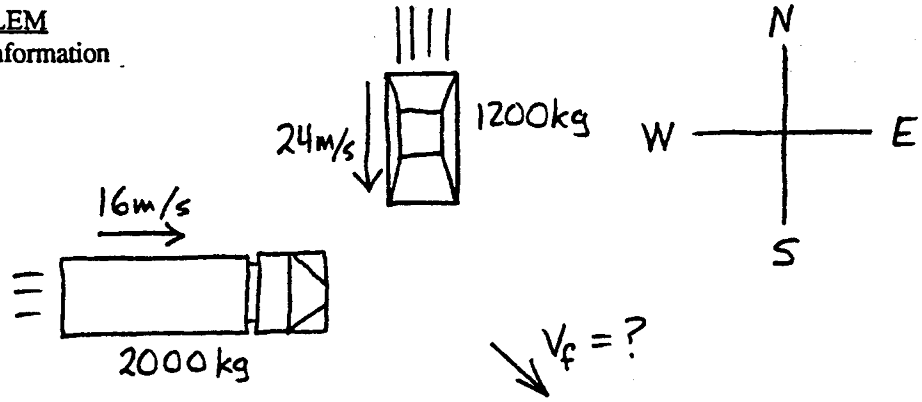
Problem #7: A 1200-kg car traveling south at 24-m/s collides with and attaches itself to a 2000-kg truck traveling east at 16-m/s. Calculate the velocity (magnitude and direction) of the two vehicles when locked together after the collision. (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 8-5)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the velocity (magnitude and direction) of the two vehicles when locked together after the collision?

Approach Use conservation of momentum. System: car and truck

Time: Initial time is the instant just prior to the collision.

Final time is the instant just after the collision.

Initial momentum due to car and truck separately.

Final momentum due to car/truck locked together.

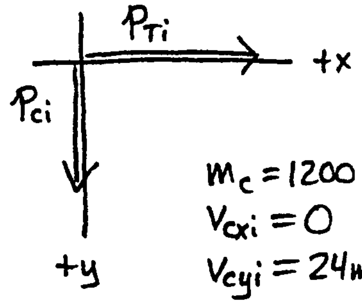
Assume no momentum transfer between system and environment.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

Momentum diagram

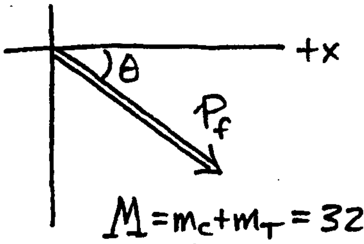
⟨initial state⟩ ⟨momentum transfer⟩ ⟨final state⟩



