

Wednesday January 19, 2005
quiz #1

DO AS MANY OF THE FOLLOWING QUESTIONS AS YOU CAN IN THE NEXT HOUR

Handy integrals

$$\int_{-a/2}^{a/2} \frac{1}{\sqrt{b^2 + y^2}} dy = \ln \left[\frac{\sqrt{a^2 + 4b^2} + a}{\sqrt{a^2 + 4b^2} - a} \right]$$

$$\int_{-\infty}^{\infty} \frac{dz'}{(a^2 + z'^2)^{3/2}} = \frac{2}{a^2}$$

$$\int_0^{2\pi} \frac{\sin^2 \phi' d\phi'}{(1 - \cos \beta \cos \phi')} = - \int_0^{2\pi} \frac{\cos^2 \phi' d\phi'}{(1 - \cos \beta \cos \phi')^2} = 2\pi \sec^2 \beta$$

$$\int_0^{2\pi} \frac{\sin \phi' d\phi'}{(1 - \cos \beta \cos \phi')} = \int_0^{2\pi} \frac{\sin \phi' \cos \phi' d\phi'}{(1 - \cos \beta \cos \phi')} = 0$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{(\vec{r} - \vec{r}') dq'}{|\vec{r} - \vec{r}'|^3}$$

$$dq' = \lambda' dl' = \sigma' da' = \rho' d\tau'$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{d\vec{b}' \times (\vec{r} - \vec{r}')}{|\vec{r} - \vec{r}'|^3}$$

$$d\vec{b}' = \lambda' d\vec{l}' = \vec{K}' da' = \vec{J}' d\tau'$$

$$\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$\rho_b = -\vec{\nabla} \cdot \vec{P}$$

$$\sigma_b = \hat{n} \cdot \vec{P}$$

$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{K}_b = \vec{M} \times \hat{n}$$

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 v_r + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} \sin \theta v_\theta + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} v_\phi$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$\oint \vec{H} \cdot d\vec{l} = I_{f,enc}$$

Problem 0: Write down Maxwell's equations in differential form.

Problem 1: The function f is given by

$$f(x, y, z) = \tanh(x^2 - \sin(yz)).$$

Evaluate the integral

$$\int_{\vec{r}_A}^{\vec{r}_B} (\nabla f) \cdot d\vec{l}$$

over the arc described by the parameterization

$$\vec{r}' = a \cos \theta \hat{i} + b \sin \theta \hat{j}, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

Problem 2: The vector field \vec{v} is given by

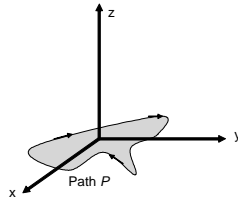
$$\vec{v} = (x^2 + y^2 + z^2 - a^2)(\cos \theta \hat{x} + \sin \theta \hat{y}) + (x^2 + y^2 + z^2)(\cos \phi \sin \theta \hat{x} + \sin \phi \sin \theta \hat{y} + \cos \theta \hat{z})$$

evaluate the integral

$$\iiint_V (\nabla \cdot \vec{v}) d\tau$$

where V is the volume of the interior of a sphere of radius a centered at the origin.

Problem 3: The path P of the figure below lies completely in the $x - y$ plane and encloses an area A .



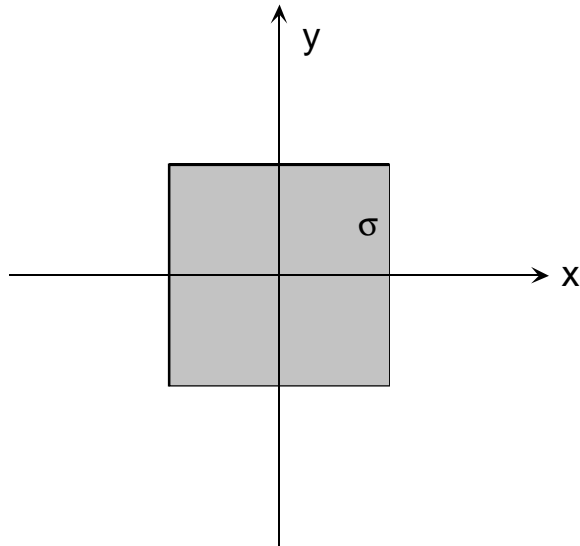
Consider the vector field

$$\vec{v} = y\hat{x} + z\hat{y} + x\hat{z}.$$

Evaluate

$$\oint_P \vec{v} \cdot d\vec{l}$$

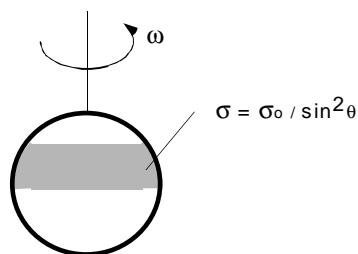
Problem 4. Find an exact expression for the field everywhere on the x -axis if a square of dimension a and uniform charge per unit area σ sits centered on the $x - y$ plane as shown:



Hint: Find the electric field directly without first computing $V(\vec{r})$ and use one of the given integrals. Do not expect this to simplify into something pretty. Sometimes the answer is just ugly.

Problem 5 (a) (a few points) Use ampere's law to find the field a distance s from a wire carrying a current I .

(b) (lots of points) A sphere of radius a carries a surface charge density $\sigma = \sigma_0 / \sin^2 \theta'$ in the range of spherical polar angles $\frac{\pi}{4} < \theta' < \frac{\pi}{2}$ and $\sigma = 0$ for all other angles. The sphere spins with an angular velocity ω about a z -axis as shown.



Use the Biot-Savart law to find \vec{B} on the z axis above the spinning sphere. (Hint: $\vec{K}' = \sigma \vec{v}'$. Express the Cartesian components of \vec{v}' in terms of $\omega, a \sin \theta'$, and ϕ' .)

(c) (a few points) Substitute $I = \sigma_0 \omega L^2$ into (a) to make sure the units of your answer to (a) agrees with the units of your solution to (b).

(d) (a few points) For large z , how fast does your expression for the \vec{B} field given in (b) fall off with z ?

Problem 6. A sphere of radius R carries a free charge density $\rho = \rho_o(r/R)^{1/2}$. The sphere is made of a dielectric material of electric susceptibility χ_e . Find

- (a) The displacement vector \vec{D} inside and outside the sphere
- (b) The polarization \vec{P} inside the sphere.
- (c) The bound charge density and bound surface charge density. (Hint: Make sure ρ_b has the units of ρ_o and σ_b has the units of $\rho_o R$.)

Problem 7. An infinitely long cylinder of radius a lies on the z axis of a coordinate system and carries a uniform magnetization $\vec{M} = M_o \hat{i}$. (NOTE: The axis of polarization is at right angles to the axis of the cylinder! You will need some of the integrals given on page 1.)

- (a) Find the bound current \vec{J}_b
- (b) Find the bound surface current \vec{K}_b
- (c) Find the \vec{B} field at a point $x\hat{i}$ directly from your answers to parts a and b. Assume $x > a$.
- (d) Find \vec{H} at a point $x\hat{i}$ with $x > a$ directly from $\rho_m = -\vec{\nabla} \cdot \vec{M}$, $\sigma_m = \vec{M} \cdot \hat{n}$ and show $\vec{B} = \mu_o \vec{H} + \vec{M}$ agrees with your answer to part c.