Context-Rich Problems

The "Context-Rich Problems: Solutions Outline" is a method of problem solving developed by Dr. Kenneth Heller at the University of Minnesota which helps solve problems that may appear very difficult. Since there are many problems in physics, and in life, like this, I will be using this outline during this class. The steps in the "Context-Rich Problems: Solutions Outline" are steps you should be using anyway. They are spelled out explicitly in the Outline. Below are some ideas about what each section should contain.

FOCUS the PROBLEM:

- In this step you should draw a picture using all given information. Once you have drawn the picture you should not have to look at the problem again to get any information. If a value for acceleration, distance, etc. is given, it should be written on the picture. The picture should not be done in "physics" language, but rather simply a picture with all the information.
- Question(s): Don't write down lots of different questions. Only write down what specifically is asked for in the problem, not how you will solve it.
- Approach: Write a brief description of how you expect to solve the problem

DESCRIBE THE PHYSICS:

- Physics Diagram(s) and Define Quantities. The "physics" picture you draw will depend on the type of problem you are solving. For a problem involving kinematics, you should draw a "motion diagram" showing *all* of the important moments, or *different* times/locations, in the problem labeled with a different subscript. The number of important times/locations will depend on the problem. For every important time, you should determine the position, the velocity, and the time, as well as the acceleration between different times.
- It is essential that every unique variable have a unique symbol, or subscript.
  - Always use a different subscript at each different time, (like t_1, x_1, and v_1, for the first time, position, and velocity, and t_2, x_2, and v_2, for the second time, position, and velocity). For every different time, you should either write what the value is (something like v_1 = 15.6 m/s) or write that you don't know the value, (like t_2 = ?). For every *different important instance* you need to have a *time, position, and velocity written with a different subscript.* (In two dimensions, you need these variables in x and y.) Between every two different times, write what the acceleration is between those two different times. It may be a constant, like a_2 = -8.0 m/s^2, or it may be zero like a_2 = 0 m/s^2.
  - If you have more than one object in the problem then the subscripts on the variable must indicate the correct object and the correct location. For instance if your problem involves a boy named Bill and a girl named Gina, then the location of Bill along the horizontal axis at the initial time might be labeled x_{B1}, and the location of Gina at the same time might be labeled x_{G1}.
- Target Variable(s): Write only what is actually being asked for in the problem, using the exact subscript(s) you defined in the "Physics Diagram and Defined Quantities." There should be no more target variables than what is actually being asked for. Make sure you have the target variable with the appropriate subscript. If you want the third time, write t_3 for the target, not just t.
- Quantitative Relations: Write the equations you will use for the problem based on the approach you described. Any general equations you might use must be written here.
- Once you finish this step you should not have to look at the problem again, or at the book or any other information, because you have everything you need to solve the problem written on this first page.

PLAN THE SOLUTION:

- Construct Specific Equations: If you don't know where to start, then start with an equation with your target variable, and with as many other variables as you know. If possible, it is often good to use the initial moment in this equation since many of those variables will be set to zero. Make sure you use appropriate subscripts as defined in "DESCRIBE THE PHYSICS." Only two subscripts may be used in any single kinematic equation at this point. For instance, you can not mix times with subscripts 2 and 1 with positions with subscripts 2 and 3.

- Make sure you use equations only when they are valid. For instance, the kinematic equations are only valid if the acceleration is a constant
- Once you have one equation with your target variable in the equation, write exactly what your unknown quantities are. These are any value used in the equation that you did not know in "DESCRIBE THE PHYSICS" (like t_2 = ?). Use the same subscripts defined that section. If you have more unknown quantities than equations write another equation and keep track of the unknowns, and rearrange the equations as necessary.
- As much as possible and reasonable, you should be doing almost all of the algebra without solving for actual numbers.
- When you have a final equation which solves for the target variable, make sure the units are correct.

EXECUTE THE PLAN:

- Just plug numbers into your final equation and get the answer.

