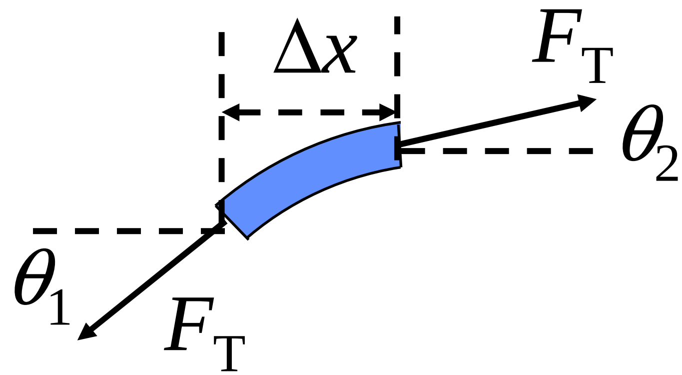
D) 1/2 E) 1/4

Problem: A string has a mass of 0.300 kg and a length of 6.00 m and is set up as shown in the picture. What is the speed of a pulse on this string?



Mathematical Description of Waves

Consider a string in the shape of a wave.



$$\sum F_y = ma_y$$
$$F_T \sin\theta_2 - F_T \sin\theta_1 = (\mu\Delta x)\partial^2 y/\partial t^2$$

If the angles are small, $\sin\theta = \tan\theta = s$ where s is the slope

$$F_T\,(s_2 - s_1) = (\mu\Delta x)\partial^2 y/\partial t^2$$
$$F_T\,(\Delta s/\Delta x) = \mu\partial^2 y/\partial t^2$$

$$F_T\,(\Delta s/\Delta x) = \mu \partial^2 y/\partial t^2$$

As Δx gets very small,

$$\Delta s/\Delta x = \partial s/\partial x = \partial(\partial y/\partial x)/\partial x = \partial^2 y/\partial x^2$$

$$\partial^2 y/\partial x^2 = (\mu/F_T)\,\partial^2 y/\partial t^2$$

$$\partial^2 y/\partial x^2 = (1/v^2)\,\partial^2 y/\partial t^2$$

Although we derived this for a string, this is a general equation that is true for all one dimensional waves. The solution to this equation is:

$$y = A\sin(kx - \omega t)$$

$$\partial^2 y/\partial x^2 = (1/v^2)\, \partial^2 y/\partial t^2$$

$$-A\, k^2 \sin(kx - \omega t) = -A\,(\omega^2/v^2)\sin(kx - \omega t)$$

$$v = \omega/k$$

This function must repeat itself every wavelength

$k\lambda = 2\pi$

$k = 2\pi/\lambda$ $\quad$ k is the "wave number"

This function must repeat itself every period

$\omega T = 2\pi$

$\omega = 2\pi/T = 2\pi f$ $\quad$ ω is angular frequency

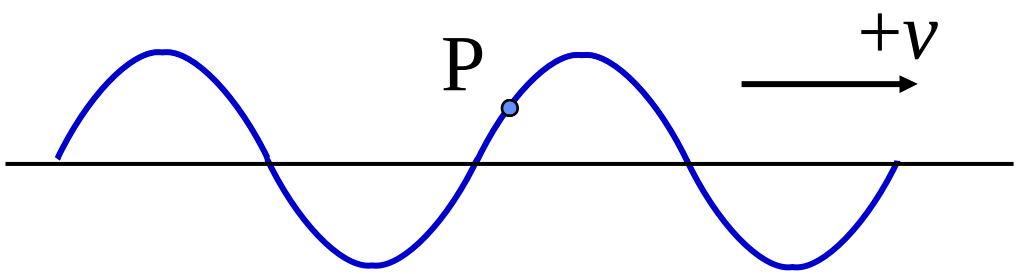
$$v = \omega/k = \lambda f$$

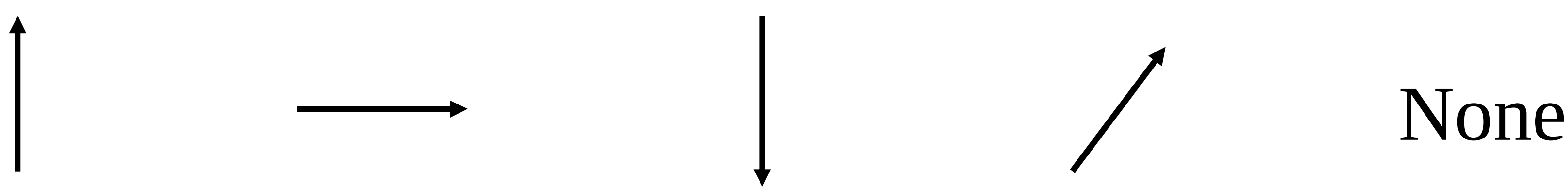
The solution can also be written to emphasize its periodicity: $y = A\sin\{2\pi(x/\lambda - t/T)\}$

or emphasize it is a function $x - vt$: $y = A\sin\{k(x - vt)\}$

Interactive Question

A wave traveling to the right on a stretched string is shown below. The direction of the instantaneous velocity of the point P on the string is:





(A) (B) (C) (D) (E)

Interactive Question

Sinusoidal waves travel on five identical strings. Four of the strings have the same tension, but the fifth has a different tension. Use the mathematical forms of the waves, given below, to identify the string with the different tension.

A) $y = (2\text{ cm})\sin(2x - 4t)$
B) $y = (2\text{ cm})\sin(4x - 10t)$
C) $y = (2\text{ cm})\sin(6x - 12t)$
D) $y = (2\text{ cm})\sin(8x - 16t)$
E) $y = (4\text{ cm})\sin(8x - 16t)$

Problem: Transverse waves on a string have a wave speed of 8.00 m/s, amplitude of 0.0700 m and a wavelength of 0.320 m. The waves travel in the $-x$ direction, and at $t = 0$, the $x = 0$ and of the string has zero displacement and is moving in the $+y$ direction.

(a) Write a wave function for this wave.

(b) What is the period of the simple harmonic motion for a particle on the string?

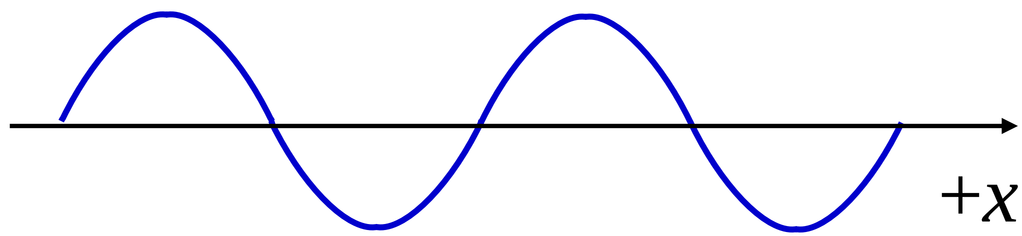
(c) What is the transverse displacement of a particle at $x = 0.360$ m at $t = 0.150$ s?

(d) At what time after $t = 0.150$ s will this point have zero displacement?

Energy and Power of a Wave

Consider a string with a wave propagating down it. Each little section of the string is oscillating in a transverse direction to the wave with simple harmonic motion at an instantaneous velocity of v'. We can see this from the wave equation at a fixed value of x.

$$y = D\sin(kx - \omega t)$$



The kinetic energy of a small part of the string with mass dm is

$$dE_{\mathrm{kin}} = (1/2)dm\, v'^2 = (1/2)dm\,(\partial y/\partial t)^2$$
$$= (1/2)dm\,(-\omega D)^2\cos^2(kx - \omega t)$$
$$= (1/2)(\mu\, dx)(-\omega D)^2\cos^2(kx - \omega t)$$

The average kinetic energy is then

$$\langle dE_{\text{kin}} \rangle = (1/2)(\mu\, dx)(-\omega D)^2 \langle \cos^2(kx - \omega t)\rangle$$
$$= (1/4)(\mu\, dx)\omega^2 D^2$$

The string oscillates up and down with SHM, so it also has an elastic potential energy:

$$E_{\text{el}}\ (\text{max}) = (1/2)kD^2 = (1/2)m\omega^2 D^2$$
$$\langle dE_{\text{el}} \rangle = (1/2)dE_{\text{el}}\ (\text{max}) = (1/4)(\mu\, dx)\omega^2 D^2$$

$$\langle dE \rangle = \langle dE_{\text{kin}} \rangle + \langle dE_{\text{el}} \rangle = (1/2)(\mu\, dx)\omega^2 D^2$$

The power in a wave is given by:

$$\langle P \rangle = \langle dE \rangle / dt$$
$$= (1/2)(\mu\, dx/dt)\omega^2 D^2$$
$$= (1/2)\mu v \omega^2 D^2$$

In 3-dimension, $dm = \rho\, dV = \rho\, A\, dx = \rho\, Av\, dt$

So substitute $\rho\, Av\, dt$ for $\mu\, dx$ and integrate to get:

$\langle E \rangle = (1/2)\rho\, Avt\omega^2 D^2$ $\langle P \rangle = (1/2)\rho\, Av\omega^2 D^2$

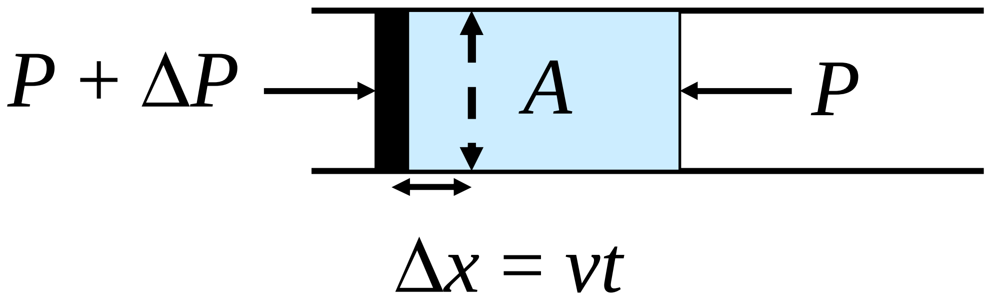
The intensity is the power per area:

$I = P/A = (1/2)\rho\, v\omega^2 D^2$

Sound Waves

Sound waves are created from a source which is vibrating. The vibrating source creates longitudinal waves in some medium which can be detected by our ear or by an instrument. Sound waves are longitudinal pressure waves. The particles in the medium that the sound propagates in will oscillate back and forth longitudinally due to a difference in pressure.

