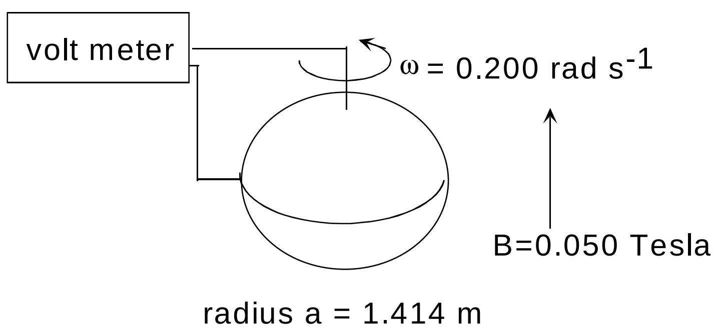
Integrals of interest

$$\frac{1}{2}\int_0^{2\pi} \frac{\cos\phi}{(1-\varepsilon\cos^2\frac{\phi}{2})^{1/2}}d\phi = \pi\, {}_2F_1(\varepsilon) - 2K(\varepsilon) \text{ (for } 0<\varepsilon<1)$$

$$\int_0^{\pi} \sin^{2n+1}\theta d\theta = \frac{2^{n+1}n!}{(2n+1)!!} = (2, \frac{4}{3}, \frac{16}{15}, \frac{32}{35}, ...) \text{ for } n = 0,1,2,3...$$

$$\int_o^{\pi} \sin^{2n}\theta d\theta = \frac{\pi(2n-1)!!}{2^n n!} = (\pi, \frac{\pi}{2}, \frac{3\pi}{8}, \frac{5\pi}{16}, ...) \text{ for } n = 0,1,2,3...$$

# Problem 1: Strange power plant.




A perfectly conducting spherical shell of radius $a$ rotates in along an axis parallel to a uniform magnetic field $\vec{B}$ at an angular frequency $\omega$. A voltmeter is hooked up to the north pole and equator (using conductive brushes.) Find an expression for the voltage read in terms of $B_o$, $\omega$ and $a$ and find the voltage for the values indicated in the figure.

# Problem 2: Not about to change space travel

We will learn in class that far way from a time-dependent neutral charge distribution, the electric and magnetic fields are often well approximated by

$$\vec{E} = \frac{\mu_o}{4\pi r}[\hat{r} \times (\hat{r} \times \frac{d^2\vec{p}}{dt^2})] \tag{1}$$

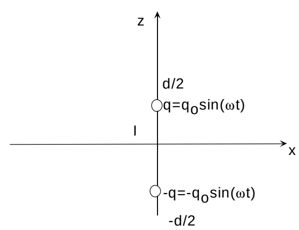
$$= \frac{\mu_o}{4\pi r}\left[\hat{r}(\hat{r} \cdot \frac{d^2\vec{p}}{dt^2}) - \frac{d^2\vec{p}}{dt^2}\right] \tag{2}$$

$$\vec{B} = \frac{-\mu_o}{4\pi r c}[\hat{r} \times \frac{d^2\vec{p}}{dt^2}] \tag{3}$$

where $\vec{p}$ is the time-dependent dipole moment of the distribution.

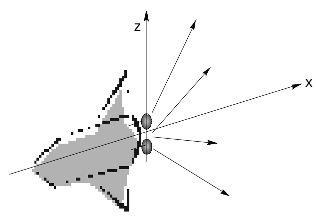
Also recall that the maxwell Stress Tensor is given by

$$T_{ij} = \varepsilon_o(E_iE_j - \frac{1}{2}\delta_{ij}E^2) + \frac{1}{\mu_o}(B_iB_j - \frac{1}{2}\delta_{ij}B^2)$$




(a) Consider a source of radiation consisting of two tiny balls located at $\pm d/2$ on the $z$ axis. The charges are hooked up to a power source so that charge on the top ball is $q_o \sin\omega t$ whereas the charge on the bottom ball is $-q_o \sin\omega t$. Find an expression for the $x, y$ and $z$ components of $\vec{E}$ and $\vec{B}$ at a time $t$ in terms of the spherical polar coordinates $\theta, \phi$,and $r$ as well as the constants $q_o$ and $\omega$.

