

Context-Rich Problems: Energy

FOCUS the PROBLEM: (This is done the same way as all context-rich problems).

- As with all the context rich problems, in this step you should draw a picture using all given information. Once you have drawn the picture you should not have to look at the problem again to get any information. The picture should not be done in "physics" language, but rather simply a picture with all the information.
- Question(s): Write down what specifically is asked for in the problem, not how you will solve it.
- Approach: Write a brief description of how you expect to solve the problem. You should include "conservation of energy." You may also need to use other approaches like kinematics or dynamics.

DESCRIBE THE PHYSICS:

- For problems involving conservation of energy you should draw a 3 part diagram. The first part should be a coordinate system showing the initial conditions, or state. This should include everything you need to determine the kinetic energy and potential energy of the system. The variables should including all values of mass, velocity, height and spring compression distances. (After all that is what you will need for kinetic energy, gravitational potential energy, and elastic potential energy. The third part of the diagram should be the same thing as the first part, but for the final state. The second (middle) part of the diagram should be an energy transfer diagram. This should show all forces that have a component along the same axis as the motion. It is from this diagram that you will determine the work done by nonconservative forces. Make sure every unique variable is uniquely labeled.
- If you also need to use kinematics draw a motion diagram. If you also need to use dynamics draw a free body diagram.
- Target Variable(s): (This is done the same way as all context-rich problems).
As always, write only what is actually being asked for in the problem, using the exact subscript(s) you defined in the "Physics Diagram and Defined Quantities." There should be no more target variables than what is actually being asked for
- Quantitative Relations: Write the equations you will use for the problem based on the approach you described. Any general equations you might use must be written here. For conservation of energy, two equations that must be written down are:
 - $W_{NC} = \Delta E = \Delta K + \Delta U$
 - $W = \int \mathbf{F} \cdot d\mathbf{l}$

PLAN THE SOLUTION: (This is done the same way as all context-rich problems.)

- Construct Specific Equations: If you don't know where to start, then start with an equation with your target variable, and with as many other variables as you know. Make sure you use appropriate subscripts as defined in "DESCRIBE THE PHYSICS."
- Once you have one equation with your target variable in the equation, write exactly what your unknown quantities are. Use the same subscripts defined in the previous section. If you have more unknown quantities than equations write another equation and keep track of the unknowns, and rearrange the equations as necessary.
- As much as possible and reasonable, you should be doing almost all of the algebra without solving for actual numbers.
- When you have a final equation which solves for the target variable, make sure the units are correct.

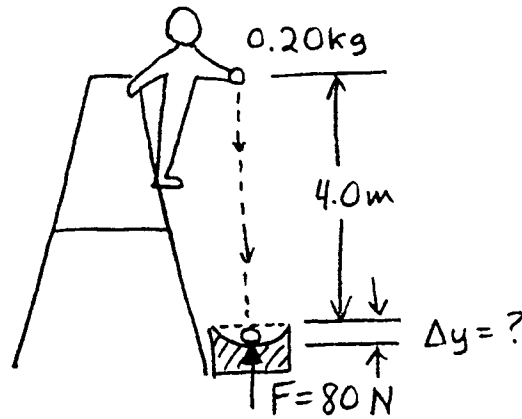
EXECUTE THE PLAN: (This is done the same way as all context-rich problems.)

EVALUATE YOUR ANSWER: (This is done the same way as all context-rich problems.)

Problem #2: A 0.20-kg egg is dropped from a ladder a vertical distance of 4.0-m. The egg will break if subjected to an impulse force greater than 80-N. Over what minimum distance must a constant force be exerted to avoid breaking the egg? (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 6.9)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the minimum distance Δy an 80 N force can be exerted on the egg to bring it to rest without breaking it?

Approach

Use conservation of energy. System: egg and Earth

Time: Initial time is the instant after egg is released.

Final time is the instant egg comes to rest.

Initial energy is gravitational potential. Final energy is zero.

Input energy is zero.