EVALUATE YOUR ANSWER:

- Have you answered the question asked. A simple numerical answer may not always answer the question that was asked.
- Is your answer unreasonable? You may want to use order-of-magnitude estimations to check this if you aren't sure, or use common sense.

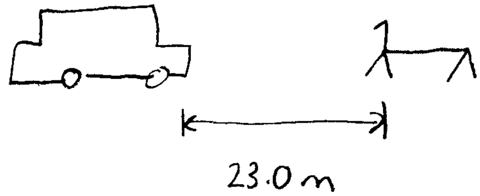
Context-Rich Problems: Example 1

While driving through a national park at 35.0 mph (15.6 m/s) a deer jumps on the road 23.0 meters in front of your car. You slam on your brakes and stop just in time. You then begin to wonder how good your reflexes were and how long it took for you to react from the time you saw the deer until the time you slammed on your brakes (your reaction time). Of course, you speculate that during that time the car continued to travel at a constant speed. You then take your car to an empty parking lot and by braking hard you determine that your car can decelerate at $-8.0\ m/s^2$. What was your reaction time?

<u>FOCUS the PROBLEM</u>

Draw a picture of the situation including ALL the information given in the problem.

$a = -8.0\ m/s$



Question(s): What is the problem asking you to find?

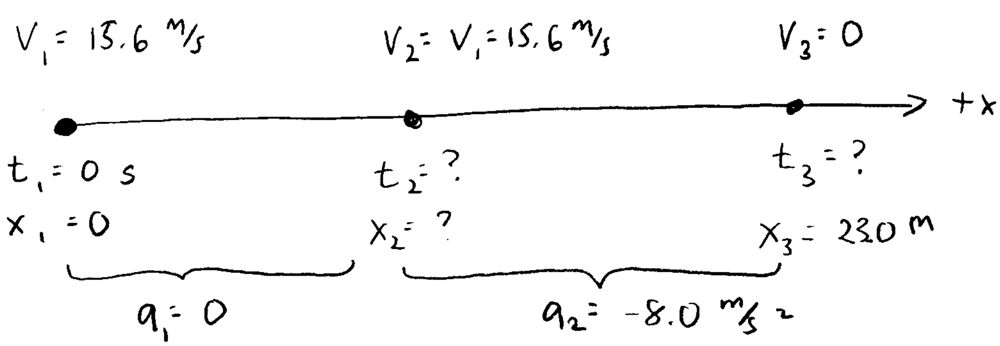
How long does it take from the time I notice the deer until the time I slam on my brakes?

Approach: Outline the approach you will use.

Use kinematic equations with a constant velocity before hitting the brakes and a constant acceleration after hitting the brakes

<u>DESCRIBE the PHYSICS</u>

Draw physics diagram(s) and define ALL quantities uniquely.



Which of your defined quantities is your Target variable(s)?

$\boxed{t_2}$ This is my reaction time

Quantitative Relationships: Write equations you will use to solve this problem.

$$x_2 - x_1 = v_1(t_2 - t_1) - \frac{1}{2}a(t_2 - t_1)^2$$

$$V_2^2 = V_1^2 + 2a(x_2 - x_1)$$

$$V_2 = V_1 + a(t_2 - t_1)$$

PLAN the SOLUTION
Construct Specific Equations (Same Number as Unknowns)

Equations — UNKNOWNS

Find t_2

$x_2 - \cancel{x_1}^{0} = v_1(t_2 - \cancel{t_1}^{0}) + \frac{1}{2}\cancel{a_1}^{0}(t_2 - t_1)^2$

