Physics 4213/5213
Lecture 19 (Part 1)

1 Introduction

Having considered the scattering of two particles that are both different and that are not particle anti-particle combinations, this lecture will examine the cases of electron-electron and electron-positron scattering. In the case of two totally dissimilar particles, the scattering process is described by a single two vertex Feynman diagram, in the cases to be considered in this lecture there will be two Feynman diagrams that have to be added together. In other words, these new processes will have amplitudes that interfere with each other.

2 Spinless Electron-Electron Scattering

The first case to be considered is the case of two identical particles scattering off each other. For the sake of taking a concrete example, the process to be examined is:

$$e^- + e^- \rightarrow e^- + e^- \tag{1}$$

Before drawing the Feynman diagram for this process, rewrite the reaction in terms of the labels A, B, C, and D:

$$A + B \rightarrow C + D \tag{2}$$

In this process it is possible to have A and C entering and leaving one vertex, but it is also possible to have A and D entering and leaving the vertex, since the electrons themselves are not distinguishable. This means that two Feynman diagrams are required to describe electron-electron scattering. Further since the two processes are indistinguishable, and the Feynman diagrams are the quantum mechanical amplitudes, the amplitudes for the two are added together. It is then the sum of the amplitudes that the magnitude squared is calculated for.

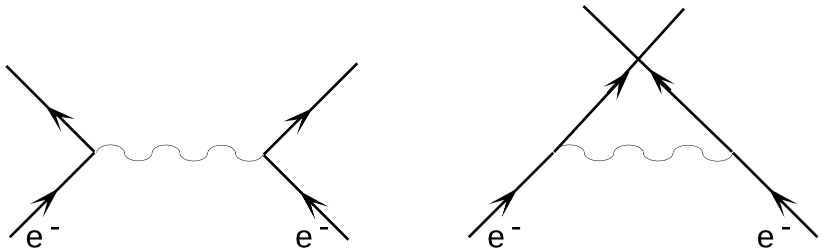
To calculate the invariant amplitude, start with the Feynman diagrams given in figure 2. The amplitude is given by:

$$-i\mathcal{M}_{e^-e^-} = -ie^2\left(-\frac{(P_A + P_C)_\mu (P_B + P_D)^\mu}{(P_D - P_B)^2} - \frac{(P_A + P_D)_\mu (P_B + P_C)^\mu}{(P_C - P_B)^2}\right) \tag{3}$$

Remember that in this version of the problem, the electrons are assumed to be spin zero identical particles. Therefore, the wave-function (amplitude) must be symmetric under particle exchange. Notice that the second term is simply the first term with particles C and D interchanged, therefore if particles A and B are interchanged, the wave-function remains the same (no change in sign).

3 Spinless Electron-Positron Scattering

Having determined the amplitude for electron-electron scattering, the amplitude for electron-positron scattering will be calculated. In this case there are also two Feynman diagrams that

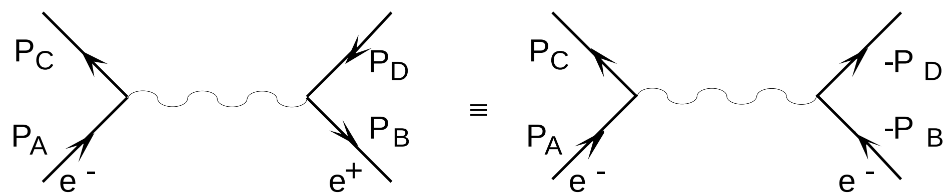


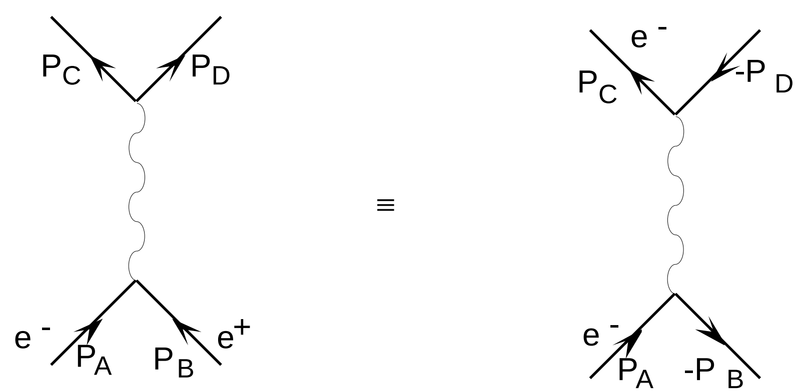
contribute to the process (see fig 3). The first diagram corresponds to the scattering of the two particles, while the second diagram represents the annihilation of the electron and positron. Recall that the Feynman rules were given for a particle with a negative charge. Therefore, the vertex factors need to be modified to accommodate the positive charges. This still leads to a problem for the annihilation diagram, since two different charges occur at a single vertex. The more interesting method, and the one that takes account of different charges at the same vertex, uses the previously realized fact that the positron is simply an electron with negative energy traveling backward in time. Therefore, in the annihilation diagram, reverse the direction of the positron line so that now a continuous flow occurs through the vertex (forward going electron to backward going electron). This also requies transforming the 4-momentum by multiplying it by a minus sign ($P \to -P$. With this transformation, the invariant amplitude is given by:

$$-i\mathcal{M}_{e^-e^+} = -ie^2\left(-\frac{(P_A+P_C)_\mu(-P_B-P_D)^\mu}{(P_D-P_B)^2} - \frac{(P_A-P_B)_\mu(-P_D+P_C)^\mu}{(P_C+P_D)^2}\right) \tag{4}$$

Finally, notice that the amplitudes for e^-e^- and e^-e^+ are related to each other. The difference being that the 4-momenta for particles B and D are of opposite sign:

$$\mathcal{M}_{e^-e^+}(P_A, P_B, P_C, P_D) = \mathcal{M}_{e^-e^-}(P_A, -P_D, P_C, -P_B) \tag{5}$$


P_C
P_A
e^-
P_D
P_B
e^+
≡
P_C
P_A
e^-
$-P_D$
$-P_B$
e^-


P_C
P_D
e^-
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e^+
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e^-
P_C
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