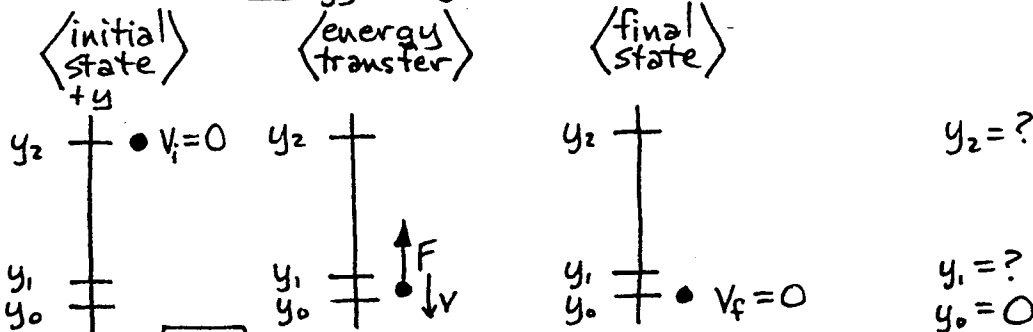
Output energy is due to the stopping force.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

Assume the egg is dropped from rest ($v_i = 0$) and lands on a pad that exerts a constant 80 N stopping force.

energy diagram



Target Quantity(ies)

Δy_1

Quantitative Relationships

$\Delta E = E_f - E_i = W_{NC}$

$E_i = mgy_2$

$W_{NC} = \int_{y_0}^{y_1} F \cdot dy$

5-12

$\therefore mgy_2 = \int_{y_0}^{y_1} F \cdot dy$

$E_f = mgy_0 + \frac{1}{2}mv_f^2 = 0$

$E_i = U_i + K_i = 0$

$m = 0.20 \text{ kg}$

$F = 80 \text{ N}$

$\Delta y_2 = y_2 - y_1 = 4.0 \text{ m}$

$\Delta y_1 = y_1 - y_0 = ?$

PLAN the SOLUTION
Construct Specific Equations

Find Δy_1

$$\Delta y_1 = y_1 \quad (1)$$

Find y_1

$$mgy_2 = \int_{y_0}^{y_1} F \cdot dy \quad (2)$$

$$mgy_2 = [Fy]_{y_0}^{y_1}$$

$$mgy_2 = F(y_1 - y_0) \rightarrow 0$$

Find y_2

$$\Delta y_2 = y_2 - y_1 \quad (3)$$

$$y_2 = \Delta y_2 + y_1$$

$$mg(\Delta y_2 + y_1) = Fy_1$$

$$Fy_1 - mgy_1 = mg\Delta y_2$$

$$y_1 = \frac{mg\Delta y_2}{(F - mg)}$$

$$\Delta y_1 = \frac{mg\Delta y_2}{(F - mg)}$$

Unknowns

$$\Delta y_1$$

$$y_1$$

$$y_2$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$\Delta y_1 = \frac{(0.20\text{kg})(9.8\text{m/s}^2)(4.0\text{m})}{(80\text{N} - (0.20\text{kg})(9.8\text{m/s}^2))}$$

$$\Delta y_1 = 0.10\text{m}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected Δy_1 is in units of length.

Is Answer Unreasonable?

No. $0.10\text{m} = 10\text{cm}$. This seems like a reasonable distance in which to stop an egg without breaking it.

Is Answer Complete?

Yes. The maximum allowed force (80N) will stop an egg in $\Delta y_1 = 10\text{cm}$. This answers the question.

(extra space if needed)

Check Units

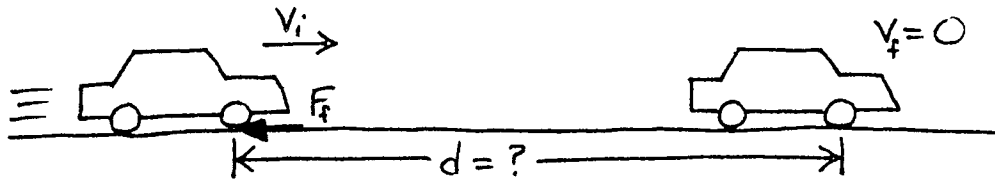
$$\frac{[\text{kg}][\text{m/s}^2][\text{m}]}{([\text{kg m/s}^2] - [\text{kg}][\text{m/s}^2])} = [\text{m}] \quad \text{OK}$$

Problem #3: Show that the minimum distance needed to stop a car traveling at speed v on a level road is $v^2/2\mu g$, where μ is the coefficient of friction between the car and the road and g is the acceleration of gravity. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.20)

FOCUS the PROBLEM

Picture and Given Information

μ is coefficient of friction between car and road



Question(s) How far does the car go before stopping?