Consider a piston, moving at speed *v*, pushing on a tube filled with air. The wave in the air will move at a speed of *v*´. We look at a small section of the air as the piston pushes



The impulse given to the section of air is:

$$\sum Ft = \Delta p$$
$$\{(P + \Delta P)A - PA\}t = \Delta m v'$$
$$\Delta PAt = \Delta m v' = \rho V v' = \rho A \Delta x \, v' = \rho A vt \, v'$$
$$\Delta P = \rho v v'$$

$$\Delta P = \rho v v'$$
$$-\Delta P\ (V/\Delta V) = -\rho v v'\ (V/\Delta V)$$
$$-\Delta P\ (V/\Delta V) = -\rho v v'\ (Avt/(-Av't)) = \rho v^2$$
$$B = \rho v^2$$
$$v = \sqrt{B/\rho}$$

This works well for liquids, but for a gas, the velocity is given by:

$$v = \sqrt{\gamma RT/M}$$

where R = 8.314 J/(mol·K) is the universal gas constant, T is the temperature in Kelvin, M is the molar mass (kg/mol), and γ is a constant that depends on the kind of gas. For a diatomic gas it is 1.4.

Problem: How fast does sound travel in air (at 20^{o} C), and in water?

Interactive Question

A guitar string is plucked and set into vibration. It disturbs the surrounding air, resulting in a sound wave. Which entry in the table below is correct?

	in string	in air
A) the wave is transverse	yes	yes
B) frequency changes if string is tightened	yes	no
C) wave is transmitted by particle vibrations	no	yes
D) wave speed increases if temperature rises	no	yes
E) wave transports energy	yes	no

Pitch

High frequencies create high pitches and low frequencies produce low pitches.

Audible: 20 Hz – 20,000 Hz
Ultrasonic: Above 20,000 Hz
Infrasonic: Below 20 Hz

Intensity

The intensity of the sound wave is a measure of how loud it is. The intensity is given by the power per unit area:

$$I \equiv P/A \quad (\mathrm{W/m^2})$$

The power is proportional the the square of the wave amplitude. The intensity must increase by about a factor of 10 for a sound to appear twice as loud to a human ear. Therefore, intensity is measured on a log scale using the units of **bel**s or one tenth of a bel, a **decibel** (β). A decibel measures the sound intensity <u>level</u> or decibel <u>level</u>, rather than the intensity.

$$\beta = 10\log(I/I_0) \qquad\qquad I_0 = 1.0\times10^{-12}\ \mathrm{W/m^2}$$

(I_0 is approximately the softest audible sound.)

Problem: What is the intensity level of a sound wave having an intensity of

(a) 1.0×10^{-12} W/m^2,

(b) 1.0×10^{-11} W/m^2

(c) 1.0×10^{-10} W/m^2

Problem: A sound is measured to have a decibel level of 30 db. If the intensity is decreased by a factor of 2, by how much does the intensity level change?

Interactive Question

Which of the following is most closely identified with the loudness of a musical note?

A) Frequency
B) Velocity
C) Phase
D) Amplitude

Longitudinal Pressure Waves

For a longitudinal wave, the wave equation is:

$$D = x_m \sin(kx - \omega t)$$

$\Delta P = -B\Delta V/V = -BA\Delta D/A\Delta x$

or in the limit as $\Delta x \to 0$

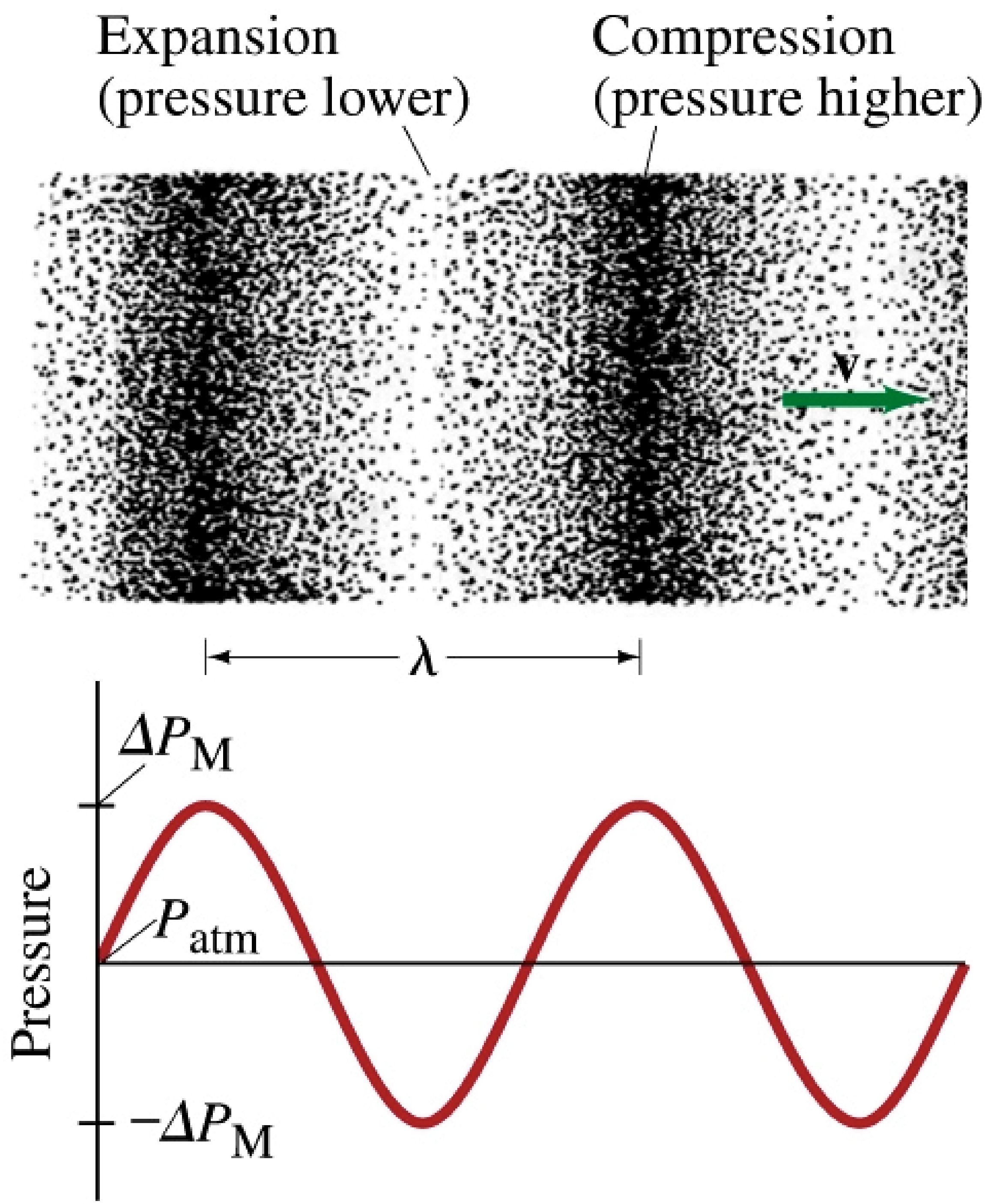
$\Delta P = -B\, \partial D/\partial x$

$$\Delta P = -Bk\, x_m \cos(kx - \omega t) = v^2 \rho k\, x_m \cos(kx - \omega t)$$

$$= v(\omega/k)\, \rho k\, x_m \cos(kx - \omega t) =$$

$$= \rho v \omega\, x_m \cos(kx - \omega t)$$

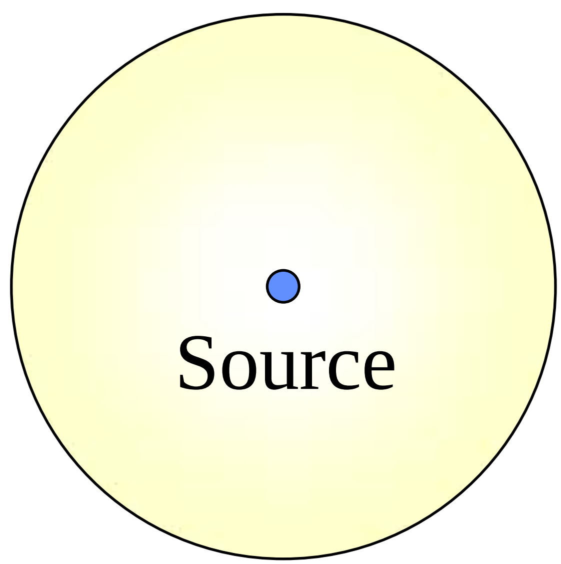
A sound wave can be thought of as a longitudinal displacement or a longitudinal pressure wave.



Expansion
(pressure lower)
Compression
(pressure higher)
v
λ
ΔP_M
P_atm
Pressure
−ΔP_M

Most sound waves travel out in all three directions from a source. The intensity is equal to the Power/Area. The total power stays the same as the sound spreads over an increasingly large spherical area given by $4\pi r^2$.

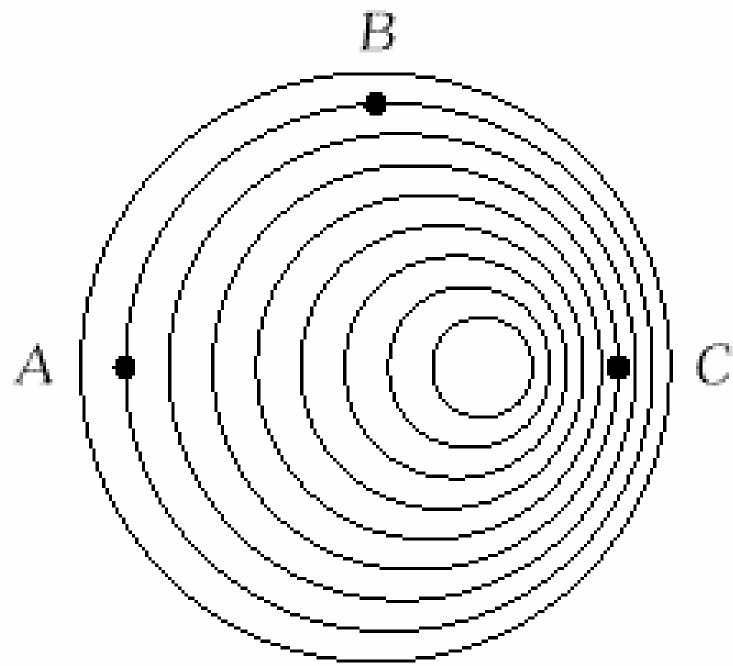
$$I = P/4\pi r^2$$



Problem: During a fireworks show a rocket explodes. To a listener who is 640 m away, the sound has an intensity of 0.10 W/m^2. What is the intensity for a listener who is 160 m away?

