Context Rich Problems: Dynamics

Some hints for solving problems with dynamics.

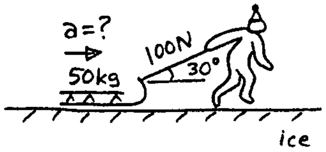
FOCUS THE PROBLEM: As usual, draw a picture with all the information from the problem. This shouldn't be a physics diagram, just a picture. Look at what is given. If things like velocities, distances, and times are given, then you will probably have to use *kinematics* somewhere in the problem. So part of the approach should include kinematic equations. If there are forces given or forces asked for, then you will have to use dynamics which includes Newton's second law. Often you will have to use kinematics and dynamics in the same problem since both involve acceleration. Once you have finished this step, you should not have to look at the problem again.

DESCRIBE THE PHYSICS: If you have to use kinematics in the problem, you will need to draw a motion diagram with each point uniquely labeled as outlined in the hints for solving Context Rich Problems involving kinematics. In addition, you will need *to draw a different free body diagram for every different object in the problem.* On the free body diagram of each object, draw all the forces which are acting *on* that object. Do not draw forces that object exerts, but only those exerted on that object from other objects. These will be things like gravity, friction, pushes, pulls, springs, and tension. To the side of the object (not on the free body diagram) draw an axis showing your positive x and y axes. It is always smart to orient the axes so that one axis points in the direction of the acceleration. In addition, draw and label an arrow showing the direction of the acceleration if you know it. *Draw this arrow to the side of the free body diagram, not on the diagram. Only forces acting on the object in question should be on the free body diagram.* Note that the acceleration will not be 9.8 m/s^2 acting down unless gravity is the only force acting on the object, and the object is near the surface of the earth. If any force is not already pointing in only the x or y direction, then divide that force vector into its components using the rules for breaking a vector into components. As always, write only what is actually being asked for in your target variable. For the equations used, write the kinematic equations if you decided that you would need kinematics in this problem, and write $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$ if the problem is in two dimensions. All equations should be just generic equations only. If you have chosen your axes wisely the acceleration in one direction will be zero, and the right hand side of one of these equations will be zero.

PLAN THE SOLUTION: The rest of the problem is solved the same way all the problems are solved. Write an equation with your target variable in it, using the exact subscripts defined in the "DESCRIBE THE PHYSICS" section. Be smart about this. Try to write an equation with as many known variables as possible, to limit your number of unknown variables. If this equation introduces new unknown variables, then write more equations until the number of unknown equations equals the number of unknown quantities. Sometimes in these equations for force, the value of the mass will actually appear in every term in the equation, and so cancel out. In such a case, you can actually solve the problems with one fewer equation than unknown, (if one of the unknown quantities is the mass and it happens to cancel out in the final equation.) So look for this possibility. Do the rest of the problem as you did problems involving kinematics.

Problem #3: A rope connected to a 50-kg sled pulls it along a frictionless sheet of ice. The tension in the rope is 100-N and the rope is oriented at an angle of 30° above a line drawn parallel to the ice. Calculate the horizontal acceleration of the sled. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 4.14)

FOCUS the PROBLEM
Picture and Given Information

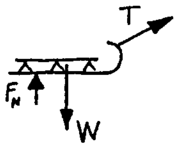


Question(s) What is the horizontal acceleration of the sled?

Approach Use dynamics. Find the horizontal and vertical forces acting on the sled. The net horizontal force will lead to a horizontal acceleration.
Since ice is slippery, assume friction can be neglected.

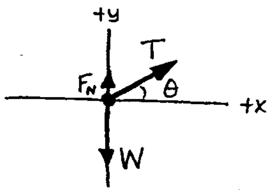
DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

free-body diagram



T: contact force of rope on sled
W: gravitational force of Earth on sled
F_N: normal force of ice on sled

force diagram



$T = 100N$ $m = 50kg$ $g = 9.8 m/s^2$

Target Quantity(ies) $\boxed{a_x}$

Quantitative Relationships

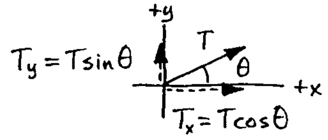
$\sum F_x = T_x = ma_x$

$\sum F_y = T_y + F_N - W = m\cancel{a_y}^{0}$

$W = mg$

$\therefore T_y + F_N - mg = 0$

components of T



PLAN the SOLUTION
Construct Specific Equations

Find a_x

$T_x = ma_x$ ①

Find T_x

$T_x = T\cos\theta$ ②

$T\cos\theta = ma_x$

$\boxed{a_x = \frac{T\cos\theta}{m}}$

unknowns

$\boxed{a_x}$

$\boxed{T_x}$

Check Units

$\frac{[N]}{[kg]} = \frac{[kg\ m/s^2]}{[kg]} = [m/s^2]$ OK

EXECUTE the PLAN
Calculate Target Quantity(ies)

$a_x = \frac{100\ N\ \cos(30°)}{50\ kg}$

$a_x = 1.7 m/s^2$

EVALUATE the ANSWER
Is Answer Properly Stated?
Yes. As expected a_x has ur
of acceleration.