$m_c = 1200$ kg
$V_{cxi} = 0$
$V_{cyi} = 24$ m/s

NONE

$m_T = 2000$ kg
$V_{Txi} = 16$ m/s
$V_{Tyi} = 0$



$M = m_c + m_T = 3200$ kg
$V_f = ?$
$\theta = ?$

Target Quantity(ies) $\boxed{V_f, \theta}$

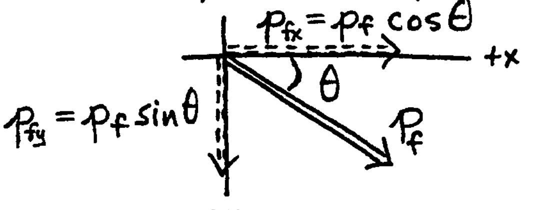
Quantitative Relationships

$p_{fx} - p_{ix} = J_x = 0$

$p_{fy} - p_{iy} = J_y = 0$

$p_f = M v_f$

Components of p_f



$p_{ix} = m_T V_{Txi}$ $\quad \therefore p_{fx} - m_T V_{Txi} = 0$

$p_{iy} = m_c V_{cyi}$ $\quad \therefore p_{fy} - m_c V_{cyi} = 0$

Also: $p^2 = p_x^2 + p_y^2$

PLAN the SOLUTION

Construct Specific Equations

Unknowns

$\boxed{v_f, \theta}$

Find v_f

$p_f = M v_f$ ① $\boxed{p_f}$

Find p_f

$p_f^2 = p_{fx}^2 + p_{fy}^2$ ② $\boxed{p_{fx}, p_{fy}}$

Find p_{fx}

$p_{fx} - m_T v_{Txi} = 0$ ③

$p_{fx} = m_T v_{Txi}$

Find p_{fy}

$p_{fy} - m_c v_{cyi} = 0$ ④

$p_{fy} = m_c v_{cyi}$ Ⓐ

$p_f^2 = (m_T v_{Txi})^2 + (m_c v_{cyi})^2$

$p_f = \sqrt{(m_T v_{Txi})^2 + (m_c v_{cyi})^2}$

$\sqrt{(m_T v_{Txi})^2 + (m_c v_{cyi})^2} = M v_f$

$$\boxed{v_f = \frac{\sqrt{(m_T v_{Txi})^2 + (m_c v_{cyi})^2}}{M}}$$

Find θ

$p_{fy} = p_f \sin\theta$ ⑤

$\sin\theta = p_{fy}/p_f$

$\theta = \arcsin(p_{fy}/p_f)$

$$\boxed{\theta = \arcsin\left(\frac{m_c v_{cyi}}{M v_f}\right)}$$

← result Ⓐ

← equation ①

↑use previous result for v_f

Check Units

$$v_f: \frac{\sqrt{([kg][m/s])^2 + ([kg][m/s])^2}}{[kg]} = \frac{[kg][m/s]}{[kg]} = [m/s] \quad \text{OK}$$

continued

$$\theta: \arcsin\left(\frac{[kg][m/s]}{[kg][m/s]}\right) = \arcsin([\,]) \quad \text{OK}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$v_f = \frac{\sqrt{(2000\,kg)^2(16\,m/s)^2 + (1200\,kg)^2(24\,m/s)^2}}{3200\,kg}$$

$$\boxed{v_f = 13.5\,m/s}$$

$$\theta = \arcsin\left(\frac{(1200\,kg)(24\,m/s)}{(3200\,kg)(13.5\,m/s)}\right) = \arcsin(0.6$$

$$\boxed{\theta = 42° \text{ South of East}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected v_f has units of velocity and θ is an angle.

Is Answer Unreasonable?

No. v_f is in the right ballpark and θ is nearly 45° as expected since the two initial perpendicular momenta are similar in size.

Is Answer Complete?

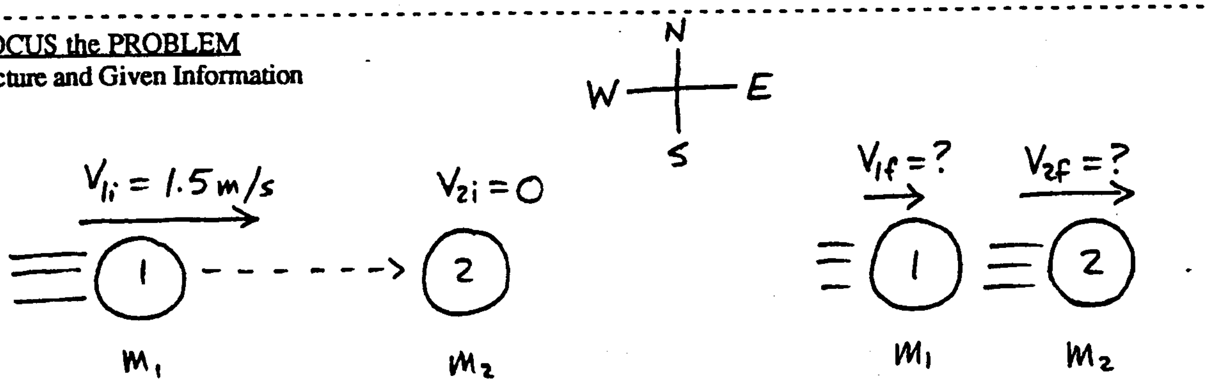
Yes. The magnitude and direction of the final velocity have been found which answers the question.

(extra space if needed)

Problem #8: A billiard ball at rest is hit head-on by a second billiard ball moving 1.5-m/s toward the east. If the collision is elastic and we ignore rotational motion, calculate the final speed of each ball. (Based on Fishbane, Gasiorowicz and Thornton 1993, example 8-8)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the final speed of each ball?

Approach Use conservation of momentum and conservation of energy.

System: ball 1 and ball 2

Time: Initial time is the instant before the collision.
Final time is the instant after the collision.

Initial mom. is due to ball 1. Final mom. is due to balls 1 and 2.

No net momentum transfer between system and environment.

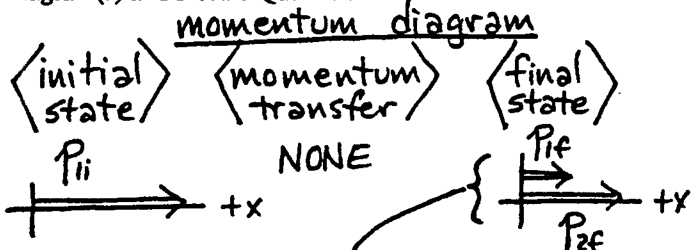
Initial energy is kinetic. Final energy is kinetic.

No energy input or output. "Elastic" collision ⇒ no kinetic energy lost.