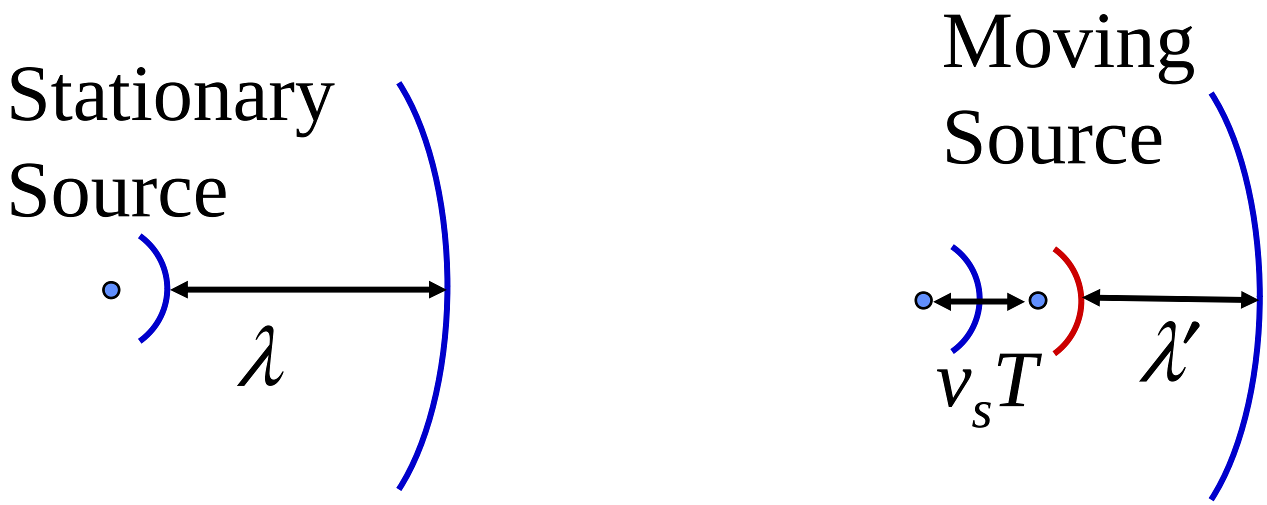
Interactive Question

Three observers, A, B, and C are listening to a moving source of sound. The diagram shows the location of the wavecrests of the moving source with respect to the three observers. Which of the following is true?



A) The wavefronts move faster at A than at B and C.
B) The wavefronts move faster at C than at A and B.
C) The frequency of the sound is highest at A.
D) The frequency of the sound is highest at B.
E) The frequency of the sound is highest at C.

Doppler Effect



$$\lambda' = \lambda - v_sT = \lambda - v_s\,\lambda/v = \lambda(1 - v_s/v)$$

$$f' = f/(1 - v_s/v) = fv/(v - v_s)$$

If the source is moving away: $f' = fv/(v + v_s)$

In a similar derivation, for the observer moving:

$f' = f(v + v_o)/v$ for the observer moving toward the source

$f' = f(v - v_o)/v$ for the observer moving away from source

All these equations can be written in a single equation:

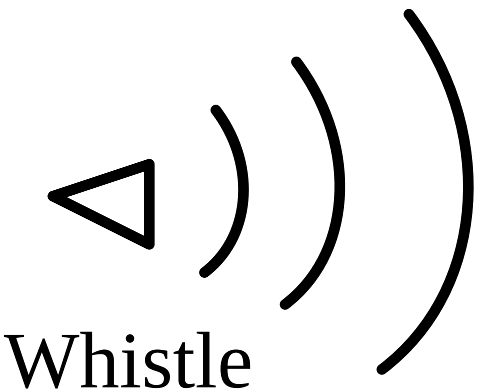
$$f' = f\left(\frac{v \pm v_o}{v \mp v_s}\right)$$

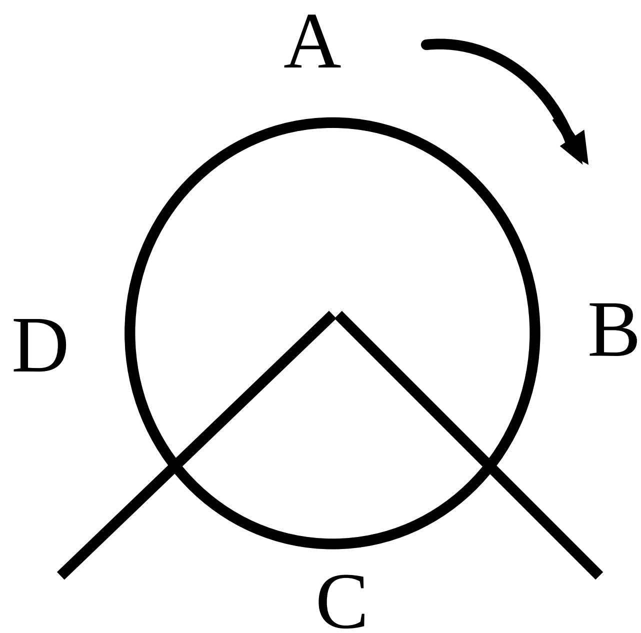
f': apparent frequency
f: original frequency
v_o: velocity of the observer
v_s : velocity of the source

The top signs apply when the source and/or object move toward each other and the bottom signs apply when they move away from each other.

Interactive Question

You are riding on a Ferris wheel and hear a loud whistle being blown from in front of you. On which point on the Ferris wheel will the pitch of the whistle sound lowest?





A) A

B) B

C) C

D) D

E) Both B and D

Problem: An ambulance travels down a highway at a speed of 75.0 mi/h (33.05 m/s) with its siren emitting a sound with a frequency of 400 Hz. What frequency is heard

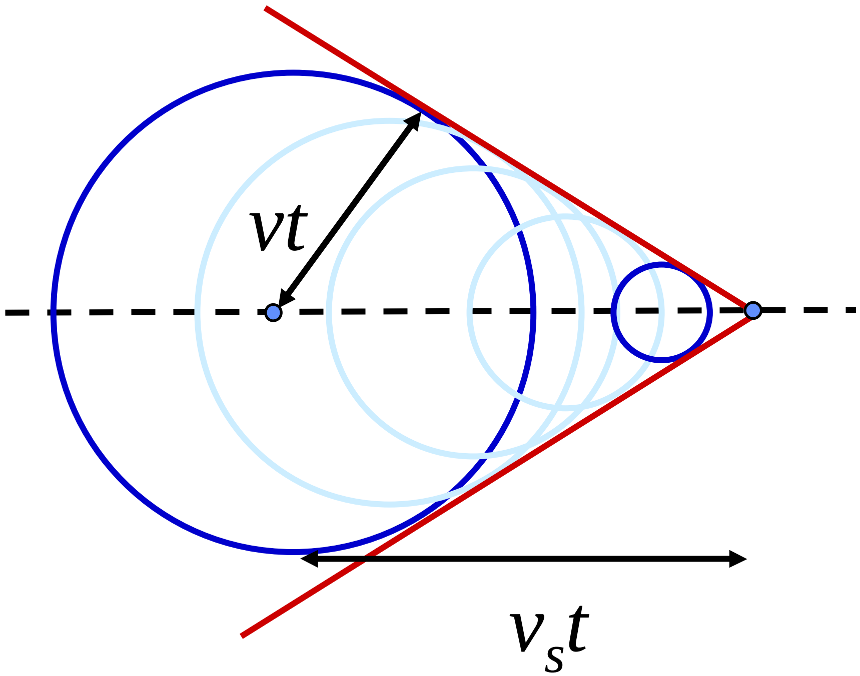
(a) by someone standing still when the ambulance approaches?

(b) by a passenger in a car traveling at 55 mi/h (24.6 m/s) in the opposite direction as it approaches the ambulance?

(c) by a passenger in a car traveling at 55 mi/h in the opposite direction as it moves away from the ambulance?

Problem: A man sitting in his car which is stopped sounds his horn that has a frequency of 500 Hz. The sound is reflected off of a car moving at 24 m/s toward the man. What frequency does the man hear?

Sonic Boom

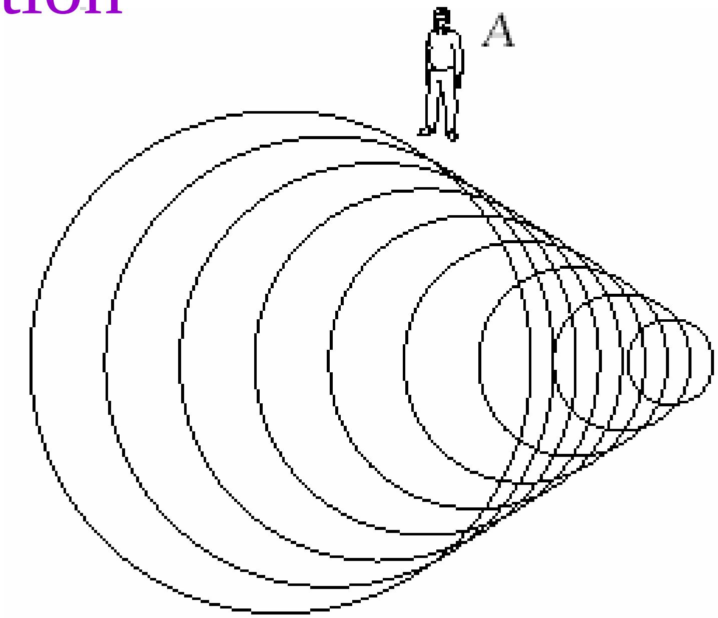


v_s is the speed of the source
v is the speed of sound

$$\sin\,\theta = vt/v_s t = v/v_s$$

Interactive Question

The figure shows the wavefronts generated by an airplane flying past an observer at a speed greater than that of sound. After the airplane has passed, the observer reports hearing



A) a sonic boom only when the airplane breaks the sound barrier, then nothing.

B) a succession of sonic booms.

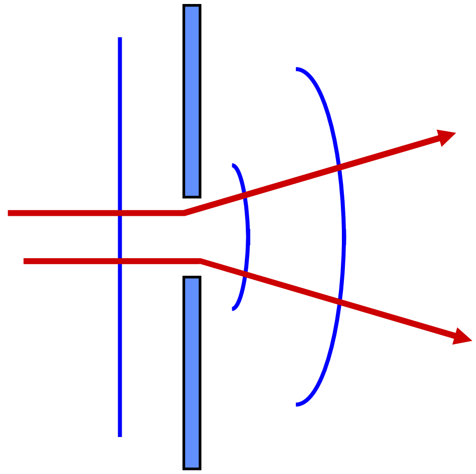
C) a sonic boom, then silence.

D) first nothing, then a sonic boom, then the sound of engines.

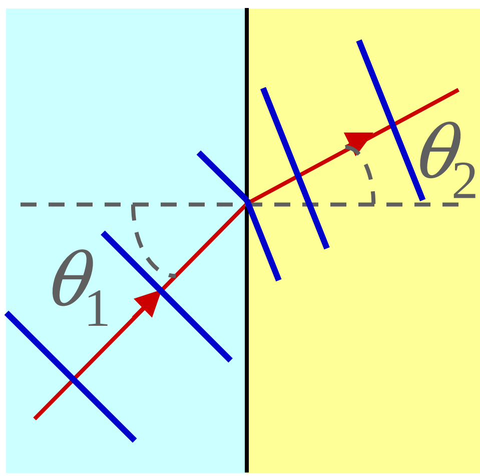
E) no sonic boom because the airplane flew faster than sound all along.