Is Answer Unreasonable?
No. This modest acceleratio
could be achieved by a sle

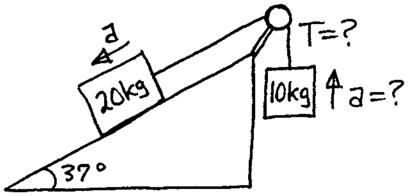
Is Answer Complete?
Yes. a_x is the horizontal acce
of the sled which answers
question.

(extra space if needed)

Problem #4: Two blocks are connected by a cable strung over a pulley, which is mounted at the top of an incline. The 10-kg block hangs over the edge of the incline. The 20-kg block rests on the frictionless incline. The "toe" of the incline is at an angle of 37° with respect to the horizontal. Find the magnitude of the acceleration and the tension in the cable. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.14)

FOCUS the PROBLEM

Picture and Given Information



Question(s)

1) What is the acceleration of the blocks?
2) What is the tension in the cable?

Approach

Use dynamics -- sum the forces and find the acceleration of each block separately. The two accelerations must be the same because the blocks are connected by a taut cable.

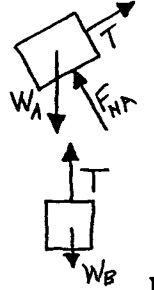
Assume no friction in pulley and between 20kg block and incline.

Assume massless pulley. Assume massless cable ⟹ tension is the same throughout the cable.

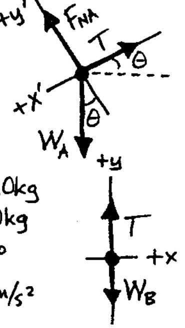
DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

free-body diagrams



force diagrams



F_{NA}: normal force of incline on block A

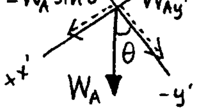
W_A, W_B: gravitational force of Earth on block A, B (also known as "weight")

T: contact force of cable on block A, B (also known as "tension")

$m_A = 20kg$
$m_B = 10kg$
$\theta = 37°$
$g = 9.8 m/s^2$

Components of W_A

$W_{Ax'} = W_A \sin\theta$ $\quad W_{Ay'} = W_A\cos\theta$



Target Quantity(ies) $\boxed{a, T}$

Quantitative Relationships

$$\sum F_{Ax'} = W_{Ax'} - T = m_A a$$

$$\sum F_{Ay'} = F_{NA} - W_{Ay'} = 0$$

$$\sum F_{Bx} = 0$$

$$\sum F_{By} = T - W_B = m_B a$$

$W_A = m_A g$

$W_B = m_B g$

$\therefore T - m_B g = m_B a, \; W_{Ax'} = m_A g\sin\theta, \; W_{Ay'} = m_A g\cos\theta$

PLAN the SOLUTION

Construct Specific Equations

Unknowns $\boxed{a, T}$

Find a

$T - m_B g = m_B a$ ①

Find T

$W_{Ax'} - T = m_A a$ ② $\quad \boxed{W_{Ax'}}$

Find $W_{Ax'}$

$W_{Ax'} = m_A g \sin\theta$ ③

$m_A g\sin\theta - T = m_A a$

$T = m_A g\sin\theta - m_A a$ Ⓐ

$m_A g\sin\theta - m_A a - m_B g = m_B a$

$g(m_A\sin\theta - m_B) = (m_A + m_B)a$

$$\boxed{a = \frac{g(m_A\sin\theta - m_B)}{(m_A+m_B)}}$$

and $\boxed{T = m_A g \sin\theta - m_A a}$ ← result Ⓐ

Check Units

a: $\frac{[m/s^2]([kg]-[kg])}{[kg]} = [m/s^2]$ OK

T: $[kg][m/s^2] - [kg][m/s^2]$
$= [kg\ m/s^2] = [N]$ OK

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$a = \frac{(9.8 m/s^2)(20kg\sin(37°) - (1}{(20kg + 10kg)}$$

$\boxed{a = 0.66\ m/s^2}$

$T = (20kg)(9.8 m/s^2)\sin(37°) - (20k$

$\boxed{T = 105 N}$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. Both the acceleration and tension came out with the
units.

Is Answer Unreasonable?

No. The acceleration is a smal
of g and the tension exceed
weight of block B just a li

Is Answer Complete?

