Distance-redshift from an optical metric that includes absorption

B. Chen[1, *] and R. Kantowski[1, †]

[1]Homer L. Dodge Department of Physics and Astronomy,
University of Oklahoma, 440 West Brooks, Room 100, Norman, OK 73019, USA
(Dated: July 29, 2009)

We show that it is possible to equate the intensity reduction of a light wave caused by weak absorption with a geometrical reduction in intensity caused by a "transverse" conformal transformation of the spacetime metric in which the wave travels. We are consequently able to modify Gordon's optical metric to account for electromagnetic properties of ponderable material whose properties include both refraction and absorption. Unlike refraction alone however, including absorption requires a modification of the optical metric that depends on the eikonal of the wave itself. We derive the distance-redshift relation from the modified optical metric for Friedman-Lemaître-Robertson-Walker spacetimes whose cosmic fluid has associated refraction and absorption coefficients. We then fit the current supernovae data and provide an alternate explanation (other than dark energy) of the apparent acceleration of the universe.

I. INTRODUCTION

The concept of an optical metric was introduced by Gordon [1] who proved that solutions to Maxwell's equations in a curved spacetime filled with a fluid whose electromagnetic properties are described by a real permittivity $\epsilon(x)$ and a real permeability $\mu(x)$ (and consequently by a real refraction index $n(x) = \sqrt{\epsilon\mu}$) could be found by solving a modified version of Maxwell's equations in a related optical spacetime with vacuum values for the permittivity and permeability, i.e., with $\epsilon(x) = 1$ and $\mu(x) = 1$ [see Eq. (64)]. There is a direct relation between solutions in these two spacetimes, the physical spacetime with an index of refraction and its physical metric, and the optical spacetime with $n = 1$ and its optical-metric. Gordon's original metric accounted for refraction only. An open question was whether or not absorption could also be accounted for by modifying the spacetime geometry. In [2] we made such a modification and showed that Gordon's optical metric could be generalized to include absorption by allowing the metric to become complex. In [2] we defined a complex refraction index $N = n + i\kappa$, and distinguished two different cases, i.e., strong and week absorption. In the case of strong absorption the real and imaginary parts of the index of refraction are of the same order ($\kappa \sim n$), the eikonal $S(x^a)$ has an imaginary part, and the optical metric is complex. In the weak absorption case ($k \ll n$), the eikonal $S(x^a)$ can be taken as real, and the real part of the optical metric (Gordon's original metric) remains as the significant geometrical structure. In this paper we show that for weak absorption another generalization of Gordon's metric exists that can account for absorption as well as refraction. The generalization amounts to a transverse conformal transformation (see Sec. II) of Gordon's original optical metric. This new optical metric remains real but does include absorption. Including absorption via a transverse conformal transformation requires an optical metric that depends on the eikonal of the wave itself.

The contents of this paper are as follows: In the next section we define transverse conformal transformations. In Sec. III we consider the geometrical optics approximation and relate wave properties in the two spacetimes, the physical and optical. In Sec. IV we compare the effects of the transverse conformal transformation with absorption and relate the conformal factor σ to optical depth τ. In Sec. V we construct the optical metric that accounts for absorption in Friedman-Lemaître-Robertson-Walker (FLRW) spacetimes. In Sec. VI we include refraction by generalizing Gordon's metric [1] to including absorption. In Sec. VII we derive the refraction/absorption corrected distance-redshift relation from the generalized Gordon's optical metric in FLRW spacetimes. In Sec. VIII we fit the current supernovae data with the Hubble curve of a cosmological model in which the cosmic fluid has both an associated refraction index n and a constant opacity parameter α, and provide an alternate interpretation of the apparent acceleration of the cosmic expansion. In Sec. IX we give our conclusions.

---

*Electronic address: Bin.Chen-1@ou.edu
†Electronic address: kantowski@nhn.ou.edu

II. THE "TRANSVERSE" CONFORMAL TRANSFORMATION

A conformal transformation [3] is defined as a rescaling of the metric and is usually written in a form similar to

$$\hat{g}_{ab} = e^{2\sigma} g_{ab}, \tag{1}$$

where $\sigma(x^a)$ is an arbitrary scalar function defined on the spacetime manifold. The vacuum Maxwell equations[1]

$$\begin{aligned} F_{[ab,c]} &= 0, \\ \nabla_b F^{ab} &= 0, \end{aligned} \tag{2}$$

are formally invariant under a conformal transformation, i.e.,

$$\begin{aligned} \hat{F}_{[ab,c]} &= 0, \\ \hat{\nabla}_b \hat{F}^{ab} &= 0, \end{aligned} \tag{3}$$

provided that the covariant electromagnetic field tensor F_{ab} transforms as

$$\hat{F}_{ab} = F_{ab}, \tag{4}$$

and hence the contravariant field transforms as

$$\hat{F}^{ab} = e^{-4\sigma} F^{ab}. \tag{5}$$

The above is a purely mathematical observation and simply says that if you have a solution to Maxwell's vacuum equations in one spacetime then you have a related solution to Maxwell's vacuum equations in any conformally related spacetime. Nothing is being said by this about the existence of any new physically relevant field. One can and does make good use of such transformations to compactify spacetimes and to analyze fields at "∞", see e.g., [4]. However, by modifying these conformal transformations to be "transverse" [see Eqs. (10) and (11)] we are able to construct new and useful solutions to Maxwell's equations.

Because the two metrics are defined on the same differentiable manifold we can make unique correspondences between events, world lines, and various fields in these two spacetimes. We call the original metric and manifold physical spacetime and the manifold with the conformally transformed metric the conformal spacetime. For example a normalized fluid 4-velocity, $\hat{u}^a(x)$, defined in the conformal spacetime would be related to the corresponding physical fluid 4-velocity by $\hat{u}^a(x) = e^{-\sigma} u^a(x)$. If we compare observed properties of radiation fields that are related as in Eqs. (4) and (5) and as seen in these two spacetimes by corresponding observers, we find that energy fluxes are diminished by a factor of $e^{-4\sigma}$. This net effect can be looked at as the result of the area of the radiation's wave front changing by a factor $e^{2\sigma}$, and the energies and rates each changing by a factor $e^{-\sigma}$. We will make use of this intensity reduction to account for absorption in physical spacetime. However, because a pure conformal transformation has the undesirable property of altering a wave's frequency and wavelength it must be appropriately modified to represent absorption, which obviously does not.

We assume that our physical spacetime is filled with two vector fields. The first is the normalized velocity field u^a ($u^a u_a = -1$) of a fluid whose linear and isotropic optical properties we know and the second is a null field k^a ($k^a k_a = 0$) corresponding to the eikonal of a given electromagnetic wave (see Sec. III for details of the geometrical optics approximation). These two fields define a 2-dimensional timelike subspace at each point of the manifold. We are hence able to decompose the tangent space at each point of the 4-dimensional spacetime manifold into two orthogonal 2-dimensional subspaces, one timelike $g_{\parallel ab}$ and spanned by the pair (u^a, k^a) and the other spacelike $g_{\perp ab}$. The full metric can be decomposed in terms of these two orthogonal pieces as

$$g_{ab} = g_{\parallel ab} + g_{\perp ab}. \tag{6}$$

We can give a simple expression for $g_{\parallel ab}$ by defining a second null vector ℓ^a lying in the two dimensional timelike subspace spanned by u^a and k^a as

$$\ell^a \equiv \frac{k^a}{2(u \cdot k)^2} + \frac{u^a}{(u \cdot k)}, \tag{7}$$

---

from which it follows that

$$\ell^a \ell_a = 0, \quad \ell^a u_a = -\frac{1}{2(u \cdot k)}, \quad \ell^a k_a = 1. \tag{8}$$

The metrics on the orthogonal 2-dimensional surfaces can be written as

$$\begin{aligned}
g_{\parallel ab} &= 2\ell_{(a} k_{b)}, \\
g_{\perp ab} &= g_{ab} - 2\ell_{(a} k_{b)}.
\end{aligned} \tag{9}$$