Diffraction and Refraction

Diffraction



Refraction

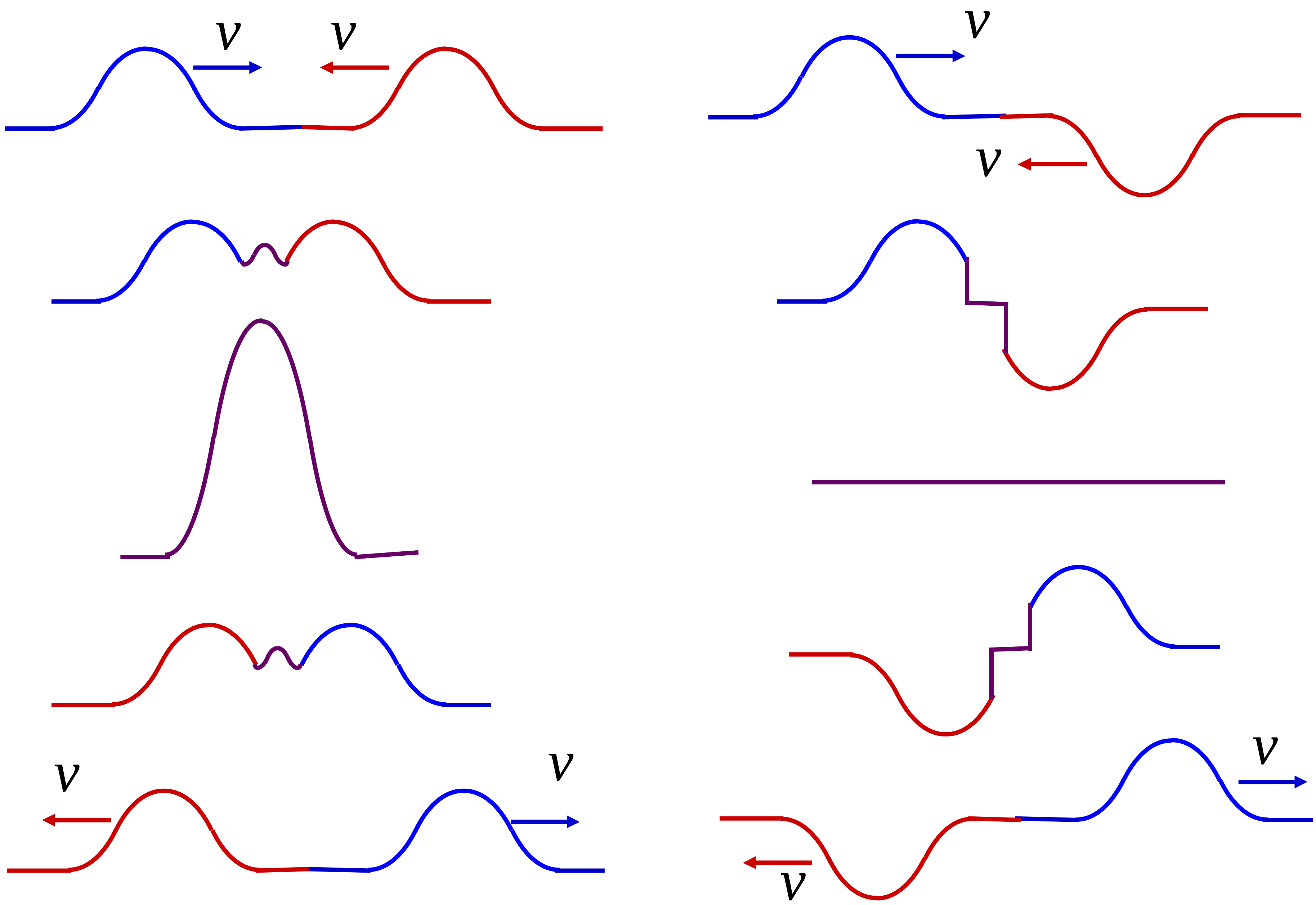


$$\sin\theta_2/\sin\theta_1 = v_2/v_1$$

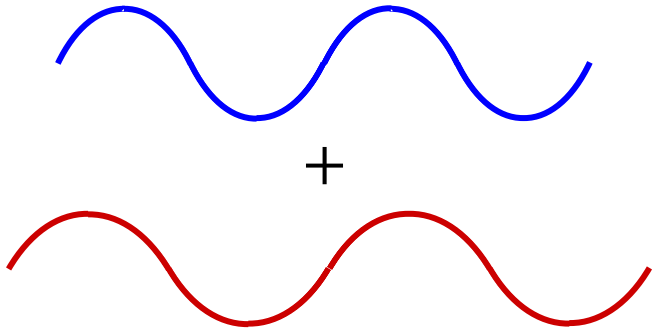
Diffraction and refraction occur for all types of waves. They are very important concepts when we talk about electromagnetic waves next semester.

Problem: A longitudinal earthquake wave strikes a boundary between two types of rock at a 25° angle. As it crosses the boundary, the specific gravity of the rock changes from 3.7 to 2.8. Assuming that the elastic modulus is the same for both types of rock, determine the angle of refraction.

Superposition and Interference



Waves moving in the same direction



$$y_1 = A\sin(k_1 x - \omega_1 t)$$
$$y_2 = B\sin(k_2 x - \omega_2 t + \delta)$$

For simplicity, set *A*=*B*

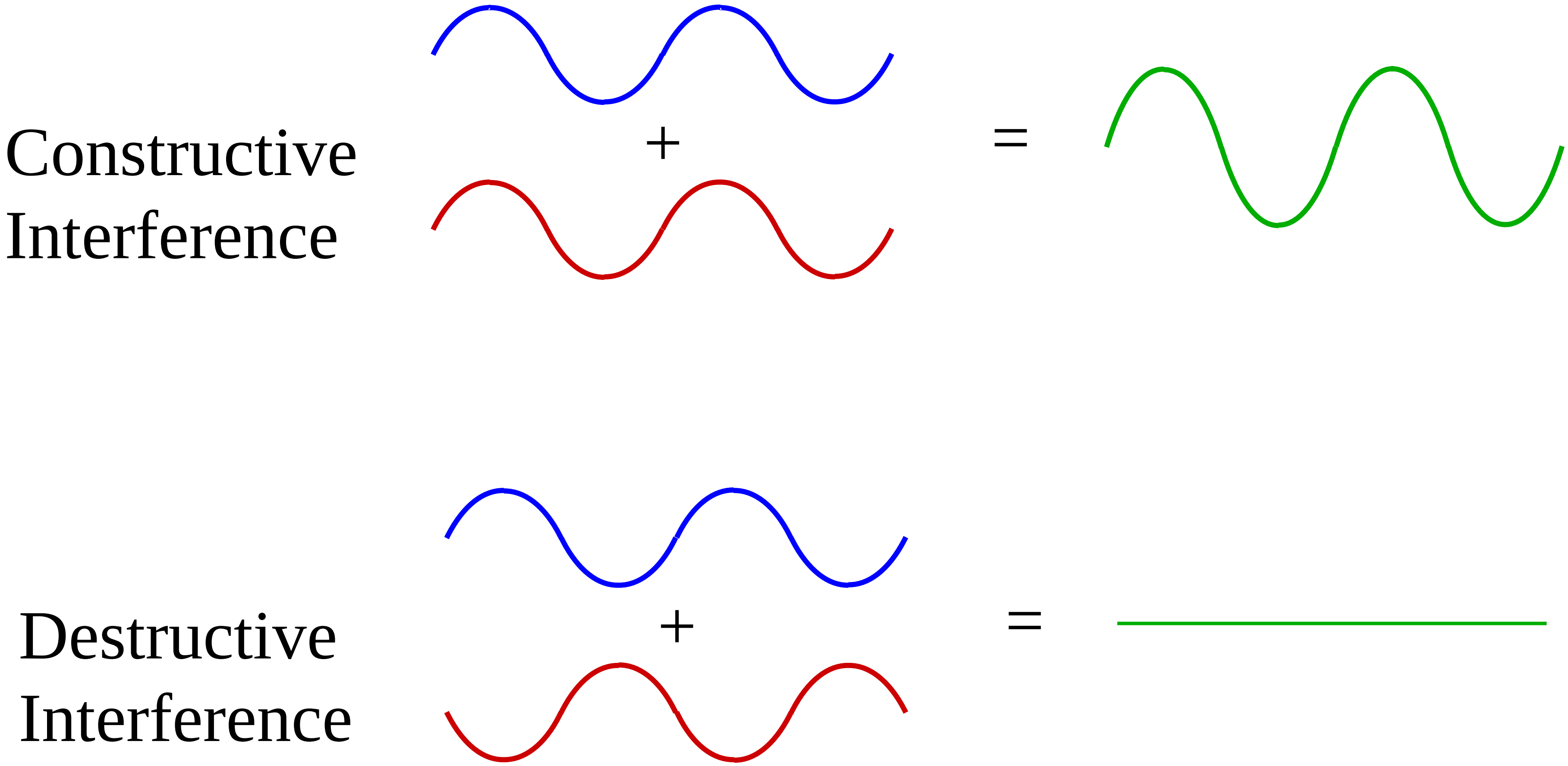
$$y = y_1 + y_2 = A\left(\sin(k_1 x - \omega_1 t) + \sin(k_2 x - \omega_2 t + \delta)\right)$$

Case 1: $k_1 = k_2$, $\omega_1 = \omega_2$

$$y = y_1 + y_2 = A\,(\sin(kx - \omega t) + \sin(kx - \omega t + \delta))$$
$$= 2A\,[\sin\{(kx - \omega t + kx - \omega t + \delta)/2\} \times$$
$$\cos\{(kx - \omega t - kx + \omega t - \delta)/2\}$$
$$y = [2A\cos(\delta/2)]\,\sin(kx - \omega t + \delta/2)$$

Amplitude that depends on the phase difference

Wave traveling in the same direction and speed as both waves



Constructive interference occurs for waves in phase

Destructive interference occurs for waves out of phase

Case 2: $\omega_1 \approx \omega_2$, $\delta = 0$, Consider a fixed point in space:

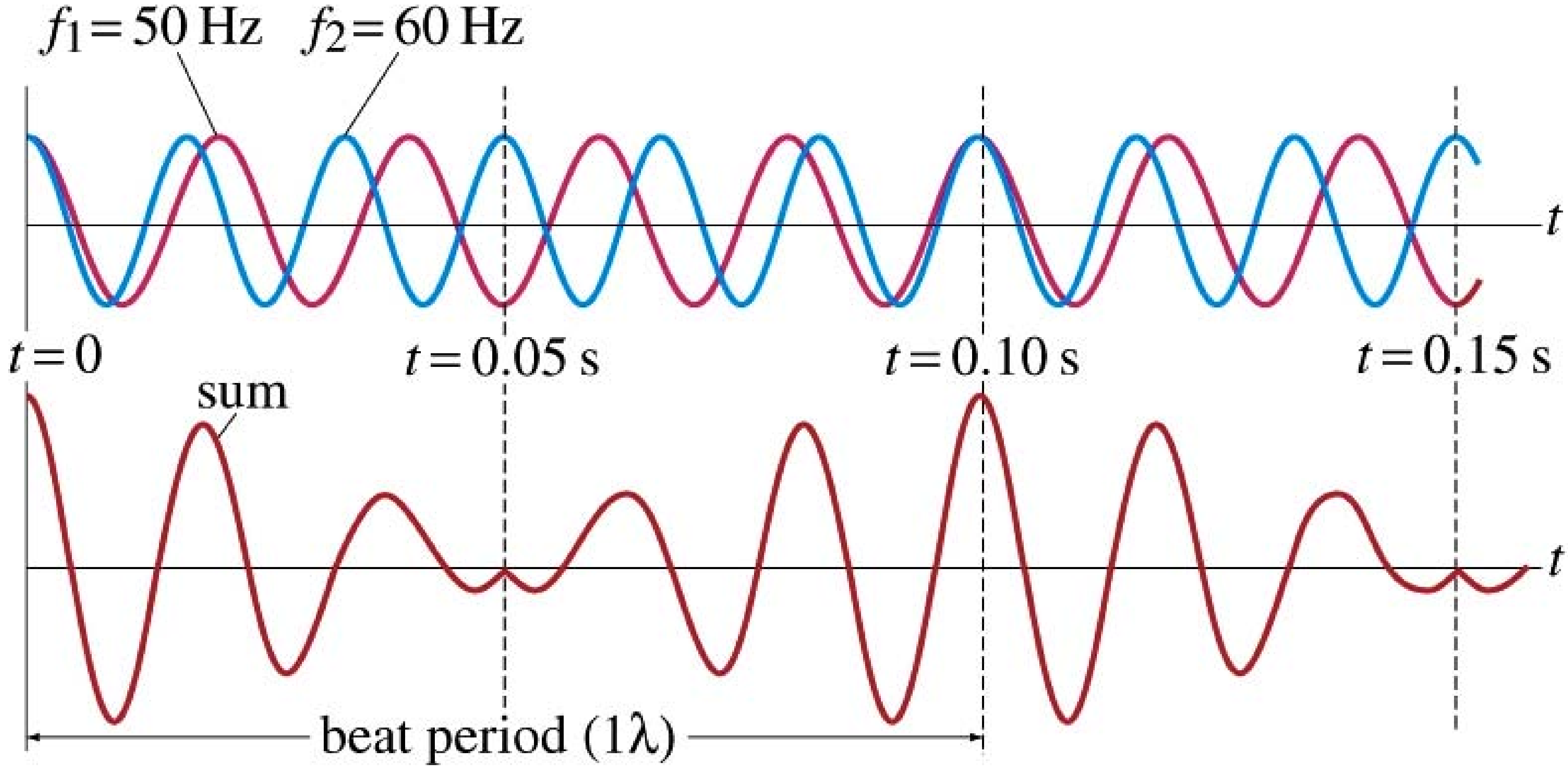
$$y = y_1 + y_2 = A(\sin(\omega_1 t) + \sin(\omega_2 t))$$
$$= 2A(\sin\{(\omega_1 t + \omega_2 t)/2\}\cos\{(\omega_1 t - \omega_2 t)/2\})$$

$$y = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\sin 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

An amplitude that changes over a time period much longer than the frequency.

A sin wave with a frequency equal to the average of the two waves.

The beat frequency is $\Delta f = |f_1 - f_2|$



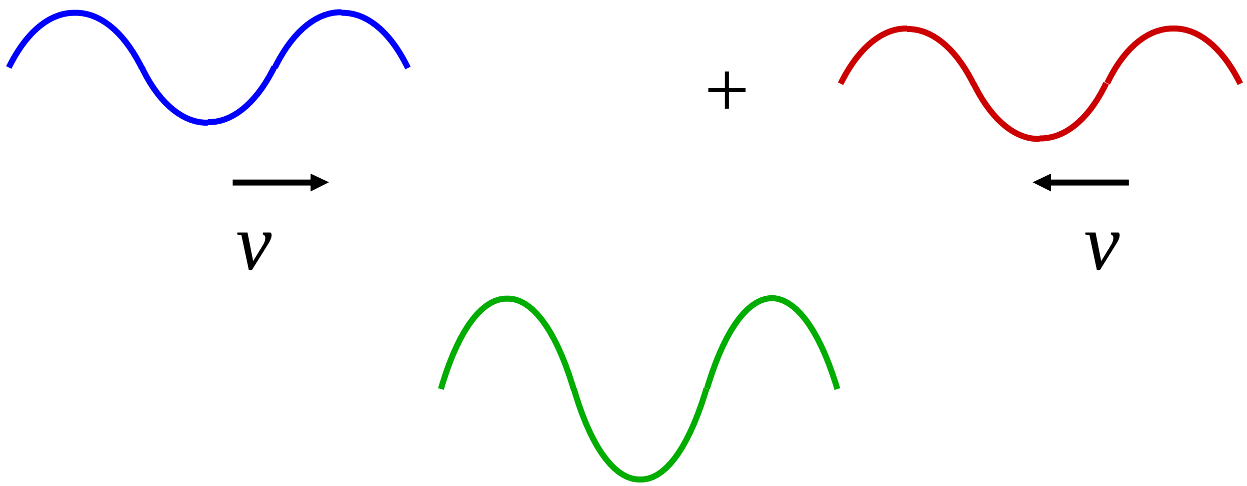
$f_1 = 50$ Hz
$f_2 = 60$ Hz
t
$t = 0$
$t = 0.05$ s
$t = 0.10$ s
$t = 0.15$ s
sum
t
beat period (1λ)

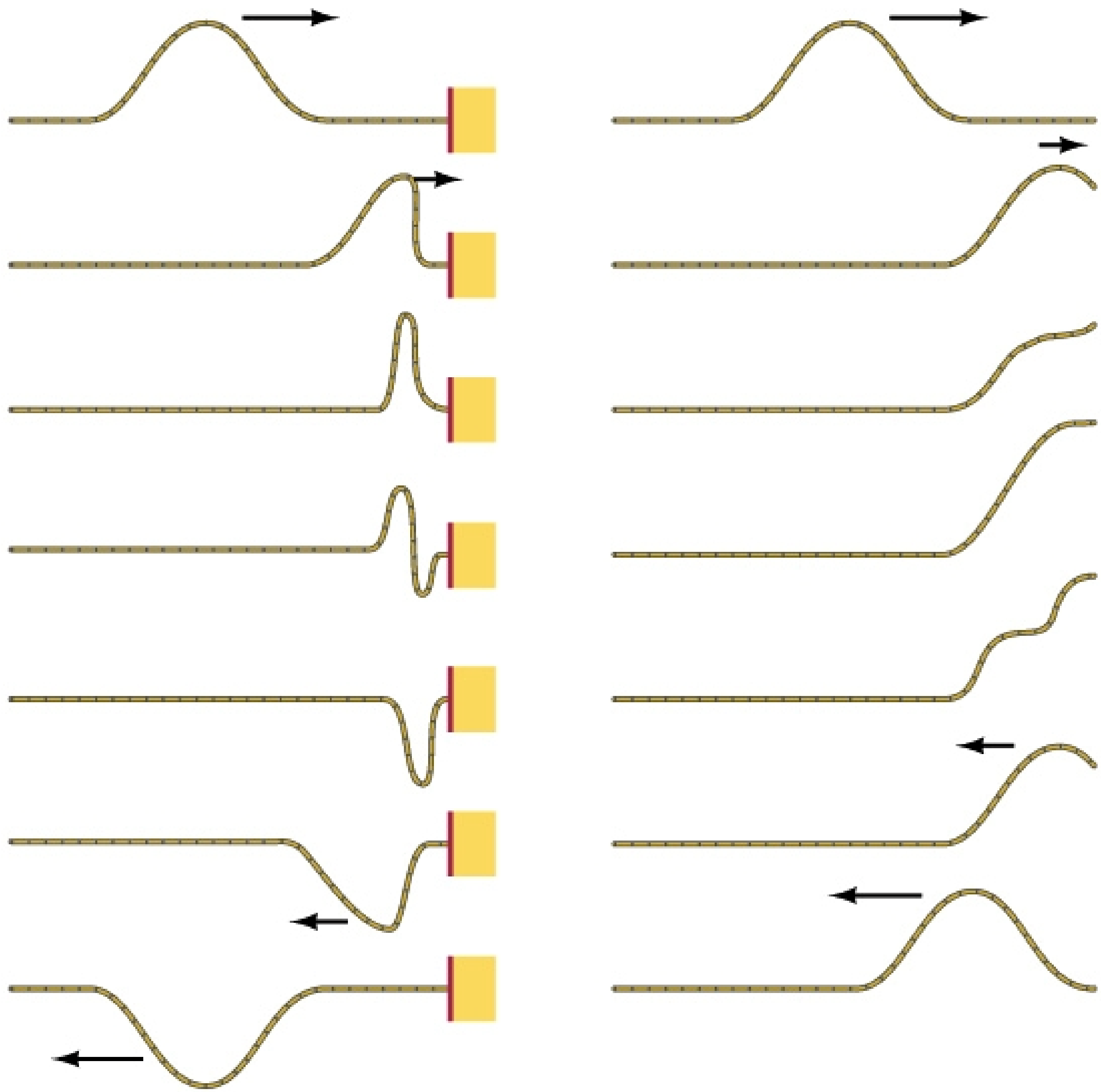
Problem: A particular piano note is supposed to vibrate at 440 Hz. A tuning fork known to vibrate at 440 Hz is sounded at the same time as the piano key is struck, and a beat frequency of 4 beats per second is heard. Find the possible frequencies the string could be vibrating.

$$f_{\text{beat}} = |f_1 - f_2|$$
$$4\text{ Hz} = |440\text{ Hz} - f|$$
f = 444 Hz or f = 436 Hz.