Approach Use dynamics to determine the forces acting on the car. Then use conservation of energy to find the distance.

System: car

Time: Initial time is the instant car begins to slow down. Final time is the instant car stops.

Initial energy is kinetic.

Final energy is zero.

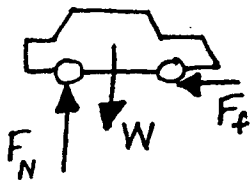
Input energy is zero.

Output energy is due to friction force.

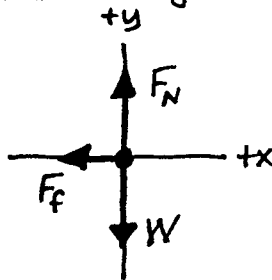
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

free-body diagram

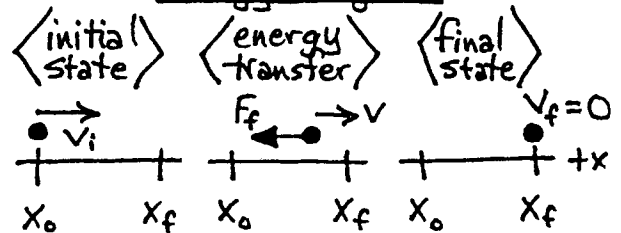


force diagram



μ -- known
 g -- known

energy diagram



$x_0 = 0$
 v_i -- known
 $m = ?$

$x_f = ?$
 $v_f = 0$

Target Quantity(ies)

x_f

Quantitative Relationships

$\sum F_y = F_N - W = 0$ $W = mg$
 $\therefore F_N - mg = 0$

$\sum F_x = -F_f = m(-a)$ $F_f = \mu F_N$
 $\therefore \mu F_N = ma$

Negative because both are in the $-x$ -direction.

$\Delta E = E_f - E_i = W_{NC}$

$E_i = K_i + U_i$

$K_i = \frac{1}{2} m v_i^2$

$W_{NC} = \int_{x_0}^{x_f} F_f \cdot dx$

$\therefore \frac{1}{2} m v_i^2 = \int_{x_0}^{x_f} F_f \cdot dx$

PLAN the SOLUTION
Construct Specific Equations

Find x_f

$$\frac{1}{2} m v_i^2 = \int_{x_0}^{x_f} F_f \cdot dx \quad (1)$$

Find F_f

$$F_f = \mu F_N \quad (2)$$

Find F_N

$$F_N - mg = 0 \quad (3)$$

$$F_N = mg$$

$$F_f = \mu mg$$

$$\frac{1}{2} m v_i^2 = \int_{x_0}^{x_f} \mu mg \cdot dx$$

$$\frac{1}{2} m v_i^2 = [\mu mg x]_{x_0}^{x_f}$$

$$\frac{1}{2} v_i^2 = \mu g (x_f - x_0) \rightarrow 0$$

Unknown m has cancelled out so
no need to "Find m "!

$$\frac{1}{2} v_i^2 = \mu g x_f$$

$$x_f = \frac{v_i^2}{2\mu g}$$

Check Units

$$\frac{[m/s]^2}{[m/s^2]} = [m] \quad \text{OK}$$

↑ μ is unitless

unknowns

x_f

m, F_f

F_N

EXECUTE the PLAN

Calculate Target Quantity(ies)

No numerical calculation required.

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected x_f has units of length.

Is Answer Unreasonable?

No. The larger v_i , the longer x_f .
And the larger μ (ie. the stronger the frictional force) the shorter x_f .
This makes sense.

Is Answer Complete?