$\boxed{x_2, t_2}$

(1) $x_2 = v_1 t_2$

Find x_2

$\cancel{v_3^2}^{0} = v_2^2 + 2a_2(x_3 - x_2)$

$-v_2^2 = 2a_2(x_3 - x_2)$

Since $v_2 = v_1$

(2) $-v_1^2 = 2a_2(x_3 - x_2)$

I now have 2 equations, 2 unknowns, so I just work backwards

From (2),

$-v_1^2 = 2a_2 x_3 - 2a_2 x_2$

$x_2 = \frac{v_1^2}{2a_2} + x_3$

Plug (1) into this

$v_1 t_2 = \frac{v_1^2}{2a_2} + x_3$

$\Rightarrow \boxed{t_2 = \frac{v_1}{2a_2} + \frac{x_3}{v_1}}$

Check Units

$$\frac{[m]/[s]}{[m]/[s]^2} + \frac{[m]}{[m]/[s]} = [s] + [s] \quad \text{ok}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$t_2 = \frac{15.6\ m/s}{2(-8.0\ m/s^2)} + \frac{23.0\ m}{15.6\ m/s}$$

$$= \boxed{0.50\ s}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, in seconds

Is Answer Unreasonable?

No, ½ second seems about the right amount of time to notice the deer and slam on my brakes

Is Answer Complete?

yes, this is the reaction time

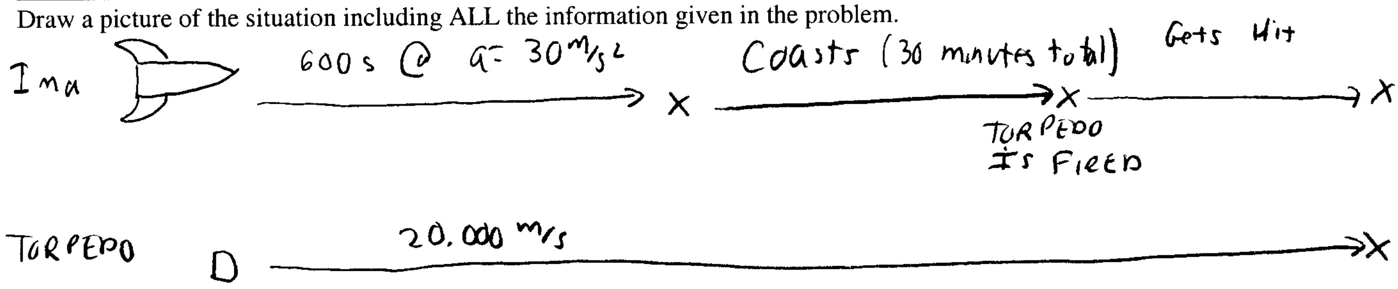
(extra space if needed)

Context-Rich Problems: Example 2

You have been hired for the summer as a technical advisor on a science fiction movie. In a certain scene, the villain, Ima Pill, escapes from a space station in deep space. She steals a small space ship and blasts off. Her ship accelerates at 30 m/s^2, but after 10 minutes all of her fuel is burned up and the ship coasts at a constant velocity. Meanwhile, the hero, Major Starr, learns of the escape and immediately rushes off to fire photon torpedoes at Ima. Once fired, the torpedo travels at 20,000 m/s. By the time the torpedoes are fired, Ima has a 30 minute head start. The director want you to tell her how long Ima will be in the spaceship before the torpedoes hit her.

FOCUS the PROBLEM

Draw a picture of the situation including ALL the information given in the problem.



Question(s): What is the problem asking you to find?