Standing Waves, Resonant Frequencies

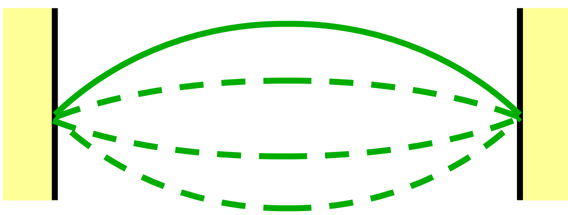
The result of waves moving back and forth with certain boundary conditions



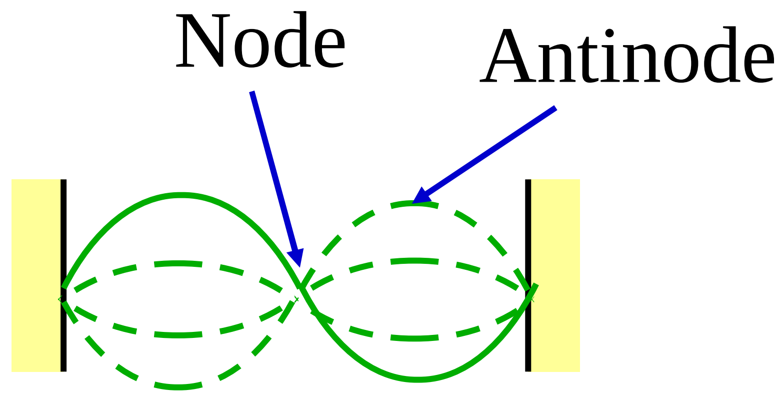


(a) (b)

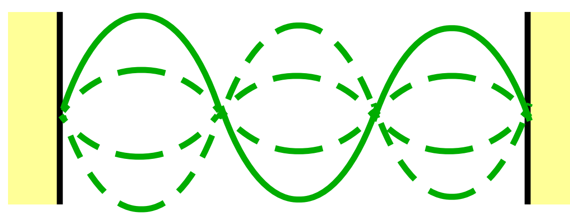
Only certain natural frequencies, or resonant frequencies, are allowed.



Fundamental or
First Harmonic



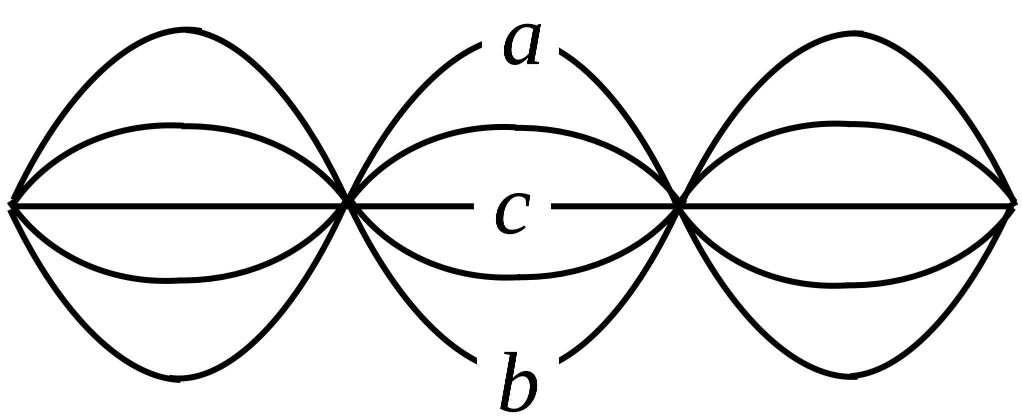
Second Harmonic or
First Overtone



Third Harmonic or
Second Overtone

Interactive Question

A string is clamped at both ends and plucked so it vibrates in a standing mode between two extreme positions *a* and *b*. Let upward motion correspond to positive velocities. When the string is in position *c*, the instantaneous velocity of points along the string:

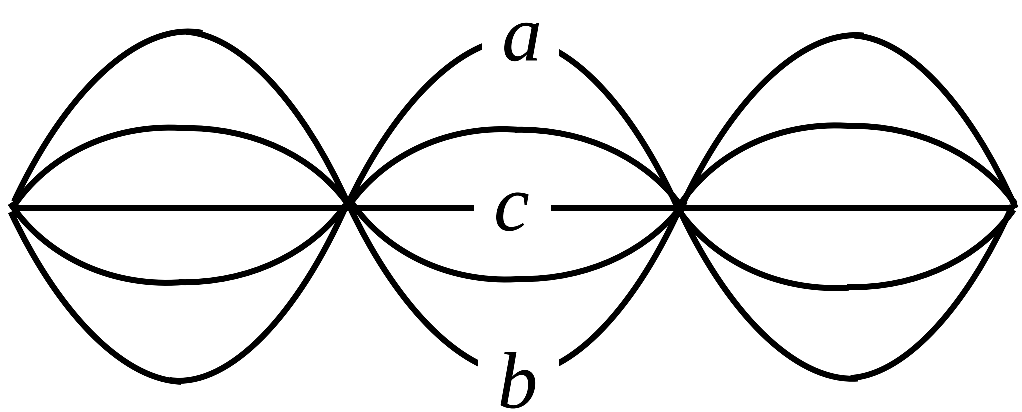


A) is zero everywhere B) is positive everywhere

C) is negative everywhere D) depends on location

Interactive Question

A string is clamped at both ends and plucked so it vibrates in a standing mode between two extreme positions *a* and *b*. Let upward motion correspond to positive velocities. When the string is in position *b*, the instantaneous velocity of points along the string:

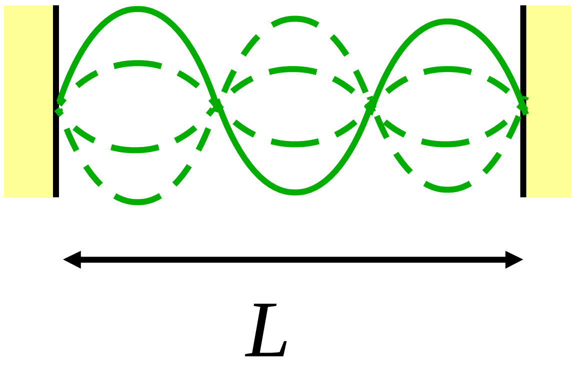


A) is zero everywhere

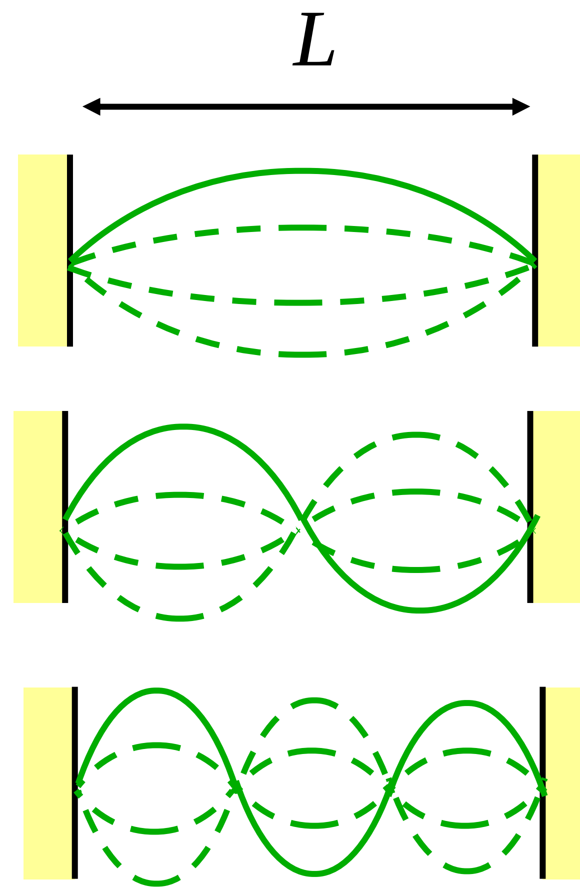
B) is positive everywhere

C) is negative everywhere

D) depends on location



$\lambda_n = 2L/n$ for the n^{th} harmonic



First Harmonic: $\lambda_1 = 2L$

Second Harmonic: $\lambda_2 = L$

Third Harmonic: $\lambda_3 = 2L/3$

Etc.

Let's derive an equation for standing waves:

A wave moving to the right: $y_1 = y_\mathrm{m} \sin(kx - \omega t)$

A wave moving to the left: $y_2 = y_\mathrm{m} \sin(kx + \omega t)$

$$
\begin{aligned}
y = y_1 + y_2 &= y_\mathrm{m} (\sin(kx - \omega t) + \sin(kx + \omega t)) \\
&= y_\mathrm{m} (\sin kx \cos \omega t - \cos kx \sin \omega t) + \\
&\qquad y_\mathrm{m} (\sin kx \cos \omega t + \cos kx \sin \omega t)
\end{aligned}
$$

$$y = 2y_\mathrm{m} \sin kx \cos \omega t$$

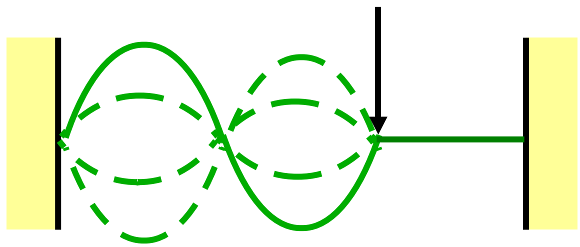
This is just a sine wave, fixed in space, with maximum amplitude $2y_\mathrm{m}$ oscillating up and down at a frequency of $\omega/2\pi$.

The standing wave is equivalent to two traveling waves moving in opposite directions, so we can still calculate the velocity and frequency of the waves moving on the string.

$v = \lambda / T = \lambda f \qquad \lambda_n = 2L/n \qquad v = \sqrt{F_{\mathrm{T}} / \mu}$

$$f_n = \frac{v}{\lambda_n} = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{F_{\mathrm{T}}}{\mu}}$$

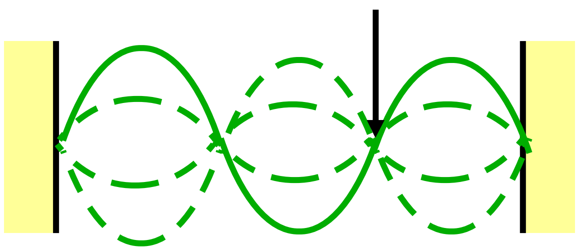
If you hold one point down and the rest doesn't vibrate, like playing a guitar:



$L' = (2/3)L$

$n = 1$

If you hold one point down, but let the rest vibrate :

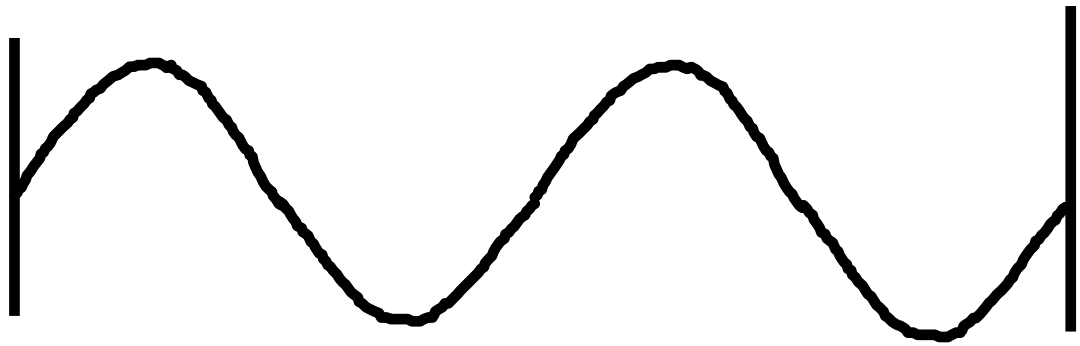


$L' = L$

$n = 3$

Interactive Question

The string shown below is fixed at each end and vibrating at 12 Hz.