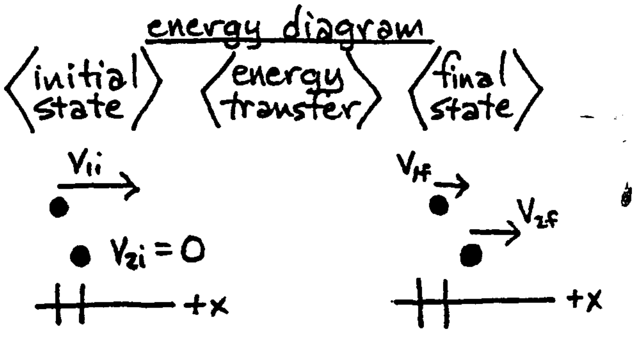
Assume the balls have the same mass.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities



Relative sizes of final momenta unknown at this point.



$V_{1i} = 1.5\,m/s$ $V_{1f} = ?$

$V_{2i} = 0$ $V_{2f} = ?$

$m_1 = m_2 = m = ?$

Target Quantity(ies) $\boxed{V_{1f}, V_{2f}}$

Quantitative Relationships

$$P_f - P_i = J = 0$$

$$p_f = mV_{1f} + mV_{2f}$$

$$p_i = mV_{1i}$$

$$\therefore \cancel{m}V_{1f} + \cancel{m}V_{2f} - \cancel{m}V_{1i} = 0$$

$$\Rightarrow V_{1f} + V_{2f} - V_{1i} = 0$$

$$E_f - E_i =$$

$$E_f = \tfrac{1}{2}mV_{1f}^2 + \tfrac{1}{2}mV_{2f}^2$$

$$E_i = \tfrac{1}{2}mV_{1i}^2$$

$$\therefore \cancel{\tfrac{1}{2}}\cancel{m}V_{1f}^2 + \cancel{\tfrac{1}{2}}\cancel{m}V_{2f}^2 - \cancel{\tfrac{1}{2}}\cancel{m}V_{1i}^2 = 0$$

$$\Rightarrow V_{1f}^2 + V_{2f}^2 - V_{1i}^2 = 0$$

PLAN the SOLUTION

Construct Specific Equations

Unknowns

$\boxed{V_{1f}, V_{2f}}$

Find V_{1f}

$V_{1f} + V_{2f} - V_{1i} = 0$ ①

$V_{1f} = V_{1i} - V_{2f}$ Ⓐ

Find V_{2f}

$V_{1f}^2 + V_{2f}^2 - V_{1i}^2 = 0$ ②

$(V_{1i} - V_{2f})^2 + V_{2f}^2 - V_{1i}^2 = 0$

used result Ⓐ

$\cancel{V_{1i}^2} + V_{2f}^2 - 2V_{1i}V_{2f} + V_{2f}^2 - \cancel{V_{1i}^2} = 0$

$\cancel{2}V_{2f}^2 = \cancel{2}V_{1i}V_{2f}$

$\boxed{V_{2f} = V_{1i}}$ assumed $V_{2f} \neq 0$

$V_{1f} = V_{1i} - V_{1i}$

$\boxed{V_{1f} = 0}$

Check Units

[m/s] OK

EXECUTE the PLAN

Calculate Target Quantity(ies)

$V_{2f} = 1.5\,m/s$

$V_{1f} = 0$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected V_{2f} has units of velocity.

Is Answer Unreasonable?

No. All the momentum and kinetic energy get transferred from ball 1 to ball 2 in the collision.

Is Answer Complete?

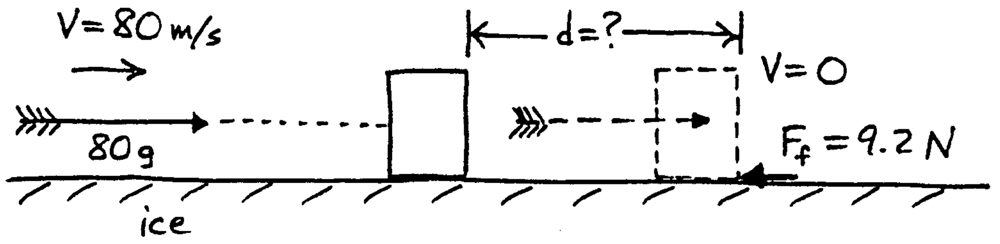
Yes. V_{1f} and V_{2f} are the final speeds of the two balls. This answers the question.

(extra space if needed)

Problem #9: An 80-g arrow moving at 80-m/s hits embeds itself in a 10-kg block resting on ice. How far does the block slide on the ice following the collision if it is opposed by a 9.2-N force? (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 8-7)

FOCUS the PROBLEM

Picture and Given Information



Question(s) How far does the block slide before coming to rest?