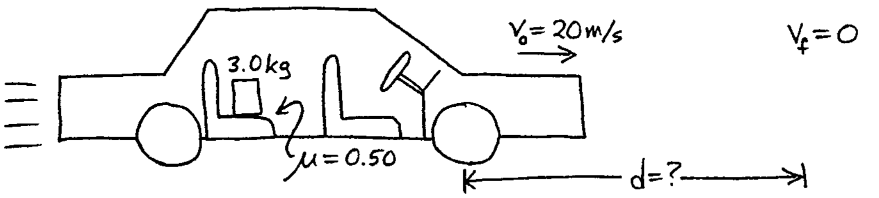
Yes. The acceleration of the b
the tension in the cable have
found. This answers the quest

(extra space if needed)

Problem #5: A 3.0-kg box sits on a horizontal surface of your car seat as you drive at a speed of 20-m/s. The coefficient of friction between the box and the seat is 0.50. You apply the brakes to stop the car. Calculate the shortest possible stopping distance so that the box does not start to slide off the seat. (Similar to Fishbane, Gasiorowicz and Thornton 1993, problem 5.30)

FOCUS the PROBLEM

Picture and Given Information



Question(s) What is the shortest distance in which the car can stop without allowing the box to slide?

Approach

Use dynamics to evaluate the forces on the box and the acceleration of the car/box--to stay together they must accelerate the same. Then use kinematics to get the distance.

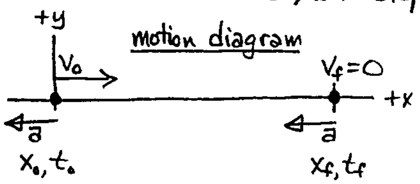
Time: Initial time is the instant you apply the brakes.
Final time is the instant the car/box stops.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

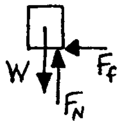
$X_0 = 0$
$V_0 = 20$ m/s
$t_0 = 0$
$a = ?$

motion diagram

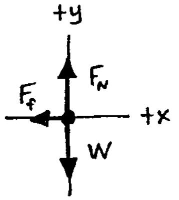


$X_f = ?$
$V_f = 0$
$t_f = ?$

free-body diagram



force diagram



W: gravitational force of Earth on box

F_N: normal force of seat on box

F_f: frictional force of seat on box

Target Quantity(ies) $\boxed{X_f}$

m = 3.0kg
$\mu = 0.50$
$g = 9.8 m/s^2$

Quantitative Relationships

$\sum F_y = F_N - W = 0$ $W = mg$

$\therefore F_N - mg = 0$

$\sum F_x = -F_f = m(-a)$ $F_f = \mu F_N$

$\therefore \mu F_N = ma$

Negative because both are in the -x-direction.

Constant acceleration (-a) in x-direction so use

$x_f = \frac{1}{2}(-a)(t_f - \overset{0}{\cancel{t_0}})^2 + v_0(t_f - \overset{0}{\cancel{t_0}}) + \overset{0}{\cancel{x_0}}$

$\Rightarrow x_f = -\frac{1}{2}a t_f^2 + v_0 t_f$

Also: $-a = -\bar{a} = \frac{(\overset{0}{\cancel{v_f}} - v_0)}{(t_f - \overset{0}{\cancel{t_0}})} = -v_0/t_f \Rightarrow a = \frac{v_0}{t_f}$

PLAN the SOLUTION

Construct Specific Equations

unknowns

Find x_f $\boxed{x_f}$

$x_f = -\frac{1}{2} a t_f^2 + v_0 t_f$ ① $\boxed{a, t_f}$

Find a

$\mu F_N = ma$ ② $\boxed{F_N}$

Find F_N

$F_N - mg = 0$ ③

$F_N = mg$

$\mu \cancel{m} g = \cancel{m} a$

$a = \mu g$ Ⓐ

$x_f = -\frac{1}{2}\mu g t_f^2 + v_0 t_f$

Find t_f

$a = v_0/t_f$ ④

$t_f = v_0/a$

$t_f = v_0/\mu g$ used result Ⓐ

$x_f = -\frac{1}{2}\mu g \left(\frac{v_0}{\mu g}\right)^2 + v_0\left(\frac{v_0}{\mu g}\right)$

$x_f = -\frac{1}{2}\frac{v_0^2}{\mu g} + \frac{v_0^2}{\mu g}$

$x_f = \frac{1}{2}\frac{v_0^2}{\mu g}$

Check Units

$\frac{[m/s]^2}{[\ ][m/s^2]} = [m]$ OK

μ is unitless

EXECUTE the PLAN

Calculate Target Quantity(ies)

$x_f = \frac{1}{2}\frac{(20 m/s)^2}{(0.50)(9.8 m/s^2)}$

$x_f = 40.8 m$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes. As expected x_f came out units of length.

Is Answer Unreasonable?

No. While stopping, the car's a
velocity is 10 m/s and, consideri
above, the stopping time is ~ 4
These values sound quite plau

Is Answer Complete?

Yes. Finding the stopping dis
x_f answers the question.

(extra space if needed)