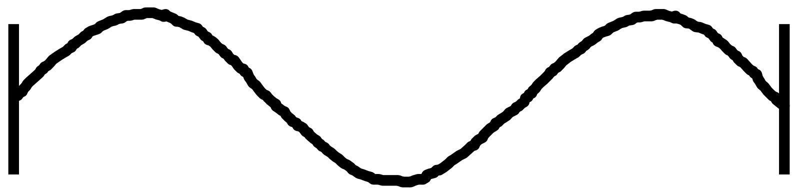


What is the fundamental frequency of this string?

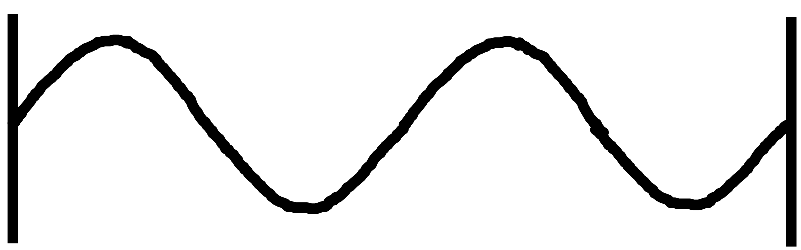
A) 48 Hz B) 24 Hz C) 8 Hz
D) 6 Hz E) 3 Hz

Interactive Question

The string shown below is fixed at each end and vibrating at 12 Hz.



If the string is then vibrated so that it produces the new pattern below, what is the frequency of vibration?



A) 48 Hz B) 36 Hz C) 16 Hz

D) 12 Hz E) 8 Hz

Problem: A violin string of length 33 cm is under a tension of 55 N. The fundamental frequency of the string is 196 Hz.

(a) At what speed do the waves travel on the string?

(b) What is the mass of the string?

(c) How far from one end of the string would you have to press the string in order that the remainder of the string have a fundamental frequency of 300 Hz?

This last problem illustrates a shortcut to solving some problems. Since $v = \sqrt{F_\mathrm{T}/\mu}$

For any string with a given tension and mass per unit length has the same velocity. So

$v = f_n\lambda_n = 2f_nL/n = f_m\lambda_m = 2f_mL/m$

$$f_n/n = f_m/m$$

where n is for one harmonic, and m is for another harmonic.

For the case when $m = 1$, $f_n = nf_1$

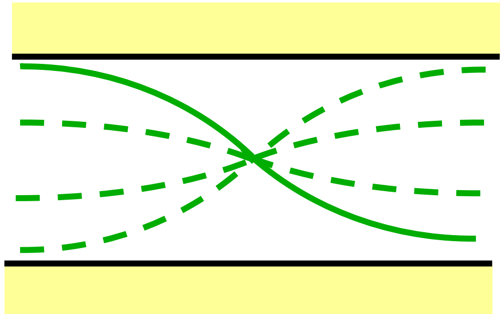
Problem: A string has its fundamental at 200 Hz. What is the difference in the frequency between the first two overtones?

Interactive Question

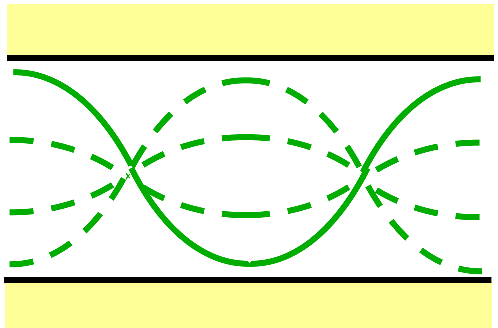
Consider the standing wave on a guitar string and the sound wave traveling through the air that is generated by the guitar string as a result of this vibration. What do the two waves have in common?

A) They have the same wavelength
B) They have the same velocity
C) They have the same frequency
D) None of the above is true.
E) More than one of the above is true.

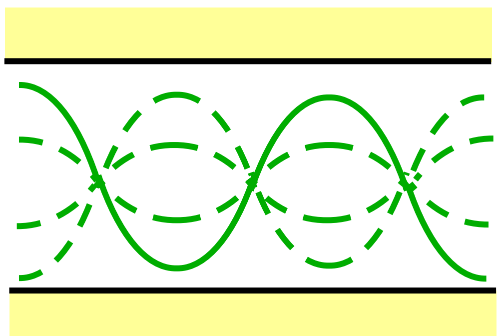
Harmonics in Tubes Open at Both Ends



Fundamental or
First Harmonic
$L = \lambda/2$, $f = v/2L$



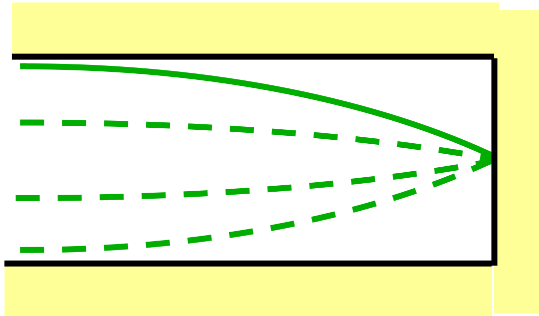
Second Harmonic or
First Overtone
$L = \lambda$, $f = v/L$



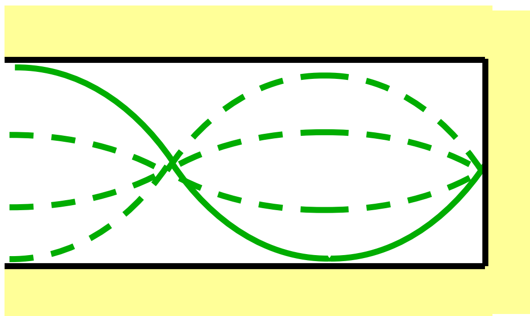
Third Harmonic or
Second Overtone
$L = 3\lambda/2$, $f = 3v/2L$

$\lambda_n = 2L/n$, $f = nv/2L$ $\quad n = 1, 2, 3, \ldots$

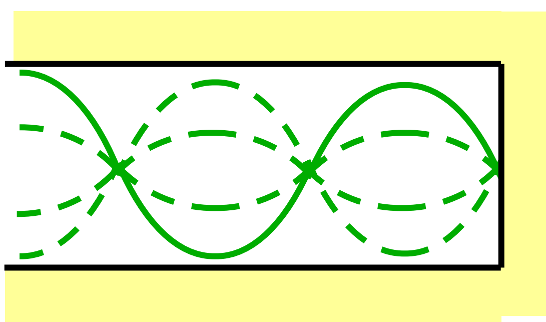
Harmonics in Tubes Open at One End



Fundamental or
First Harmonic
$L = \lambda/4$, $f = v/4L$



Second Harmonic or
First Overtone
$L = 3\lambda/4$, $f = 3v/4L$



Third Harmonic or
Second Overtone
$L = 5\lambda/4$, $f = 5v/4L$

$\lambda_n = 4L/n$, $f = nv/4L$ $\quad n = 1, 3, 5, \ldots$

Interactive Question

The lowest tone to resonate in an open pipe of length L is it fundamental frequency of 200 Hz. Which one of the following frequencies will not resonate in the same pipe?

A) 400 Hz
B) 500 Hz
C) 600 Hz
D)They will all resonate in the pipe.

Interactive Question

In a resonating pipe which is open at one end and closed at the other, there

A) are displacement nodes at each end.
B) are displacement antinodes at each end.
C) is a displacement node at the open end and a displacement antinode at the closed end.
D) is a displacement node at the closed end and a displacement antinode at the open end.

Problem: A flute with all of the holes closed can be considered as a tube with both ends open. It has a fundamental frequency of 261.6 Hz (which is middle C). If the air temperature is 20° C, how long is the flute?

Problem: A particular organ pipe can resonate at 264 Hz, 440 Hz, and 616 Hz, but not at any other intermediate frequencies.

(a) is the pipe open or closed?

(b) What is the fundamental frequency of this pipe?

Interactive Question

The lowest tone to resonate in an open pipe of length L is 400 Hz. What is the frequency of the lowest tone that will resonate in an open pipe of length $2L$?

A) 100 Hz
B) 200 Hz
C) 400 Hz
D) 800 Hz
E) 1600 Hz

Interactive Question

An open pipe of length L is resonating at its fundamental frequency. Which statement is true?

A) The wavelength is 2L and there is a displacement node at the pipe's midpoint.

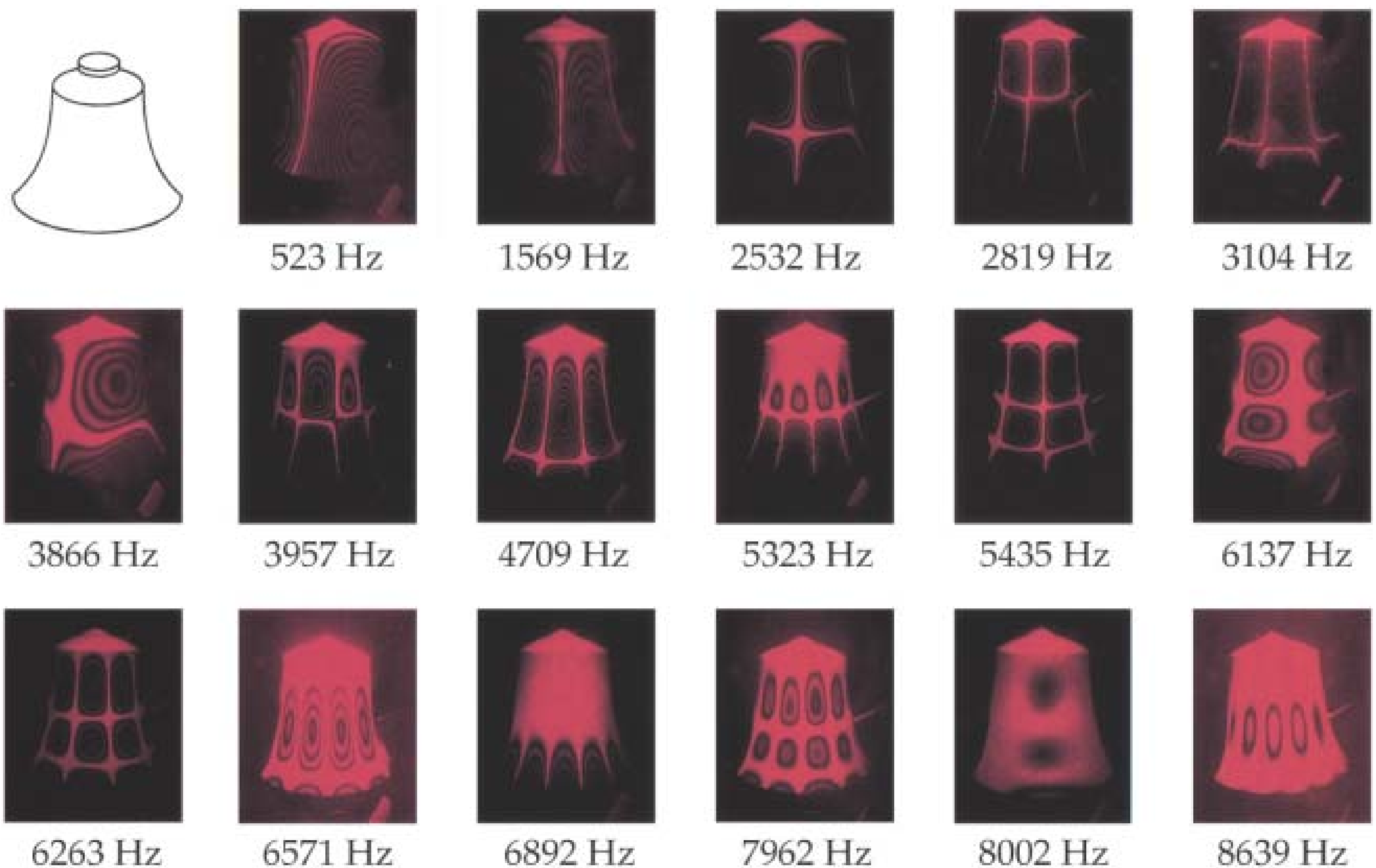
B) The wavelength is 2L and there is a displacement antinode at the pipe's midpoint.