Approach Use conservation of momentum to determine the recoil velocity of the block/arrow just after the collision. System: block + arrow

Then use conservation of energy to determine how far the block/arrow slides following the collision. Same system as above.

Time: Initial time is the instant just before arrow strikes block.
Middle time is the instant just after the collision.
Final time is the instant block/arrow comes to rest.

Just before collision momentum due to arrow. Assume arrow motion is is strictly horizontal when it strikes the block.

Just after collision momentum due to block/arrow.

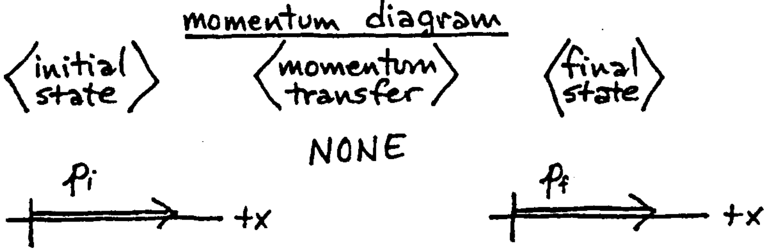
No net momentum transfer between system and environment.

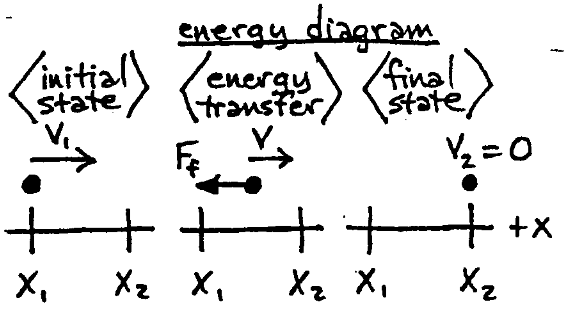
Energy just after collision is kinetic. Final energy is zero.

Input energy is zero. Output energy due to frictional force.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities





$V_{0a} = 80$ m/s $V_{0b} = 0$ $V_1 = ?$

$m_a = 80g = 0.080kg$ $m_b = 10kg$ $M = m_a + m_b = 10.08kg$ $X_1 = 0$ $X_2 = ?$

$F_f = 9.2N$

Target Quantity(ies) $\boxed{X_2}$

Quantitative Relationships

$$p_f - p_i = J = 0$$
$$p_f = Mv_1$$
$$p_i = m_a v_{0a}$$
$$\therefore\ Mv_1 - m_a v_{0a} = 0$$

$$\overset{0}{\cancel{E_f}} - E_i = W_{NC}$$
$$E_i = \tfrac{1}{2}Mv_1^2$$
$$W_{NC} = \int_{x_1}^{x_2} F_f \cdot dx$$
$$\therefore\ \tfrac{1}{2}Mv_1^2 = \int_{x_1}^{x_2} F_f \cdot dx$$

PLAN the SOLUTION
Construct Specific Equations

unknowns

$\boxed{x_2}$

$\boxed{v_1}$

Find x_2

$$\frac{1}{2}Mv_1^2 = \int_{x_1}^{x_2} F_f \cdot dx \quad ①$$

$$\frac{1}{2}Mv_1^2 = \left[F_f\, x\right]_{x_1}^{x_2}$$

$$\frac{1}{2}Mv_1^2 = F_f\left(x_2 - \cancelto{0}{x_1}\right)$$

Find v_1

$$Mv_1 - m_a v_{0a} = 0 \quad ②$$

$$Mv_1 = m_a v_{0a}$$

$$v_1 = \frac{m_a v_{0a}}{M}$$

$$\frac{1}{2}M\left(\frac{m_a v_{0a}}{M}\right)^2 = F_f x_2$$

$$\frac{m_a^2 v_{0a}^2}{2M} = F_f x_2$$

$$\boxed{x_2 = \frac{m_a^2 v_{0a}^2}{2MF_f}}$$

Check Units

$$\frac{\left([kg][m/s]\right)^2}{[kg][N]} = \frac{[kg]^2[m/s]^2}{[kg][kg\ m/s^2]}$$

$$= [m] \qquad \text{OK}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$x_2 = \frac{(0.080\,kg)^2(80\,m/s)^2}{2(10.08\,kg)(9.2\,N)}$$

$$\boxed{x_2 = 0.22\,m}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. As expected x_2 has units of length.

Is Answer Unreasonable?

No. The block is quite massive so it doesn't slide back very far.

Is Answer Complete?

Yes. x_2 is the distance the block slides which answers the question.

(extra space if needed)