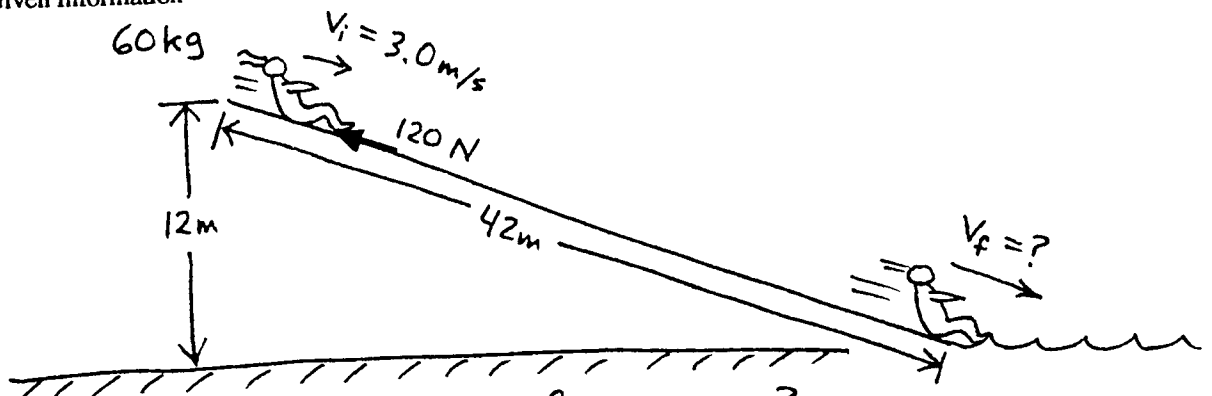
Yes. As requested we have shown that $x_f = v_i^2 / 2\mu g$.

(extra space if needed)

Problem #5: A water slide is 42-m long and has a vertical drop of 12-m. If a 60-kg person starts down the slide with a speed of 3.0-m/s, calculate his or her speed at the bottom. A 120-N average friction force opposes the motion. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 6.63)

FOCUS the PROBLEM

Picture and Given Information



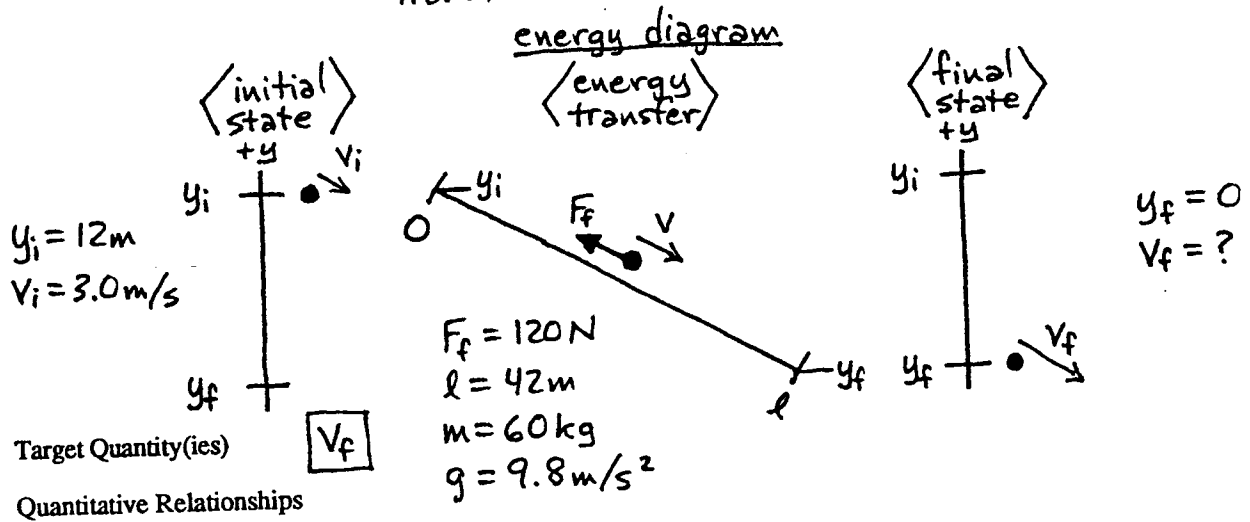
Question(s) What is the final velocity of the person?

Approach Use conservation of energy. System: person and Earth
 Time: Initial time is the instant person starts down the slide.
 Final time is the instant person reaches the bottom.
 Initial energy is kinetic and gravitational potential.
 Final energy is kinetic.
 Input energy is zero.
 Output energy is due to motion against frictional force.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

The normal force is perpendicular to the direction of motion so it leads to no energy transfer.