We conformally transform the 2-dimensional spacelike subspace only and arrive at the desired optical metric

$$\begin{aligned}
\tilde{g}_{ab} &\equiv g_{\parallel ab} + e^{2\sigma} g_{\perp ab}, \\
&= e^{2\sigma} g_{ab} + (1 - e^{2\sigma}) 2\ell_{(a} k_{b)},
\end{aligned} \tag{10}$$

with inverse

$$\begin{aligned}
\tilde{g}^{ab} &\equiv g_\parallel^{ab} + e^{-2\sigma} g_\perp^{ab}, \\
&= e^{-2\sigma} g^{ab} + (1 - e^{-2\sigma}) 2\ell^{(a} k^{b)}.
\end{aligned} \tag{11}$$

By straightforward tensor algebra we find that

$$\det \tilde{g} = e^{4\sigma} \det g. \tag{12}$$

We would have obtained $\det \hat{g} = e^{8\sigma} \det g$ were we conformally transforming the entire metric g_{ab} as in Eq. (1). We call these transformations transverse-conformal because they scale directions transverse to a wave's propagation direction k^a as seen by an observer u^a moving with the optical fluid. We find that the vacuum Maxwell equations (2) remain invariant in form under the transverse conformal transformation of Eq. (10), i.e.,

$$\begin{aligned}
\tilde{F}_{[ab,c]} &= 0, \\
\tilde{\nabla}_b \tilde{F}^{ab} &= 0,
\end{aligned} \tag{13}$$

provided that the covariant electromagnetic field tensor F_{ab} transforms as

$$\tilde{F}_{ab} = F_{ab}, \tag{14}$$

and that the contravariant field tensor $\tilde{F}^{ab}$ defined by

$$\tilde{F}^{ab} \equiv \tilde{g}^{ac} \tilde{g}^{bd} \tilde{F}_{cd} \tag{15}$$

is constrained to satisfy

$$\tilde{F}^{ab} = e^{-2\sigma} F^{ab}. \tag{16}$$

This constraint requires that two of the following three terms, i.e., F_0^{ab} and F_2^{ab}, originating from Eq. (15) vanish:

$$\tilde{F}^{ab} = F_0^{ab} + e^{-2\sigma} F_1^{ab} + e^{-4\sigma} F_2^{ab}, \tag{17}$$

where

$$\begin{aligned}
F_0^{ab} &\equiv 2\ell^{[a} k^{b]} \left(F_{cd} k^c \ell^d \right), \\
F_1^{ab} &\equiv -2k^{[a} F^{b]}{}_c \ell^c - 2\ell^{[a} F^{b]}{}_c k^c - 2F_0^{ab}, \\
F_2^{ab} &\equiv F^{ab} - F_0^{ab} - F_1^{ab}.
\end{aligned} \tag{18}$$

Consequently F_{ab} must satisfy

$$F_{cd} k^c \ell^d = 0, \tag{19}$$

and

$$F^{ab} = -2k^{[a} F^{b]}{}_c \ell^c - 2\ell^{[a} F^{b]}{}_c k^c. \tag{20}$$

As we will see in the next section [see Eq. (32)] a radiation field whose eikonal generates the null field k^a satisfies these constraints. For a familiar example, choose the physical metric to be Minkowskian, i.e.,

$$ds^2 = -c^2 dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2), \tag{21}$$

with a stationary optical fluid $u^a = (1/c, 0, 0, 0)$, and radial null geodesics $k^a = k\,(1/c, 1, 0, 0)$. From Eq. (7) we find

$$\ell^a = \frac{1}{2k}\left(-\frac{1}{c}, 1, 0, 0\right). \tag{22}$$

Equations (19) and (20) then give

$$F_{01} = -F_{10} = 0, \; F_{23} = -F_{32} = 0, \tag{23}$$

i.e., F^{ab} is a transverse field, hence motivating the designation of Eq. (10) as a "transverse" conformal transformation. Equations (19) and (20) simply express the transversality condition for an arbitrary wave. In the following section we review the geometrical optics approximation and relate electromagnetic quantities in the physical and optical spacetimes.

III. GEOMETRICAL OPTICS APPROXIMATION

Following [2, 5, 6] we write the covariant (and metric independent) field tensor as

$$F_{ab} = \mathrm{Re}\left\{ e^{iS/\lambdabar_0}\left(A_{ab} + \frac{\lambdabar_0}{i} B_{ab} + O(\lambdabar_0{}^2)\right)\right\}, \tag{24}$$

where $\lambdabar_0$ is a wavelength related parameter, $S(x^a)$ is the so-called eikonal function and is real, and $\mathrm{Re}\{\cdot\}$ stands for the real part. The A_{ab} term represents the geometrical optics approximation and the B_{ab} term is its first order correction. Defining the unitless (also metric independent) wave vector $k_a = \partial_a S$ and inserting Eq. (24) into the vacuum Maxwell equations we obtain to order $\lambdabar_0{}^{-1}$

$$\begin{aligned} A_{[ab}k_{c]} &= 0, \\ A^{ab}k_b &= 0, \end{aligned} \tag{25}$$

and to order $\lambdabar_0{}^0$

$$\begin{aligned} \partial_{[a}A_{bc]} + k_{[a}B_{bc]} &= 0, \\ \nabla_b A^{ab} + B^{ab}k_b &= 0. \end{aligned} \tag{26}$$

Equations (25) tell us that $k^a \equiv g^{ab}k_b$ is tangent to null geodesics of g_{ab}, i.e.,

$$\begin{aligned} k^a k_a &= 0, \\ k^b \nabla_b k^a &= 0, \end{aligned} \tag{27}$$

and that A_{ab} is of the form:

$$A_{ab} = -2k_{[a}\mathcal{E}_{b]}, \tag{28}$$

where $\mathcal{E}_a$ is spacelike and constrained by $\mathcal{E}_a k^a = 0$ with the remaining gauge freedom (to order $\lambdabar_0{}^0$) $\mathcal{E}_a \to \mathcal{E}_a + f(x)k_a$. Since $k^a \ell_a = 1$, we can use this freedom to choose $\mathcal{E}_a$ such that $\mathcal{E}_a \ell^a = 0$ or equivalently $\mathcal{E}_a u^a = 0$. For this choice, $\mathcal{E}^a$ is (up to a factor ω^{-1}) the amplitude of the electric field seen by observers at rest with respect to the fluid u^a. It is the geometrical optics approximation, i.e., Eq. (28) that makes the Maxwell field of Eq. (24) satisfy the needed transverse constraints of Eqs. (19) and (20) and hence allows us to introduce the transverse conformal transformation of Eq. (10). Before doing so we first finish the geometrical optics approximation for F^{ab} in the physical spacetime.

Equation (26) tells us that the order $\lambdabar_0{}^1$ correction to geometrical optics is of the form

$$B_{ab} = 2(\mathcal{E}_{[a,b]} - k_{[a}\mathcal{D}_{b]}), \tag{29}$$

with a remaining gauge freedom, to $O(\lambdabar_0{}^1)$, $\mathcal{D}_a \to \mathcal{D}_a + g(x)k_a$ and also gives the propagation equation for $\mathcal{E}^a$

$$\dot{\mathcal{E}}^a + \theta\mathcal{E}^a = \frac{k^a}{2}(\nabla_b\mathcal{E}^b + k_b\mathcal{D}^b), \tag{30}$$

where '·' is the affine parameter rate of change along the null ray, $\dot{\mathcal{E}}^a \equiv k^b \nabla_b \mathcal{E}^a$, and θ is the expansion rate of the null congruence, $\theta \equiv \nabla_a k^a / 2$. By splitting $\mathcal{E}^a = \mathcal{E} e^a$ into a scalar amplitude $\mathcal{E}$ and a unit polarization vector $e^a \overset{*}{e}_a = 1$ (where $*$ is complex conjugation), the transport equation for the amplitude $\mathcal{E}$ becomes

$$\dot{\mathcal{E}} + \mathcal{E}\theta = 0. \tag{31}$$

The geometrical optics approximation, i.e., the $O(\lambdabar_0{}^0)$ term in Eq. (24), hence simplifies to

$$F^{ab} = -2 \operatorname{Re}\{\mathcal{E} \, e^{iS/\lambdabar_0} k^{[a} e^{b]}\}. \tag{32}$$