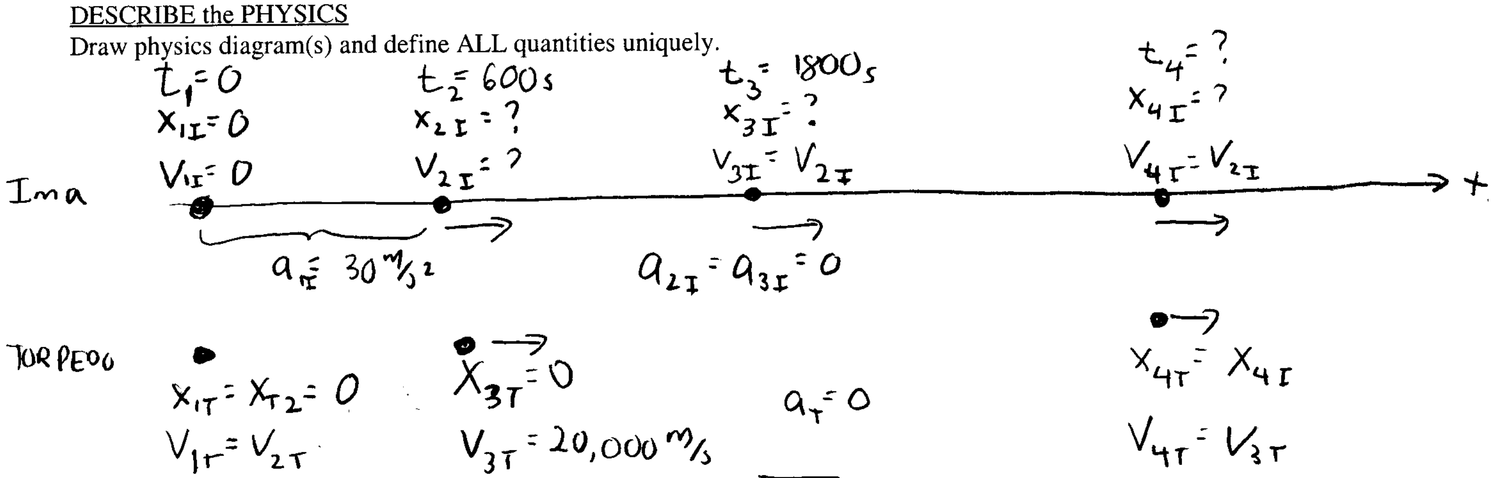
How long does Ima have in the ship before getting hit by the torpedo

Approach: Outline the approach you will use.

Use kinematic, constant acceleration equations. Torpedo has a constant velocity. Ima has acceleration for 10 minutes then a constant velocity

DESCRIBE the PHYSICS

Draw physics diagram(s) and define ALL quantities uniquely.



Which of your defined quantities is your Target variable(s)? $\boxed{t_4}$

Quantitative Relationships: Write equations you will use to solve this problem.

$$x_2 - x_1 = V_1(t_2 - t_1) + \tfrac{1}{2}a(t_2 - t_1)^2$$

$$V_2^2 = V_1^2 + 2a(x_2 - x_1)$$

$$V_2 = V_1 + a(t_2 - t_1)$$

PLAN the SOLUTION

Construct Specific Equations (Same Number as Unknowns)

Equations | Unknowns

Find t_4

$$x_{4I} - x_{2I} = v_{2I}(t_4 - t_2) + \frac{1}{2}\cancel{a_{2I}}^{0}(t_4 - t_2)^2$$

① $x_{4I} - x_{2I} = v_{2I}(t_4 - t_2)$ — t_4, x_{4I}, v_{2I}, x_{2I}

Find v_{2I}

$$v_{2I} = a_{1I}(t_2 - \cancel{t_1}^{0})$$

② $v_{2I} = a_{1I} t_2$

Find x_{2I}

$$x_{2I} - \cancel{x_{1I}}^{0} = \cancel{v_{1I}}^{0}(t_2 - t_1) + \frac{1}{2} a_{1I}(t_2 - \cancel{t_1}^{0})^2$$

③ $x_{2I} = \frac{1}{2} a_{1I} t_2^2$

Since $x_{4I} = x_{4T}$, Find x_4 using torpedo

$$x_{4T} - \cancel{x_{3T}}^{0} = v_{3T}(t_4 - t_3) + \frac{1}{2}\cancel{a_T}^{0}(t_4 - t_3)^2$$