Target Quantity(ies) V_f

Quantitative Relationships

$$\Delta E = E_f - E_i = W_{NC}$$

$$E_f = \frac{1}{2} m v_f^2 + m g y_f$$

$$E_i = \frac{1}{2} m v_i^2 + m g y_i$$

$$W_{NC} = \int_0^l \vec{F}_f \cdot d\vec{r}$$

$$\therefore \frac{1}{2} m v_f^2 - \left(\frac{1}{2} m v_i^2 + m g y_i \right) = - \int_0^l F_f \cdot dr$$

PLAN the SOLUTION
Construct Specific Equations

unknowns

Find V_f

V_f

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 - mgy_i = -\int_0^l F_f \cdot dr \quad (1)$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 - mgy_i = -[F_f r]_0^l$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 - mgy_i = -F_f l$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i - F_f l$$

$$V_f^2 = V_i^2 + 2gy_i - \frac{2F_f l}{m}$$

$$V_f = \sqrt{V_i^2 + 2gy_i - \frac{2F_f l}{m}}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$V_f = \sqrt{(3.0 \frac{m}{s})^2 + 2(9.8 \frac{m}{s^2})(12m) - \frac{2(120N)(42m)}{60kg}}$$

$$V_f = 8.7 \text{ m/s}$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected V_f has units of velocity.

Is Answer Unreasonable?

No. The person has picked up a modest amount of speed by the time they reach the bottom.

Is Answer Complete?

Yes. V_f is the velocity of the person at the bottom of the slide which answers the question.

(extra space if needed)

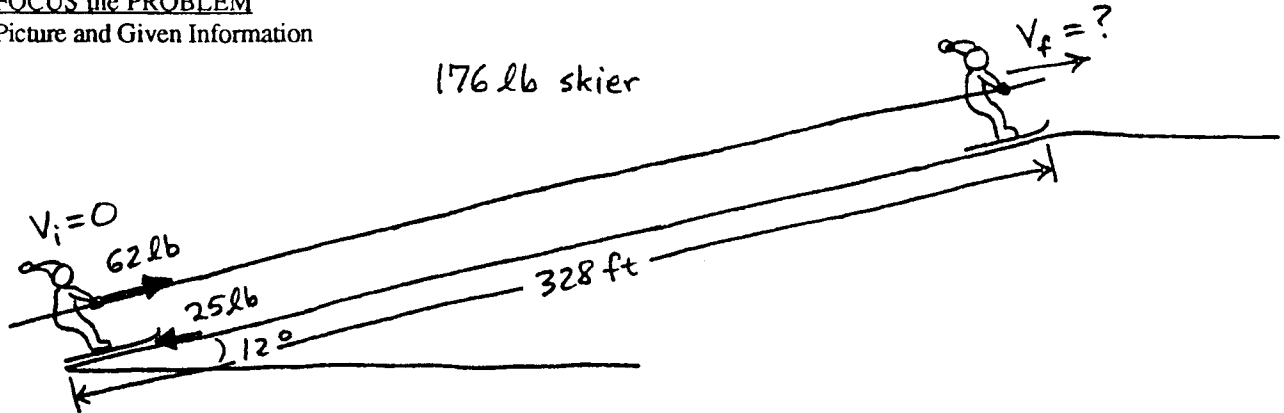
Check Units

$$\sqrt{\frac{[m/s]^2 + [m/s^2][m] - \frac{[kg \cdot m/s^2][m]}{[kg]}}{[kg]}}$$
$$= \sqrt{[m/s]^2 + [m/s]^2 - [m/s]^2}$$
$$= [m/s] \quad \text{OK}$$

Problem #6: Every winter you hold an annual ski party. Most of your friends are good skiers and can handle the tow rope which is used to go from the lodge to the first chair lift. The tow rope is somewhat unusual in that it pulls skiers with a constant force rather than pulling them at a constant speed. One of your friends, who weighs 176 pounds, usually loses his balance when a tow rope pulls him faster than 5.0-mph. This tow rope pulls people up a 12 degree hill that is 328-ft long. The tow rope exerts a 62-lb force on the skier, and the 4.0-mph wind together with the sticky snow exert a 25-lb force that opposes motion up the hill. Will your friend fall?

FOCUS the PROBLEM

Picture and Given Information



Question(s) Will the speed of the skier be greater than 5mph before he reaches the top of the hill? (If so, he will fall.)

Approach

Use conservation of energy. System: skier and Earth

Time: Initial time is the instant rope starts to pull skier.

Final time is the instant skier reaches top of the hill.

Initial energy is zero. Final energy is kinetic and grav. potential.

Input energy is due to the tow rope. Output energy is due to wind and snow resistive forces → assumed constant.