As we have indicated above, because both the physical metric g_{ab} and optical metric $\tilde{g}_{ab}$ of Eq. (10) are defined on the same manifold we can compare properties of a common field such as F_{ab} in both spacetimes. For the above geometrical optics field all covariant quantities such as $\tilde{F}_{ab}, \tilde{k}_a, \tilde{\ell}_a$, and $\tilde{\mathcal{E}}_a$ in the optical spacetime are exactly the same as F_{ab}, k_a, ℓ_a, and $\mathcal{E}_a$ in the physical spacetime. All contravariant components that lie in the k-ℓ plane are also unchanged, i.e., $\tilde{k}^a = k^a, \tilde{\ell}^a = \ell^a$, and $\tilde{u}^a = u^a$. Consequently, quantities such as affine parameters, frequencies and wavelengths of the waves are the same in both spacetimes. However, transverse contravariant components, i.e., components in the orthogonal 2-dimensional spacelike surface are scaled by the conformal factor $e^{-2\sigma}$, e.g., $\tilde{\mathcal{E}}^a = e^{-2\sigma} \mathcal{E}^a$. This changes the magnitude to $\tilde{\mathcal{E}} = e^{-\sigma} \mathcal{E}$ with a new unit polarization vector $\tilde{e}^a = e^{-\sigma} e^a$ and the expansion parameter θ of Eq. (31) to $\tilde{\theta} = \theta + \dot{\sigma}$.

The time averaged 4-flux seen by an observer moving with the optical fluid is in general

$$S^a \equiv \frac{c}{8\pi} \operatorname{Re}\left\{\overset{*}{H}{}^{ac} F_{cb} - \frac{1}{4} \delta_b^a \overset{*}{H}{}^{dc} F_{cd}\right\} u^b. \tag{33}$$

In the physical and optical spaces they are related by

$$\tilde{S}^a = e^{-2\sigma} S^a = -e^{-2\sigma} \frac{c}{8\pi}(u^b k_b)\mathcal{E}\overset{*}{\mathcal{E}} k^a, \tag{34}$$

and from the 3-D Poynting vector $S_\perp^a \equiv (g^{ab} + u^a u^b) S_b$ we find magnitudes related by

$$\tilde{S}_\perp = e^{-2\sigma} S_\perp = e^{-2\sigma} \frac{c}{8\pi} \mathcal{E}\overset{*}{\mathcal{E}}(u^b k_b)^2. \tag{35}$$

This result says that the intensity of a monochromatic wave can be reduced by a factor $e^{-2\sigma}$ at any point in spacetime by a transverse conformal transformation without altering the wave's frequency or wavelength. In the next section we equate this reduction of energy flux with absorption. In Sec. VI when we combine absorption with refraction we reverse the process and assume that physical spacetime possesses the absorbing material and the transverse conformal transformation to the optical spacetime removes it.

IV. A TRANSVERSE CONFORMAL TRANSFORMATION VERSUS ABSORPTION

The attenuation coefficient κ_ν (cm$^2 \cdot$ g^{-1}) is defined by looking at the amount of energy absorbed, dE_ν, in time dt by a pencil beam of radiation as it passes through a small slab of material of density ρ (g $\cdot$ cm^{-3}), cross-section dA and length dl (see e.g. [7])

$$dE_\nu = (\rho\kappa_\nu)I_\nu d\Omega d\nu dt dA dl. \tag{36}$$

All quantities are defined in the local co-moving frame of the optical fluid u^a. The specific intensity I_ν ($\times d\Omega d\nu$) measures the wave's intensity directed into solid angle $d\Omega$ and within the frequency range ν to $\nu + d\nu$. In the absence of emission the specific intensity $I_\nu(\tau)$, after traveling an optical depth $\tau \equiv \int \rho\kappa_\nu dl$ along a "characteristic" direction, differs from the value $I_\nu(0)$ it would have without absorption by

$$I_\nu(\tau) = I_\nu(0)e^{-\tau}. \tag{37}$$

Evaluating τ can be complicated because the frequency in the integrand is continually Doppler shifted due to the non-static nature of the optical fluid and/or the curvature of spacetime. The specific flux S_ν at any point is the first moment of specific intensity I_ν, i.e.,

$$S_\nu = 2\pi \int_{-1}^{1} \mu I_\nu d\mu. \tag{38}$$

For any I_ν that has a delta function dependence on direction, e.g., a geometrical optics wave, the corresponding value of the flux $S_\nu(\tau)$ seen by the observer in the presence of absorption is similarly related to the non-absorption value, i.e.,

$$S_\nu(\tau) = S_\nu(0)e^{-\tau}. \tag{39}$$

Comparing Eqs. (35) and (39) we find that a conformal factor σ reduces the flux by the exact same amount as absorption if

$$\sigma = \frac{1}{2}\tau. \tag{40}$$

If this conformal factor is to be unique, then either a single frequency is present at any spacetime point or the attenuation is "grey", i.e., the opacity $\alpha \equiv \rho\kappa_\nu$ is frequency independent. Even if there is only a single frequency present the frequency dependence in the absorption makes the calculation complicated. We must follow the frequency of each wave, starting from the source, as it is red/blue shifted and selectively absorbed while moving through the optical medium. In general a different conformal factor would exist for each source frequency. However, for the "grey" case the $\sigma(x^a)$ is unique. For a geometrical optics wave emanating from the world line of a point source, $\sigma(x^a)$ is defined on forward light cones (surfaces on which the eikonal S remains constant), i.e.,

$$\sigma(x^a) = \frac{1}{2} \int_{x_s^a}^{x^a} \rho(x'^b)\kappa_\nu(x'^b)dl. \tag{41}$$

The integration is performed along the null geodesic connecting the emitting event x_s^a and the spacetime point x^a. The density, attenuation coefficient, and spatial length element, respectively ρ, κ_ν, and dl, are measured in a sequence of local inertial frames ($u^a \propto \delta_0^a$) which are at rest with respect to the material along the null geodesic. We caution the reader that $\sigma(x^a)$ might not be globally defined, or even uniquely defined. The integral above is only defined for x^a which lie on forward light cones of the source. If the light source is turned on/off at some time (star birth/death) σ will only be defined on part of the spacetime manifold. However, its value can be taken as zero on the remainder. Furthermore, we might have multiple null geodesics connecting the emitting event S and receiving event O. For an example, when an Einstein ring occurs in a gravitational lensing configuration we have, in principle, an infinite number of null geodesics connecting S and O. These rays might pass through regions with different ρ's and κ_ν's and therefore give us different σ's. Rather than limiting the domain over which the conformal transformation is defined to keep it single valued, we can extend the manifold to multiple layers and make the conformal factor unique on each layer. This is in direct analogy with Riemann's extension of the domain of a multi-valued complex function on R^2 to a Riemann surface on which its value is unique. This extended domain could be truly convoluted as for example in an inhomogeneous cosmology where strong lensing is prevalent.

The simplest example to illustrate a transverse conformal transformation is a plane electromagnetic wave traveling in Minkowski spacetime filled with stratified weakly absorbing gas. We suppose a monochromatic light source is lying infinitely far away ($z = -\infty$) and is producing a plane wave propagating in the $+z$ direction. The needed properties of a stratified gas filling the spacetime are summarized by the density $\rho(z)$ and the attenuation coefficient $\kappa_\nu(z)$. The physical metric is

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2, \tag{42}$$

and the vector fields k^a and ℓ^a in Eq. (7) are trivially ($u^a = \delta_t^a/c$)

$$k^a = k\left(\frac{1}{c}, 0, 0, 1\right),$$

$$\ell^a = \frac{1}{2k}\left(-\frac{1}{c}, 0, 0, 1\right). \tag{43}$$

Equation (10) then gives

$$d\tilde{s}^2 = -c^2dt^2 + e^{2\sigma(z)}(dx^2 + dy^2) + dz^2, \tag{44}$$

where the conformal factor is related to the integral of the opacity by

$$\sigma(z) = \frac{1}{2}\int_{-\infty}^{z} \alpha(z')dz'. \tag{45}$$

We can now give a geometrical interpretation of Eq. (35). Because the rays in Eq. (43) are all running parallel to the z-axis the cross-sectional area of any bundle of rays is expanded by a factor $e^{2\sigma(z)}$ for the conformally transformed spacetime, Eq. (44), relative to initial the physical spacetime, Eq. (42). From the definition of flux, we can conclude that it will be reduced by the reciprocal factor $e^{-2\sigma}$. The reason the conformal factor was not applied to the whole 4-D metric now becomes clear: a $e^{2\sigma}$ factor in front of the entire physical metric would produce an undesirable time dilation factor e^{σ} as well as a frequency shift factor $e^{-\sigma}$. These two combined would have contributed another factor of $e^{-2\sigma}$ to the wave's flux in the conformal spacetime. We can also look at the electric field $\mathbf{E}$ and the magnetic field $\mathbf{H}$ in both spacetimes and see that in the conformal spacetime they are each diminished by a factor $e^{-\sigma}$. Consequently the flux, $\mathbf{E} \times \mathbf{H}$, is reduced by $e^{-2\sigma}$. (This reduction in amplitude/intensity matches that of weak absorption given in [2] but, not unexpectedly, differs from that of strong absorption, i.e., Eqs. (28)-(30) of [2]).