C) The wavelength is L and there is a displacement node at the pipe's midpoint.

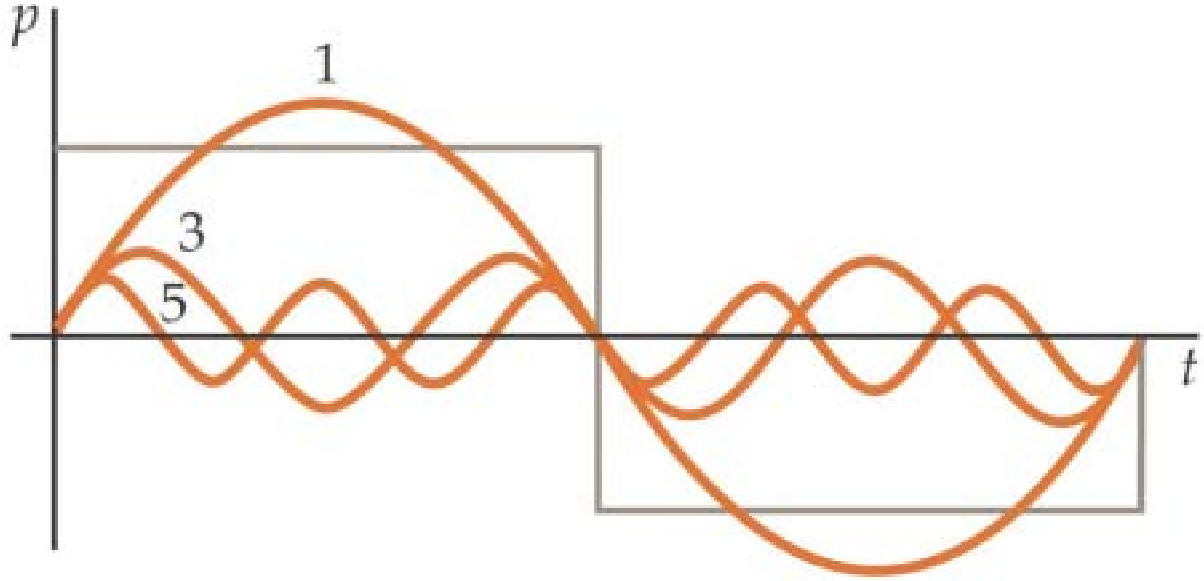
D) The wavelength is L and there is a displacement antinode at the pipe's midpoint.

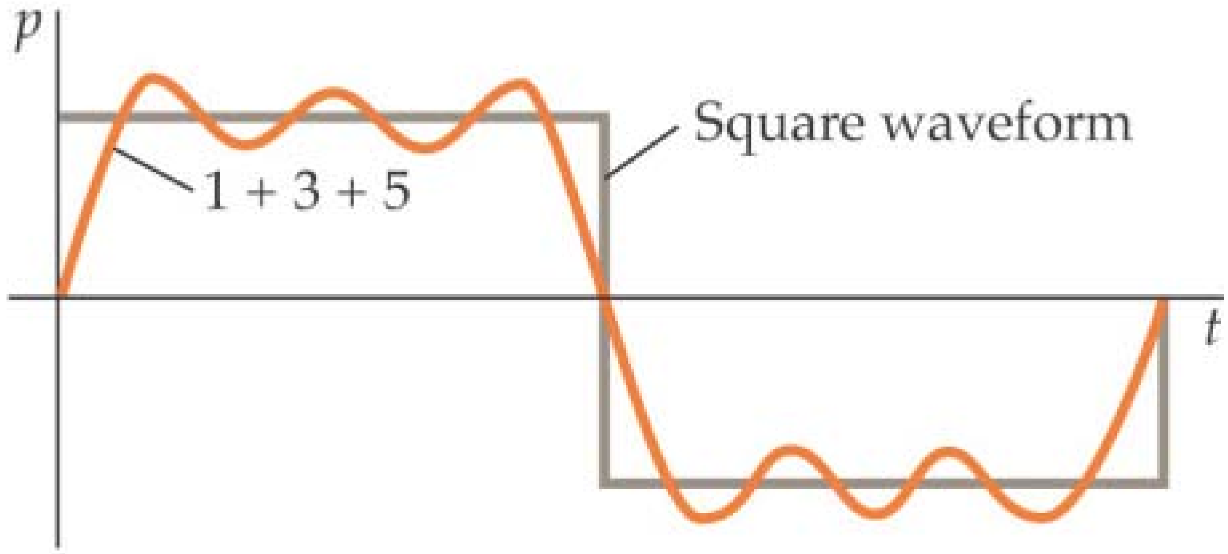
E) The wavelength is 4L and there is a displacement antinode at the pipe's midpoint.

Problem: A tuning fork is set into vibration above a vertical open tube filled with water. The water level is allowed to drop slowly. As it does so, the air in the tube above the water level is heard to resonate with the tuning fork when the distance from the tube opening to the water level is 0.125 m and again at 0.375 m. What is the frequency of the tuning fork?

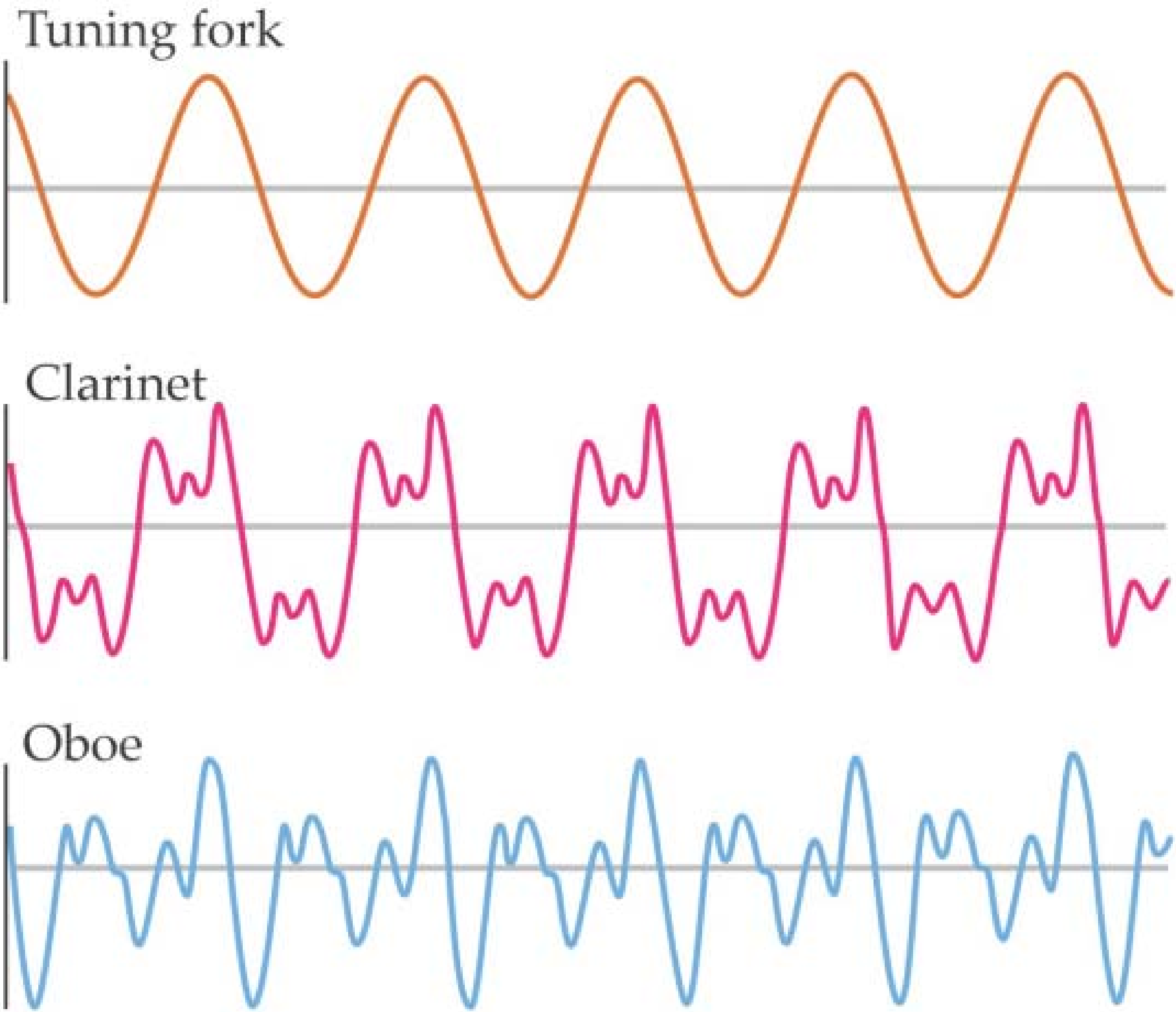

523 Hz
1569 Hz
2532 Hz
2819 Hz
3104 Hz
3866 Hz
3957 Hz
4709 Hz
5323 Hz
5435 Hz
6137 Hz
6263 Hz
6571 Hz
6892 Hz
7962 Hz
8002 Hz
8639 Hz

Fourier Analysis





Quality of Sound and Overtones



Overtones for Different Instruments