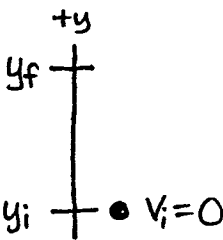
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

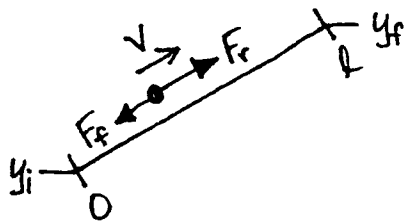
The normal force is perpendicular to the direction of motion so it leads to no energy transfer.

energy diagram

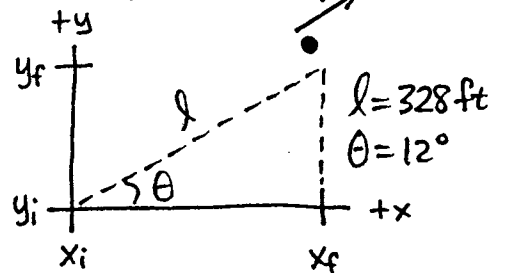
⟨initial state⟩



⟨energy transfer⟩



⟨final state⟩



Target Quantity(ies) V_f

Quantitative Relationships

$W = 176 \text{ lb}$
 $g = 32 \text{ ft/s}^2$

$v_i = 0$
 $v_f = ?$
 $F_r = 62 \text{ lb}$
 $F_f = 25 \text{ lb}$

$x_i = 0$
 $x_f = l \cos \theta$
 $y_i = 0$
 $y_f = l \sin \theta$

$\Delta E = E_f - E_i = W_{NC}$

$E_f = \frac{1}{2} m v_f^2 + m g y_f$

$W_r = \int_0^l \vec{F}_r \cdot d\vec{r}$

$W_f = \int_0^l \vec{F}_f \cdot d\vec{r}$

$E_i = m g y_i + \frac{1}{2} m v_i^2$

$\therefore \frac{1}{2} m v_f^2 + m g y_f = \int_0^l \vec{F}_r \cdot d\vec{r} - \int_0^l \vec{F}_f \cdot d\vec{r}$

Also: $W = mg$

$W_{NC} = W_r + W_f$

PLAN the SOLUTION
Construct Specific Equations

unknowns

Find v_f

$$\frac{1}{2}mv_f^2 + mgy_f = \int_0^l F_r \cdot dr - \int_0^l F_f \cdot dr \quad (1)$$

$$\frac{1}{2}mv_f^2 + mgy_f = [F_r r]_0^l - [F_f r]_0^l$$

$$\frac{1}{2}mv_f^2 + mgy_f = F_r l - F_f l$$

Find y_f

$$y_f = l \sin \theta \quad (2)$$

$$\frac{1}{2}mv_f^2 + mgl \sin \theta = (F_r - F_f)l$$

Find m

$$W = mg \quad (3)$$

$$m = W/g$$

$$\frac{1}{2} \frac{W}{g} v_f^2 + Wl \sin \theta = (F_r - F_f)l$$

$$\frac{1}{2} \frac{W}{g} v_f^2 = (F_r - F_f)l - Wl \sin \theta$$

$$v_f^2 = \frac{2gl}{W} (F_r - F_f - W \sin \theta)$$

$$v_f = \sqrt{\frac{2gl}{W} (F_r - F_f - W \sin \theta)}$$

Check Units

$$\sqrt{\frac{[ft/s^2][ft]}{[lb]} ([lb] - [lb] - [lb])}$$

$$= \sqrt{\frac{[ft]^2}{[s]^2}} = [ft/s] \quad \text{OK}$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$v_f = \sqrt{\frac{2(32ft/s^2)(328ft)}{176lb}} (62lb - 25lb - 176lb \sin(12^\circ))$$

$$v_f = 7.0 \frac{ft}{s} \left(\frac{1 \text{ mile}}{5280 \text{ ft}} \right) \left(\frac{3600 \text{ s}}{1 \text{ hour}} \right)$$

$$v_f = 4.8 \text{ mi/hr}$$

This is less than 5mph so the skier probably will not fall.

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected v_f has units of velocity.

Is Answer Unreasonable?

No. A skier might easily be pulled by a rope at speeds like 7 ft/s.

Is Answer Complete?

Yes. The skier's speed at the top of the hill has been found to be less than 5mph. This answers the question.

(extra space if needed)