V. ABSORPTION IN FLRW SPACETIMES

The interesting example for cosmology is absorption by the Intergalactic Medium (IGM) which is naturally modeled as absorption by the cosmological fluid in Friedman-Lemaître-Robertson-Walker (FLRW) spactimes. The familiar Robertson-Walker metric can be written in co-moving coordinates ($u^a = \delta_t^a/c$) as

$$ds^2 = -c^2\, dt^2 + R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right\}, \tag{46}$$

where $k = 1, 0, -1$ for a closed, flat or open universe, respectively. The eikonal of Eq. (32) for a point source located at $r = 0$ (see [2]) is

$$S(t, r) = R_0 \left(-\int_{t_e}^{t} \frac{c\, dt'}{R(t')} + \text{sinn}^{-1}[r] \right), \tag{47}$$

where

$$\text{sinn}[r] \equiv \begin{cases} \sin[r] & k = +1 \\ r & k = 0 \\ \sinh[r] & k = -1, \end{cases} \tag{48}$$

and R_0 is the radius of the universe at the observation time t_0. The corresponding radial null geodesics are

$$k^a = R_0 \left(\frac{1}{c\, R(t)}, \frac{\sqrt{1 - kr^2}}{R^2(t)}, 0, 0 \right), \tag{49}$$

for which Eq. (7) gives

$$\ell^a = \frac{R(t)}{2R_0} \left(-\frac{1}{c}, \frac{\sqrt{1 - kr^2}}{R(t)}, 0, 0 \right), \tag{50}$$

and from which it follows that

$$2\ell_{(a}k_{b)} = \text{diag} \left[-c^2, \frac{R^2(t)}{1 - kr^2}, 0, 0 \right]. \tag{51}$$

Equation (10) then gives the optical metric

$$d\tilde{s}^2 = -c^2\, dt^2 + R^2(t) \left\{ \frac{dr^2}{1 - kr^2} + e^{2\sigma} r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \right\}. \tag{52}$$

An observer at (r, t) will see the source redshifted an amount z as a function of co-moving radius r and observing instant t, which can be obtained by integrating Eq. (49). If the dynamics of a Robertson-Walker metric is determined by general relativity, and the gravity source is a mixture of non-interacting perfect fluids including a cosmological constant, the cosmology is called FLRW and has a redshift which can be found by inverting

$$r(t, z) = \text{sinn} \left[\frac{c}{R(t)H(t)} \int_0^z \frac{dz'}{h(z')} \right], \tag{53}$$

where

$$h(z) = \sqrt{\Omega_\Lambda + \Omega_k(1+z)^2 + \Omega_m(1+z)^3 + \Omega_r(1+z)^4}. \tag{54}$$

Here $\Omega_\Lambda, \Omega_m, \Omega_r$ are the respective density parameters for the assumed vacuum energy, cold matter and radiation content of the current universe.

For spatially homogeneous and nondispersive (grey) absorption the differential optical depth $d\tau$ is related to the opacity α by

$$
\begin{aligned}
d\tau &= -\alpha(cdt) \\
&= \alpha \frac{c}{H} \frac{1}{(1+z)} \frac{dz}{h(z)},
\end{aligned}
\tag{55}
$$

and the total optical depth $\tau(r,t)$ of a source at $(0, t_e(r,t))$ seen by an observer at (r,t) is the integral

$$\tau(z) = \frac{c}{H(t)} \int_0^z \frac{\alpha(z')}{(1+z')h(z')} dz'. \tag{56}$$

The conformal factor σ of Eq. (52) is therefore one-half this value.

The presence of absorption changes the distance modulus-redshift relation $\mu(z)$, e.g., the magnitudes of supernovae are corrected for absorption by the IGM before drawing a Hubble diagram. From the definition of distance modulus

$$\mu \equiv 5 \log \frac{d_L}{1\mathrm{Mpc}} + 25, \tag{57}$$

and luminosity distance d_L in terms of flux S

$$d_L = \left(\frac{L}{4\pi S} \right)^{1/2}, \tag{58}$$

we immediately obtain via Eq. (39) the absorption-corrected luminosity distance

$$\tilde{d}_L = e^{\tau/2} d_L, \tag{59}$$

and the corrected distance modulus $\tilde{\mu}$

$$\tilde{\mu} = \mu + \frac{5}{2\ln 10} \tau. \tag{60}$$

This is the normal interpretation of the increase in luminosity distance caused by light absorption. If we use the optical metric to compute luminosity distance (see e.g. [8]) we find that compared with the physical metric the cross-sectional area of the ray bundle is expanded by a factor $e^{2\sigma}$ which reduces the received flux by a factor of $e^{-2\sigma}$, i.e., from Eq. (52)

$$\tilde{d}_L = e^{\sigma(r_o, t_o)}(1+z) R_0 r = e^\sigma d_L. \tag{61}$$

We see that by identifying σ with $\tau/2$ we have the same luminosity distance in the optical spacetime with no absorption as in the physical spacetime with absorption.

VI. INCORPORATING BOTH REFRACTION AND ABSORPTION INTO THE OPTICAL METRIC

Up to now, we have been considering only absorption, however, we know that besides having its flux reduced, a light wave's path and speed can be altered (refracted) due to the presence of a polarizable material. An interesting and useful tool to study refraction in curved spacetime was developed by Gordon [1] (see also Ehlers [9]). Gordon modified Einstein's physical spacetime metric to include the effects of a nondispersive refractive material on Maxwell's theory. The theory accounts for any polarizable material whose constitutive properties are summarized by two scalar functions, a permittivity $\epsilon(x^a)$ and a permeability $\mu(x^a)$. His optical metric $\bar{g}_{ab}$ [1] is defined on the same differentiable manifold as Einstein's spacetime metric g_{ab} and is related to it by

$$\bar{g}_{ab} = g_{ab} + \left(1 - \frac{1}{n^2}\right) u_a u_b, \quad \bar{g}^{ab} = g^{ab} + \left(1 - n^2\right) u^a u^b, \tag{62}$$

where $n(x^a) = \sqrt{\epsilon\mu}$ is the refractive index and u^a is the 4-velocity of the optical fluid, normalized using the physical spacetime metric g_{ab}. Null geodesics of the $\bar{g}_{ab}$ metric are identical to light paths (timelike) in physical spacetime filled with a refracting material of index $n(x)$. Even though we can incorporate absorption of only geometrical optics waves into Gordon's metric, his index of refraction theory applies to all Maxwell fields, see [2] for some related examples. Gordon showed that solutions to Maxwell's theory (F_{ab}, H^{ab}) in the presence of such a material can be found by solving a slightly modified set of vacuum ($\epsilon = \mu = 1$) Maxwell equations for $(\bar{F}_{ab}, \bar{F}^{ab})$ in his optical spacetime. The fields are connected by

$$
\begin{aligned}
H^{ab} &= \frac{1}{\mu}\bar{F}^{ab}, \\
F_{ab} &= \bar{F}_{ab},
\end{aligned}
\tag{63}
$$

and the modified Maxwell equations are [2]

$$
\begin{aligned}
\bar{F}_{[ab,c]} &= 0, \\
\bar{\nabla}_b\left(\sqrt{\epsilon/\mu}\,\bar{F}^{ab}\right) &= 0.
\end{aligned}
\tag{64}
$$