④ $x_{4T} = x_{4I} = v_{3T}(t_4 - t_3)$

Plug ④, ③, ②, into ①

$$v_{3T}(t_4 - t_3) - \frac{1}{2} a_{1I} t_2^2 = a_{1I} t_2 (t_4 - t_2)$$

$$v_{3T} t_4 - v_{3T} t_3 - \frac{1}{2} a_{1I} t_2^2 = a_{1I} t_2 t_4 - a_{1I} t_2^2$$

$$t_4(v_{3T} - a_{1I} t_2) = v_{3T} t_3 - \frac{1}{2} a_{1I} t_2^2$$

$$\boxed{t_4 = \frac{v_{3T} t_3 - \frac{1}{2} a_{1I} t_2^2}{v_{3T} - a_{1I} t_2}}$$

Check Units

$$\frac{\frac{[m]}{[s]}[s] - \frac{[m]}{[s]^2 2}[s]^2}{\frac{[m]}{[s]} - \frac{[m]}{[s]^2}[s]} = \frac{(m) - (m)}{\frac{[m]}{[s]}}$$

$$= [s] \text{ ok}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$t_4 = \frac{(20{,}000\ \text{m/s})(600\ \text{s}) - \frac{1}{2}(30\ \text{m/s}^2)(600\ \text{s})^2}{(20{,}000\ \text{m/s}) - (30\ \text{m/s}^2)(600\ \text{s})}$$

$$= 15{,}300\ \text{s} \times \left(\frac{1\ \text{min}}{60\ \text{sec}}\right) \times \left(\frac{1\ \text{hour}}{60\ \text{min}}\right)$$

$$= \boxed{4.3\ \text{hours}}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes, this is the time it takes for the photon torpedo to hit the target

Is Answer Unreasonable?

No, the ship had a 30 minute head start and is traveling at $v_2 = a_1 t_2 = 18{,}000\ \text{m/s}$, almost as fast the torpedo

Is Answer Complete?

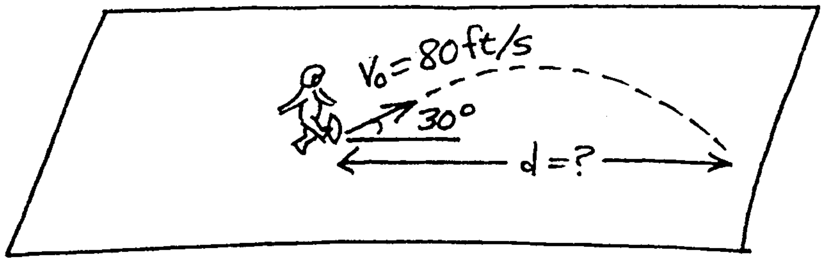
No, tell the director that if the torpedo doesn't go faster this will be one long movie.

(extra space if needed)

Problem #2: A football player kicks off from the 40-yd line. How far will the ball travel before hitting the ground if its initial speed is 80-ft/s and the ball leaves the ground at an angle of 30°? (Assume that air resistance can be ignored.) (Similar to Fishbane, Gasiorowicz and Thornton 1993, example 3-7)

FOCUS the PROBLEM

Picture and Given Information



Question(s) How far will the ball travel before hitting the ground?

Approach Use kinematics; handle vertical and horizontal motion separately.

-- horizontal motion at constant velocity

-- vertical motion at constant acceleration

Time: Initial time is the instant after ball is kicked.

Final time is the instant ball lands.

Assume air resistance can be neglected.