(b) Assuming 1 and 3 correct, find an expression for $\vec{T}_x = (\overleftrightarrow{T}_{xx}, \overleftrightarrow{T}_{xy}, \overleftrightarrow{T}_{xz})$ and $\vec{T}_x \cdot \hat{r} = \vec{T}_x \cdot (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$




(c) A new rocket ship is designed to be propelled by expelling photons from a dipole antenna. Half of the radiation from the antenna is allowed to be spewed out the back side of the ship whereas the radiation emitted in the direction of the ship is absorbed as shown in the figure. USE YOUR ANSWER TO PART (b) to find the average force $< F_x >$ on the rocket ship. You may assume that the second integral in the expression

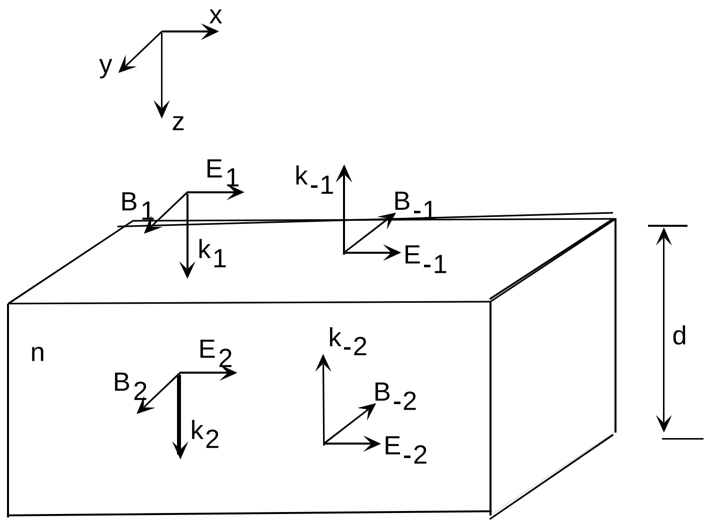
$$\frac{d\vec{P}_{mech}}{dt} = \oint_S \overleftrightarrow{T} \cdot d\vec{a} - \frac{1}{c^2}\frac{d}{dt}\int \vec{S} d\tau$$

will time average to zero.

(d) Assuming that the rock ship is traveling at a velocity $v << c$ and the power supply moving charge from the ball at $z = d/2$ to the ball at $z = -d/2$ has an internal resistance $R$, determine an expression for the efficiency of the propulsion method. (.i.e. the ratio of power $< F_x > v$ going to propell the ship to the average power $< P >$ used by the supply.

(e) Assuming $R = 1\Omega$ and given $\varepsilon_o = 8.85 \times 10^{-12} s/m/\Omega$ and assuming $\omega d \approx v \approx 1000 m s^{-1}$, how efficient is the scheme?

# Problem 3: Thin window transmission: Etalon that great.




(a) Light of vacuum wavelength $\lambda = 2\pi/k$ travels through vacuum to a flat glass surface (index of refraction $n$). The light is incident normal to the surface. We eventually will calculate the percent of light transmitted as a function of the thickness $d$ of the plate. To make sure this does not turn into an algebraic nightmare for you, please fill complete the following table in terms of $E_{\pm 1}, E_{\pm 2}$, and the indices of refraction.

| | Electric Field | Magnetic Field |
|---|---|---|
| incident #1 | $\vec{E}_{+1} = E_{+1}\widehat{x}e^{i(k_1 z - \omega t)}$ | $\vec{B}_{+1} = \frac{E_{+1}}{c}\widehat{y}e^{i(k_1 z-\omega t)}$ |
| reflected #1 | $\vec{E}_{-1} = E_{-1}\widehat{x}e^{i(-k_1 z - \omega t)}$ | |
| transmitted #2 | $\vec{E}_{+2} = E_{+2}\widehat{x}e^{i(k_2 z - \omega t)}$ | |
| reflected #2 | $\vec{E}_{-2} = E_{-2}\widehat{x}e^{i(-k_2 z - \omega t)}$ | $\vec{B}_{-2} = \frac{-nE_{-2}}{c}\widehat{y}e^{i(-k_2 z-\omega t)}$ |
| transmitted #3 | $\vec{E}_3 = E_3\widehat{x}e^{i(k_3 z - \omega t)}$ | |

(b) The static boundary value conditions

$$\begin{array}{ll} E_1^{||} = E_2^{||} & \varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp} \\ B_1^{||}/\mu_1 = B_2^{||}/\mu_2 & B_1^{\perp} = B_2^{\perp} \end{array}$$

accurately determine the ratio of reflected to transmitted light. For the study here, we may assume that $\mu_1 = \mu_2 = \mu_3$, making the two relevant boundary value conditions simple. Find $\vec{E}_3$ in terms of $\vec{E}_1$.

(c) Find the reflection coefficient $R$ = [Power reflected : Power in] as a function of $\alpha = k_2 d$ and $n$.

# Problem 4: Mutual Inductance

Find an expression for the mutual inductance $M_{ab}$ between to cocentric wire loops, one of radius $a$ and the other of radius $b$. Show explicity that $M_{ab} = M_{ba}$.