We now generalize Gordon's optical metric to include effects of an additional isotropic and frequency independent (grey) attenuation coefficient κ_ν on a radiation field described by the geometrical optics approximation. To obtain this result we simply apply a transverse conformal transformation of Eq. (10) to Gordon's metric Eq. (62). The geometrical optics theory in Gordon's optical spacetime is almost identical to the geometrical optics theory of the physical spacetime given in Sec. III (see Sec. III of [2]), e.g., the contravariant components of the Maxwell field are correctly given by Eq. (32)

$$
\bar{F}^{ab} = -2\,\mathrm{Re}\{\mathcal{E}\,e^{iS/\lambda_0}\bar{k}^{[a}\bar{e}^{b]}\}.
\tag{65}
$$

Here

$$
\bar{k}^a \equiv \bar{g}^{ab}\partial_b S,
\tag{66}
$$

is tangent to null geodesics of the optical metric (and tangent to the corresponding timelike "slower than light" curves of the physical metric), $\bar{e}^b$ is the unit (in the optical metric) polarization vector, and S is the eikonal function. The significantly different equation is the transport equation for the amplitude of the wave given by Eq. (31), which becomes

$$
\dot{\mathcal{E}} + \theta\mathcal{E} + \dot{\phi}\mathcal{E} = 0,
\tag{67}
$$

(see Eq. (18) of [2]). The presence of the additional term $\phi \equiv (1/4)\ln(\epsilon/\mu)$ is due to the afore mentioned modification of Maxwell's equations [see Eq. (64)] in Gordon's optical spacetime.

From Eqs. (10) and (11) the new optical metric becomes

$$
\tilde{g}_{ab} = e^{2\sigma}\bar{g}_{ab} + (1 - e^{2\sigma})2\bar{\ell}_{(a}\bar{k}_{b)}, \quad \tilde{g}^{ab} = e^{-2\sigma}\bar{g}^{ab} + (1 - e^{-2\sigma})2\bar{\ell}^{(a}\bar{k}^{b)}.
\tag{68}
$$

where $\bar{\ell}^a$ from Eq. (7) is a null vector field in Gordon's optical spacetime defined by

$$
\bar{\ell}^a = \frac{\bar{k}^a}{2(\bar{u}_b\bar{k}^b)^2} + \frac{\bar{u}^a}{(\bar{u}_b\bar{k}^b)},
\tag{69}
$$

and $\bar{u}^a$ the fluid's 4-velocity normalized using Gordon's optical metric

$$
\bar{u}^a = nu^a, \quad \bar{u}_a = \frac{1}{n}u_a.
\tag{70}
$$

---

[2] We thank John Ralston for pointing out to us that the modified equations can be derived from the Lagrangian $\mathcal{L} = -\frac{1}{4}\sqrt{-\det\bar{g}}\sqrt{\epsilon/\mu}\bar{F}^{ab}F_{ab}$.

The reader should observe that the transverse conformal transformation does not alter the orthogonality of the two 2-D subspaces nor does it alter the metric structure of the timelike 2-D space spanned by $\bar{u}^a$ and $\bar{k}^a$. Rewriting the new optical metric Eq. (68) in terms of the physical metric g_{ab} and physical observer u_a we have

$$\tilde{g}_{ab} = e^{2\sigma}g_{ab} + e^{2\sigma}\left(1 - \frac{1}{n^2}\right)u_a u_b + \frac{1 - e^{2\sigma}}{n^2(u\cdot k)^2}\left[k_a k_b + 2(u\cdot k)k_{(a}u_{b)}\right], \tag{71}$$

with inverse

$$\tilde{g}^{ab} = e^{-2\sigma}g^{ab} + (1-n^2)e^{-2\sigma}u^a u^b + \frac{1 - e^{-2\sigma}}{n^2(u\cdot k)^2}[\tilde{k}^a\tilde{k}^b + 2n^2(u\cdot k)u^{(a}\tilde{k}^{b)}], \tag{72}$$

where

$$\tilde{k}^a \equiv \tilde{g}^{ab}S_{,b} = \bar{g}^{ab}S_{,b} = g^{ab}S_{,b} + (1-n^2)u^a(u^b S_{,b}), \tag{73}$$

and determinant

$$\det\tilde{g} = \frac{e^{4\sigma}}{n^2}\det g. \tag{74}$$

The new Maxwell field

$$\begin{aligned}\tilde{F}^{ab} &= \mu e^{-2\sigma}H^{ab}, \\ \tilde{F}_{ab} &= F_{ab},\end{aligned} \tag{75}$$

satisfies the same equations as $\bar{F}^{ab}$, i.e., Eq. (64), but $\tilde{g}_{ab}$ has the advantage of incorporating both refraction and absorption. We now have a correspondence of geometrical optics waves in two spacetimes, the physical and the optical. In the physical spacetime the wave travels at a reduced speed c/n with an intensity that is reduced by absorption as in Eq. (37) whereas in the optical spacetime the wave travels at speed c with no extinction.

VII. THE OPTICAL METRIC IN FLRW COSMOLOGY WITH BOTH REFRACTION AND ABSORPTION

The formalism developed above appears complicated when applied to an arbitrary spacetime, however, for specific cases the formalism is more transparent. For example we reconsider FLRW spacetimes, but this time with both refraction n and absorption κ associated with the cosmic fluid. The result is elegantly simple. Gordon's optical metric for the Robertson-Walker metric [2] is

$$d\bar{s}^2 = -\frac{c^2}{n^2}dt^2 + R^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right\}. \tag{76}$$

This metric does not measure distance and time, that is reserved for Eq. (46). A dynamical theory that produced a gravitational field containing the index of refraction as given in Eq. (76) would be quite strange in that it would be partially sourced by electromagnetic polarization densities; which is not stress, energy, or momentum. The significant property of the optical metric that we use here is that its null geodesics are identical with the speed c/n photons of Eq. (46).

For spherical waves the radial null geodesics are

$$\bar{k}^a = R_0\left(\frac{n}{cR}, \frac{\sqrt{1-kr^2}}{R^2}, 0, 0\right), \tag{77}$$

and from Eq. (69) we find

$$\bar{\ell}^a = \frac{R}{2R_0}\left(-\frac{n}{c}, \frac{\sqrt{1-kr^2}}{R}, 0, 0\right). \tag{78}$$

Immediately we find

$$2\bar{\ell}_{(a}\bar{k}_{b)} = \text{diag}\left(-\frac{c^2}{n^2}, \frac{R^2}{1-kr^2}, 0, 0\right), \tag{79}$$

which gives us the new optical metric

$$d\tilde{s}^2 = -\frac{c^2}{n^2}dt^2 + R^2(t)\left\{\frac{dr^2}{1-kr^2} + e^{2\sigma}r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right\}. \tag{80}$$

With dynamics supplied by the FLRW solutions the scalar function σ in the above is

$$\begin{aligned}\sigma(r,t) &\equiv \frac{1}{2}\int_{t_e}^{t}\alpha\,\frac{c\,dt'}{n}\\ &= \frac{1}{2}\frac{c}{H(t)}\int_0^{z(z_n)}\frac{\alpha(z')}{n(z')(1+z')h(z')}dz'.\end{aligned} \tag{81}$$

Note the difference between wavelength redshift z and frequency redshift z_n (see [2]):

$$1 + z = \frac{R_0}{R(t_e)},\ 1 + z_n = \frac{n(t_o)}{n(t_e)}\frac{R_0}{R(t_e)}. \tag{82}$$

We have discussed the impact of light refraction on the distance-redshift relation in [2]. Now incorporating absorption we find from Eq. (80) that the apparent size distance $\tilde{d}_A$ to the source in the new optical metric is

$$\tilde{d}_A = R(t_e)e^{\sigma(0,t_e)}r = R(t_e)r = \bar{d}_A, \tag{83}$$

is the same as the apparent size distance in Gordon's spacetime

$$\bar{d}_A(z_n) = \frac{1}{[1+z(z_n)]}\frac{c}{H_0}\frac{1}{\sqrt{|\Omega_k|}}\text{sinn}\left[\sqrt{|\Omega_k|}\int_0^{z(z_n)}\frac{dz}{n(z)h(z)}\right]. \tag{84}$$

Finally the luminosity distance $\tilde{d}_L$ is (see [2, 8])

$$\tilde{d}_L = (1+z_n)(1+z)e^{\sigma}d_A = e^{\tau/2}\bar{d}_L. \tag{85}$$

We have thus obtained the same luminosity-redshift relation we obtained in [2] for the real eikonal case (refer to Eq. (66) of [2]) by changing the geometry rather than absorbing some of the wave's intensity. Just as in [2], Eq. (85) differs from the standard reciprocity relation [8, 10, 11]

$$d_L(z) = (1+z)^2 d_A(z), \tag{86}$$

in two aspects: first, the existence of refraction causes photon orbits to deviate from null geodesics in the physical spacetime, this giving the $1 + z_n$ factor instead of $1 + z$; second, light absorption (expressed by the conformal factor σ) violates the photon number conservation law. For an interesting discussion about using reciprocity relation as a probe of acceleration/exotic physics, see [12].