DESCRIBE the PHYSICS

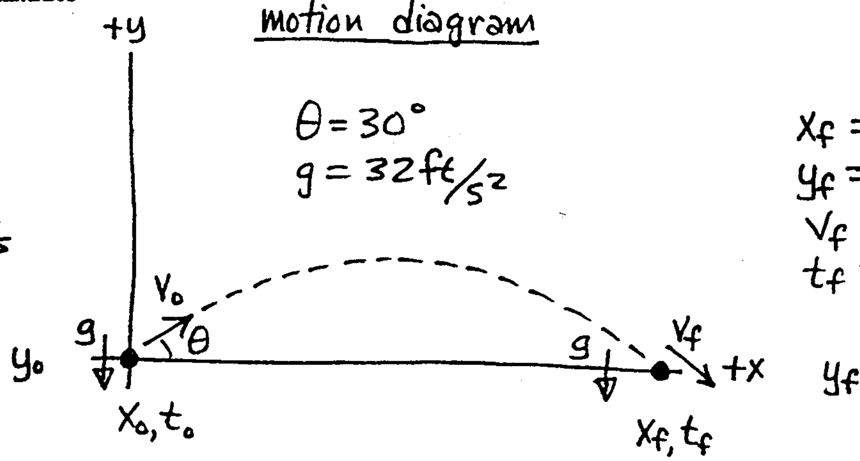
Diagram(s) and Define Quantities

motion diagram

$x_0 = 0$
$y_0 = 0$
$V_0 = 80\ \text{ft/s}$
$t_0 = 0$

$\theta = 30°$
$g = 32\ \text{ft/s}^2$

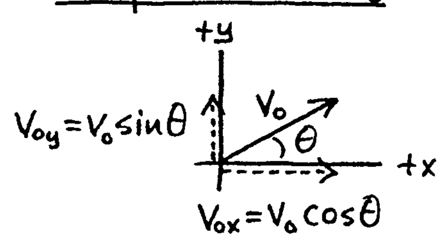
$x_f = ?$
$y_f = 0$
$V_f = ?$
$t_f = ?$



Target Quantity(ies) $\boxed{x_f}$

Quantitative Relationships

components of V₀



Constant velocity in x-direction so use

$$V_{ox} = \bar{V}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - \cancel{x_0}^{\to 0}}{t_f - \cancel{t_0}^{\to 0}} \Rightarrow V_{ox} = x_f/t_f$$

Constant acceleration (-g) in y-direction so use

$$\cancel{y_f}^{\to 0} = \frac{1}{2}(-g)(t_f - \cancel{t_0}^{\to 0})^2 + V_{oy}(t_f - \cancel{t_0}^{\to 0}) + \cancel{y_0}^{\to 0} \Rightarrow 0 = -\frac{1}{2}g t_f^2 + V_{oy} t_f$$

PLAN the SOLUTION
Construct Specific Equations

Unknowns

$\boxed{x_f}$

Find x_f

$v_{ox} = x_f / t_f$ (1)

$\boxed{t_f, v_{ox}}$

Find v_{ox}

$v_{ox} = v_o \cos\theta$ (2)

$v_o \cos\theta = x_f / t_f$

Find t_f

$0 = -\frac{1}{2} g t_f^2 + v_{oy} t_f$ (3)

$\boxed{v_{oy}}$

Find v_{oy}

$v_{oy} = v_o \sin\theta$ (4)

$0 = -\frac{1}{2} g t_f^2 + v_o \sin\theta \, t_f$

$\frac{1}{2} g t_f^{\cancel{2}} = v_o \sin\theta \, \cancel{t_f}$

$t_f = \frac{2 v_o \sin\theta}{g}$

$v_o \cos\theta = x_f \frac{g}{2 v_o \sin\theta}$

$$\boxed{x_f = \frac{2 v_o^2 \cos\theta \sin\theta}{g}}$$

Check Units

$\frac{[ft/s]^2}{[ft/s^2]} = [ft]$ OK

EXECUTE the PLAN
Calculate Target Quantity(ies)

$x_f = \frac{2(80 ft/s)^2 \cos(30°) \sin(30°)}{32 ft/s^2}$

$x_f = 173 \text{ ft}$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected x_f has units of length.

Is Answer Unreasonable?

No. 173 ft is nearly 58 yards -- a good kick off.

Is Answer Complete?

Yes. 173 ft is the distance down field the ball travels which answers the question.

(extra space if needed)