VIII.   AN APPLICATION: FLRW SPACETIME WITH BOTH REFRACTION AND ABSORPTION

In [2, 13] we have interpreted the observed apparent increase in the universe's expansion rate as caused by light refraction and absorption respectively, instead of by a cosmological constant Λ. In [13] we used a two parameter pure refraction model, i.e., $n(z) = 1 + pz^2 + qz^3$, without absorption to fit the supernovae gold sample [19]. In [2] we fit the sample with a pure absorption model, i.e., we took $n = 1$, and the opacity $\alpha(z) \equiv \rho(z)\kappa(z) = \text{const}$. In this section, we fit the same data set [14, 15, 16, 17, 18] with a cold dark matter model containing both refraction and absorption. We use simple expressions $n(z) = 1 + pz^2$ and $\alpha(z) = \text{const}$ that depend on only two parameters. Since we are concerned with the matter dominated era, we have excluded radiation ($\Omega_r = 0$) and since we are trying to only emulate acceleration we take $\Lambda = 0$. We choose the current Hubble constant to be $H_0 = 65\,\text{km/s/Mpc}$ and fit our two parameter (α, p) model for different choices of Ω_m. The scaled Hubble function $h(z)$ in this case simplifies to

$$h(z) = (1+z)\sqrt{1+\Omega_\text{m}z}. \tag{87}$$

The refraction and absorption corrected distance-redshift relation is now written as

$$\tilde{d}_L(z_n) = (1+z_n)\frac{c}{H_0}\frac{e^{\sigma(z_n)}}{\sqrt{|\Omega_k|}}\text{sinn}\sqrt{|\Omega_k|}\int_0^{z(z_n)}\frac{dz'}{n(z')h(z')}, \tag{88}$$

where

$$\sigma(z_n) = \frac{\alpha}{2} \frac{c}{H_0} \int_0^{z(z_n)} \frac{dz'}{(1+z')n(z')h(z')}, \tag{89}$$

and

$$1 + z_n = \frac{1+z}{1+pz^2}. \tag{90}$$

We use the 178 supernovae from the gold sample [19] with redshifts greater than $cz = 7000$ km/s to avoid any Hubble bubble. Our results are shown in Figures 1 and 2. In Fig. 1, we show the $\Delta\mu(z)$ curves for different model parameters. Here $\Delta\mu(z) \equiv \mu(z) - \mu_F(z)$, where $\mu_F(z)$ is the distance modulus of the fiducial, dark matter only model (horizontal black dashed curve in Fig. 1), i.e., $\Omega_\Lambda = 0, \Omega_m = 0.3, \alpha = 0, p = 0$. In each of the four frames the green curve (grey in black & white) is the concordance model, $\Omega_\Lambda = 0.7, \Omega_m = 0.3, \alpha = 0, p = 0$. In the upper left panel we show absorption models with no refraction index ($\Omega_m < 1$ for all curves in this panel). The dotted red curve has: $\Omega_\Lambda = 0, \Omega_m = 0.05, \alpha = 0.7 \times 10^{-4}\text{Mpc}^{-1}, n = 1$. The short-dashed red curve has: $\Omega_\Lambda = 0, \Omega_m = 0.3, \alpha = 1.3 \times 10^{-4}\text{Mpc}^{-1}, n = 1$. The solid black curve has: $\Omega_\Lambda = 0, \Omega_m = 0.73, \alpha = 2.2 \times 10^{-4}\text{Mpc}^{-1}, n = 1$. In the upper right panel the models are flat. The blue curve (bottom curve) has: $\Omega_\Lambda = 0, \Omega_m = 1.0, \alpha = 2.5 \times 10^{-4}\text{Mpc}^{-1}, p = 0$. The red curve (top curve) has: $\Omega_\Lambda = 0, \Omega_m = 1.0, \alpha = 2.5 \times 10^{-4}\text{Mpc}^{-1}, p = 0.024$. In the bottom left panel the blue curve (bottom curve) has: $\Omega_\Lambda = 0, \Omega_m = 1.5, \alpha = 3.3 \times 10^{-4}\text{Mpc}^{-1}, p = 0$. The red curve (top curve) has: $\Omega_\Lambda = 0, \Omega_m = 1.5, \alpha = 3.3 \times 10^{-4}\text{Mpc}^{-1}, p = 0.042$. In the bottom right panel the blue curve (bottom curve) has: $\Omega_\Lambda = 0, \Omega_m = 2.0, \alpha = 4.1 \times 10^{-4}\text{Mpc}^{-1}, p = 0$. The red (top curve) curve has: $\Omega_\Lambda = 0, \Omega_m = 2.0, \alpha = 4.1 \times 10^{-4}\text{Mpc}^{-1}, p = 0.049$.

In Fig. 2 we show the confidence contours (68.3%, 95.4%, and 99.73%) of our two parameter (α, p) model with different choices of Ω_m. We show 6 different cases with $\Omega_m = 0.05$ (baryonic matter only), $\Omega_m = 0.3$ (dark matter only), $\Omega_m = 0.73$ (our best least χ^2 pure absorption model, see [2]), $\Omega_m = 1.0$ (flat universe), $\Omega_m = 1.5$ and 2.0 (closed models). We restrict the parameter p to be nonnegative to keep the light speed $v = c/n$ less than c. For the first three cases, i.e., $\Omega_m = 0.05, 0.3$ and 0.73, the best fits occur for $p = 0$, which suggests that introducing additional refraction parameters would not significantly improve the fitting when absorption is present. However, as Ω_m increases, nonvanishing p values do give better fits. For $\Omega_m = 1.0$, the best fitting parameters ($\chi^2 = 1.04$) are $\alpha = 2.5 \times 10^{-4}\text{Mpc}^{-1}, p = 0.024$. For $\Omega_m = 1.5$, the best fitting ($\chi^2 = 1.05$) parameters are $\alpha = 3.3 \times 10^{-4}\text{Mpc}^{-1}, p = 0.042$. For $\Omega_m = 2.0$, the best fitting ($\chi^2 = 1.06$) parameters are $\alpha = 4.1 \times 10^{-4}\text{Mpc}^{-1}, p = 0.049$. This can also be seen from Fig. 1: In the upper right and two bottom frames, the blue and red curves have the same absorption coefficient α, the difference is that the red curve has p nonzero whereas the blue curve has $p = 0$. The inclusion of p for these large Ω_m cases improves the fits.

The Supernova data is currently considered to be the most compelling evidence for the existence of dark energy because of the apparent acceleration observed in the expansion of the universe (see e.g. [20, 21]). Competing interpretations have been proposed, e.g., evolutionary effects [22, 23], local Hubble bubbles [24, 25], absorption [26, 27, 28], modified gravity [29, 30, 31] and others such as slowly changing fundamental constants [32, 33].

For supernovae there are at least four different sources of opacity; the Milky Way, the hosting galaxy, intervening galaxies, and the IGM that should be taken into account. The Galactic absorption has been studied extensively [34, 35] and early constraints on properties of IGM were often obtained assuming the applicability of Galactic dust properties (see e.g. [36]). The luminosity of high redshift supernovae have been corrected for host galaxy absorption using the Galactic reddening law, see e.g., [20, 21, 37, 38]. As has been pointed out caution should be exercised when applying Galactic dust properties to the IGM, since the composition, size, shape, and alignment of intergalactic dust could be significantly different than that of Milky Way dust. Aguirre [26, 27] introduced a carbon needle model and showed that dust grains of larger size ($\geq 1\,\mu m$), which should be preferentially ejected by star burst galaxies, would have relatively higher opacities (e.g. $\kappa \sim 10^5$ cm$^2\,g^{-1}$) and much greyer absorption curves, and therefore might escape the reddening censorship based on Galactic reddening law. Evidence against this model appeared in [39, 40, 41]. Goobar et al. [42] introduced a replenishing dust model in which the dilution caused by cosmic expansion is continually replenished ($\rho = \rho_0(1+z)^3$ for $z < 0.5$, $\rho = \rho_0$ for $z \geq 0.5$). This dust model is indistinguishable from Ω_Λ and cannot be ruled out by Supernovae data alone (see table 5 of [15]). Bassett & Kunz [12] claimed to rule out the replenishing model at more than 4σ by considering violations of distance duality (reciprocity relation). Östman & Mörtsell [43] further constrained the magnitude of grey dust absorption from Quasar colors and spectra and claimed that for a wide range of intergalactic dust models, extinction larger than 0.2 mag is ruled out (see also [44, 45]). A new grey dust model is proposed in [46]. More complicated and fine tuned dust models will keep emerging in the future until dark matter/energy has been identified, if in fact it exists.
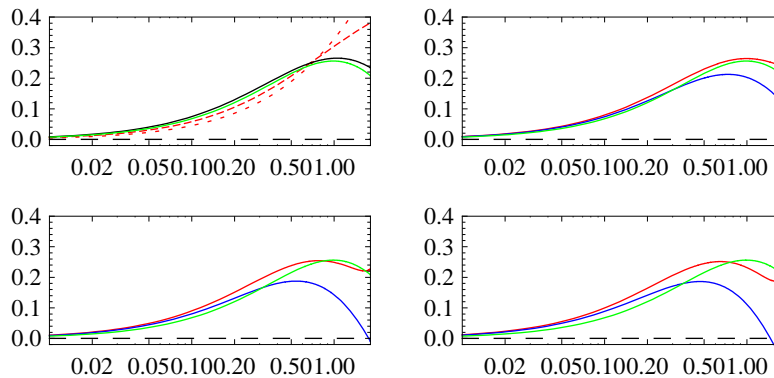
FIG. 1: $\Delta\mu$ versus z. In each of the four frames the fiducial model (horizontal black dashed) is the now disfavored dark matter only model, i.e., $\Omega_\Lambda = 0, \Omega_m = 0.3, \alpha = 0, p = 0$, and the green curve (gray in black & white) is the concordance model, $\Omega_\Lambda = 0.7, \Omega_m = 0.3, \alpha = 0, p = 0$. The upper left panel contains pure absorption models with no refraction: the dotted red curve has $\Omega_\Lambda = 0, \Omega_m = 0.05, \alpha = 0.7 \times 10^{-4} \mathrm{Mpc}^{-1}, n = 1$; the short-dashed red curve has $\Omega_\Lambda = 0, \Omega_m = 0.3, \alpha = 1.3 \times 10^{-4} \mathrm{Mpc}^{-1}, n = 1$; the solid black curve has $\Omega_\Lambda = 0, \Omega_m = 0.73, \alpha = 2.2 \times 10^{-4} \mathrm{Mpc}^{-1}, n = 1$. In the upper right panel: the blue (bottom) curve has $\Omega_\Lambda = 0, \Omega_m = 1.0, \alpha = 2.5 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0$; the red (top) curve has $\Omega_\Lambda = 0, \Omega_m = 1.0, \alpha = 2.5 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0.024$. In the bottom left panel: blue (bottom) curve has $\Omega_\Lambda = 0, \Omega_m = 1.5, \alpha = 3.3 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0$; The red (top) curve has $\Omega_\Lambda = 0, \Omega_m = 1.0, \alpha = 3.3 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0.042$. In the bottom right panel: blue (bottom) curve has $\Omega_\Lambda = 0, \Omega_m = 2.0, \alpha = 4.1 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0$; the red curve has $\Omega_\Lambda = 0, \Omega_m = 2.0, \alpha = 4.1 \times 10^{-4} \mathrm{Mpc}^{-1}, p = 0.049$.

IX. DISCUSSION

In this paper, we introduced the concept of a "transverse" conformal transformation, which allowed us to equate the intensity reduction of light caused by absorption with a geometrical reduction caused by the expansion of the cross-sectional area of light ray bundles without changing other desirable properties of waves, such as frequency and wavelength. This application of conformal transformations is new. We used it to generalize Gordon's optical metric to include light absorption via such a conformal transformation. This generalization is fundamentally different from [2] in which we included light absorption by making Gordon's metric complex. In the complex metric formalism, we distinguished two different cases: strong and weak absorption. The strong case is where a non-negligible amount of absorption occurs on a wavelength scale and the weak case is where absorption is significant only over multitudes of wavelengths. In the strong case the eikonal remains complex but in the weak case it can be taken as real. In this paper we were able to replace the effects of weak absorption by a special conformal transformation. The disadvantage of this formalism is that the optical metric depends on the eikonal of the wave itself whereas in [2] it did not. We used this new optical metric to derive the absorption and refraction corrected distance-redshift relation for FLRW spacetimes and obtained the same expression as in [2] for weak absorption. In [13] we fit supernovae data with a pure refraction model, and in [2] we fit it with a pure absorption model. In Sec. VIII we fit this data with a cosmological model possessing both refraction and absorption. We have shown that a single parameter polynomial approximation for the refraction index, i.e., $n(z) = 1 + pz^2$, together with a constant opacity parameter α fits the data well. More realistic z dependent models for α and n would be appropriate. We have not proposed a physical source for the needed refraction and/or absorption. What we have done here is to construct a geometry modification whose effects are equivalent to absorption. It is not a new absorption theory only a new way of looking at its effects. It equates the decrease in intensity of a wave caused by absorption with a decrease caused by a change in the underlying spacetime, i.e., a transverse conformal transformation. We used this modified geometry to compute the distance-redshift relations in FLRW and applied it to the Hubble curve for type SNe-Ia. In this application we ignored effects of inhomogeneities including the possibility of having a multi-valued conformal factor. We assumed that none of the SN Ia used were strongly lensed. Since the flat concordance model is consistent with other observations, e.g., the angular position of the first acoustic peak as measured by WMAP (Wilkinson Microwave Anisotropy Probe) team [47, 48], and baryonic acoustic oscillations (BAO) detected in galaxy surveys [49, 50, 51], an absorption and/or refraction theory cannot be on firm ground unless it is also consistent with these additional observations. We leave these and other applications to future efforts.
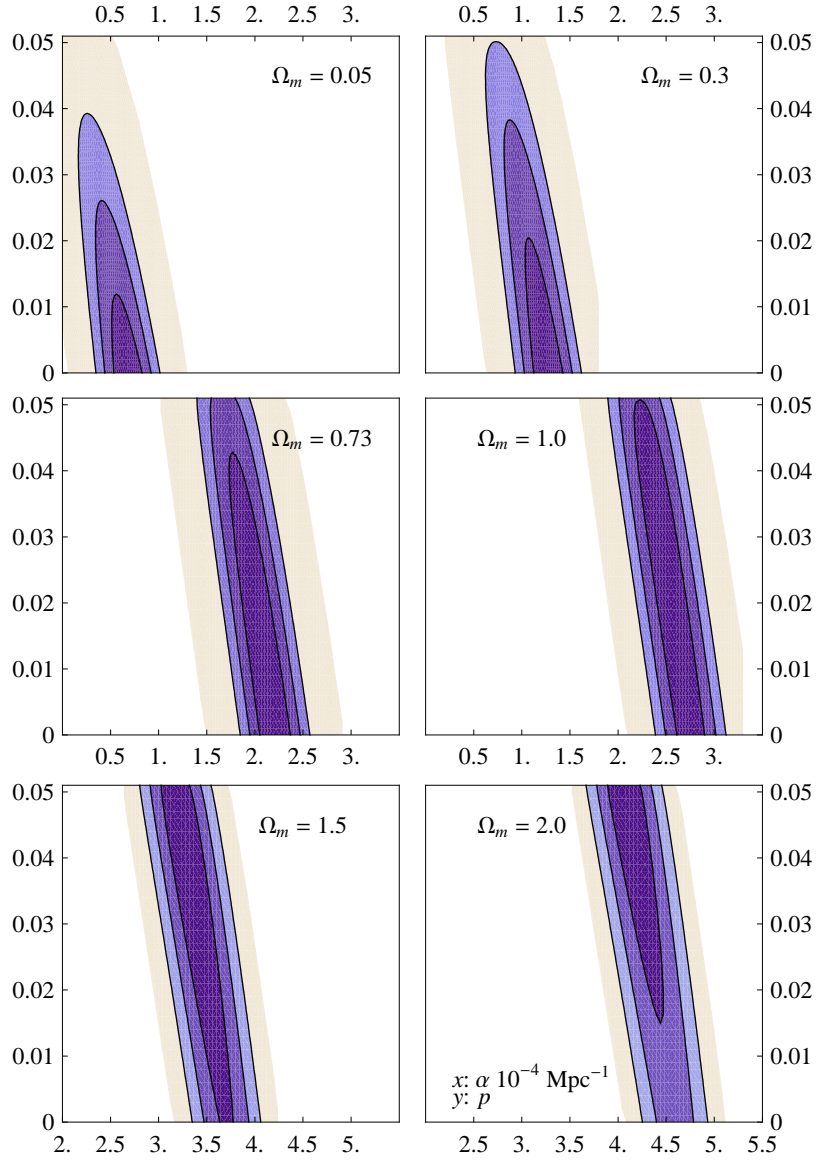
FIG. 2: Confidence contours (68.3%, 95.4%, and 99.73%) for each fixed Ω_m model. The x coordinate is the absorption coefficient α in unit of $10^{-4}\mathrm{Mpc}^{-1}$, and the y-axis is the refraction index parameter p $(n = 1 + pz^2)$. In the top left panel, $\Omega_m = 0.05$, with least chi-square (per degree of freedom) $\chi^2 = 1.09$, the best fitting parameters are $\alpha = 0.7 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0$. In the top right panel, $\Omega_m = 0.3$, least $\chi^2 = 1.05$, the best fitting parameters are $\alpha = 1.3 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0$. In the middle left panel, $\Omega_m = 0.73$, least $\chi^2 = 1.035$, the best fitting parameters are $\alpha = 2.2 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0$. In the middle right panel, $\Omega_m = 1.0$, least $\chi^2 = 1.042$, at $\alpha = 2.5 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0.024$. In the bottom left panel, $\Omega_m = 1.5$, least $\chi^2 = 1.05$, the best fitting parameters are $\alpha = 3.3 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0.042$. I the bottom right panel, $\Omega_m = 2.0$, least $\chi^2 = 1.06$, the best fitting parameters are $\alpha = 4.1 \times 10^{-4}\mathrm{Mpc}^{-1}$, $p = 0.049$.

X.   ACKNOWLEDGMENTS

[1] W. Gordon, Ann. Phys. (Leipzig) 72, 421 (1923).
[2] B. Chen and R. Kantowski, Phys. Rev. D 79, 104007 (2009).
[3] L. P. Eisenhart, *Riemannian Geometry* (Princeton Univ., Princeton, 1926).

[4] R. Penrose, *An Analysis of the Structure of Space-Time, Adams Prize Essay* (Princeton Univ., Princeton, 1967).

[5] J. Ehlers, Z. Naturforsch. 22a, 1328 (1967).

[6] R. Sachs, Proc. R. Soc. A 264, 309 (1961).

[7] D. Mihalas, *Stellar Atmospheres* (W. H. Freeman and Company, San Fransisco, 1970).

[8] G. F. R. Ellis, in *General Relativity and Cosmology*, edited by R. K. Sachs (Academic, New York, 1971), p 104.

[9] J. Ehlers, in *Perspectives in Geometry and Relativity*, edited by B. Hoffmann (Indiana Univ., Indiana, 1966), p. 127.

[10] I. M. H. Etherington, Philos. Mag. 15, 761 (1933).

[11] B. Bertotti, Proc. R. Soc. A. 294, 195 (1966).

[12] B. A. Bassett & M. Kunz, Phys. Rev. D 69, 101305(R) (2004).

[13] B. Chen and R. Kantowski, Phys. Rev. D 78, 044040 (2008).

[14] A. G. Riess et al., Astrophys. J. 607, 665 (2004).

[15] A. G. Riess et al., Astrophys. J. 659, 98 (2007).

[16] P. Astier et al., Astron. Astrophys. 447, 31 (2006).

[17] T. M. Davis et al., Astrophys. J. 666, 716 (2007).

[18] W. M. Wood-Vasey et al., Astrophys. J. 666, 694 (2007).

[19] http://braeburn.pha.jhu.edu/∼ariess/R06/.

[20] A. G. Riess et al., Astron. J. 116, 1009 (1998).

[21] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).

[22] P. S. Drell, T. J. Loredo, and I. Wasserman, Astrophys. J. 530, 593 (2000).

[23] F. Combes, New Astron. Rev. 48, 583 (2004).

[24] A. Conley et al., Astrophys. J. 664, L13 (2007).

[25] I. Zehavi et al., Astrophys. J. 503, 483 (1998).

[26] A. N. Aguirre, Astrophys. J. 512, L19 (1999).

[27] A. N. Aguirre, Astrophys. J. 525, 583 (1999).

[28] M. Rowan-Robinson, Mon. Not. R. Astron. Soc. 332, 352 (2002).

[29] M. Ishak, A. Upadhye & D. N. Spergel, Phys. Rev. D 74, 043513 (2006).

[30] M. Kunz & D. Sapone, Phys. Rev. Lett 98, 121301 (2007).

[31] E. Bertschinger & P. Zukin, Phys. Rev. D 78, 024015 (2008).

[32] A. Albrecht and J. Magueijo, Phys. Rev. D 59, 043516 (1999).

[33] J. D. Barrow, Phys. Rev. D 59, 043515 (1999).

[34] L. Spitzer, *Physical Processes in the Interstellar Medium* (John Wiley & Sons, New York, 1978).

[35] J. S. Mathis, Ann. Rev. Astron. Astrophys. 28, 37 (1990).

[36] E. L. Wright, Astrophys. J. 250, 1 (1981).

[37] A. G. Riess, W. H. Press, and R. P. Kirshner, Astrophys. J. 473, 88 (1996).

[38] A. G. Riess, W. H. Press, and R. P. Kirshner, Astrophys. J. 473, 588 (1996).

[39] R. A. C. Croft, R. Davé, L. Hernquist and N. Katz, Astrophys. J. 534, L123 (2000).

[40] R. Knop et al. Astrophys. J. 598, 102 (2003).

[41] M. Sullivan et al. Mon. Not. R. Astron. Soc. 340, 1057 (2003).

[42] A. Goobar, L. Bergström, and E. Mörtsell, Astron. Astrophys. 384, 1 (2002).

[43] L. Östman & E. Mörtsell, J. Cosmol. Astropart. Phys. 2, 005 (2005).

[44] A. K. Inoue & H. Kamaya, Mon. Not. R. Astron. Soc. 350, 729 (2004).

[45] P. S. Corasaniti, Mon. Not. R. Astron. Soc. 372, 191 (2006).

[46] A. R. Robaina & J. Cepa, Astron. Astrophys. 464, 465 (2007).

[47] D. N. Spergel et al. Astrophys. J. Suppl. Ser. 148, 175 (2003).

[48] D. N. Spergel et al. Astrophys. J. Suppl. Ser. 170, 377 (2007).

[49] M. Tegmark et al. Phys. Rev. D 69, 103501 (2004).

[50] M. Tegmark et al. Astrophys. J. 606, 702 (2004).

[51] D. J. Eisenstein et al. Astrophys. J. 633, 560 (2005).