Physics 4213/5213
Lecture 14 (Part 2)

1 Introduction

This lecture examines what happens when one of the partial waves dominates over the others, and if the partial wave undergoes a rapid phase change of $\pi/2$ (change over a small energy interval). In this case the cross section will exhibit a peak with a well defined energy, width and angular momentum. In fact if other quantum numbers exist, these will also be well defined over this region. In other words, this will behave just like a particle. In fact this what defines a particle in a production experiment.

2 Breit-Wigner Resonance Formula

To arrive at an expression for the cross section, in the region where it changes its phase by $\pi/2$, start with the expression for elastic scattering. It will be assumed that there is no absorption η_ℓ and that in this region, only one of the partial waves contributes. In this case the elastic cross section is:

$$\sigma_{\text{el}} = \frac{4\pi}{k^2}(2\ell+1)\left|\frac{e^{2i\delta_\ell}-1}{2i}\right|^2 \tag{1}$$

Notice that when $\delta_\ell = \pi/2$ the expression above reaches its maximum value—the other terms do not have maxima in the sense that the first derivative is zero. Extracting out the term containing the phase and rewriting that expression gives:

$$\frac{e^{2i\delta_\ell}-1}{2i} = e^{i\delta_\ell}\sin\delta_\ell = \frac{1}{\cot\delta_\ell - i} \tag{2}$$

The phase itself depends on the energy, assuming a rapid change, the $\cot\delta_\ell$ can be expanded about the peak value in the cross section:

$$\cot\delta(E) = \cot\delta(E_R) + (E-E_R)\left[\frac{d}{dE}\cot\delta(E)\right]_{E=E_R} + \cdots \tag{3}$$

where E_R corresponds to the local maximum in the cross section, which means that $\delta(E_R)=\pi/2$ and therefore $\cot\delta(E_R)=0$, giving:

$$\cot\delta(E) \approx -(E-E_R)\frac{2}{\Gamma} \tag{4}$$

where:

$$-\frac{2}{\Gamma} \equiv \left[\frac{d}{dE}\cot\delta(E)\right]_{E=E_R} \tag{5}$$

and the approximation is valid for values $\Gamma \ll E_R$ This expression is then reintroduced into the phase term, yielding

$$\frac{1}{\cot \delta_\ell - i} \approx \frac{\Gamma/2}{(E_R - E) - i\Gamma/2} \tag{6}$$

Finally, the elastic cross section is given by:

$$\sigma_{\text{el}} = \frac{4\pi}{k^2}(2\ell + 1)\left|\frac{\Gamma/2}{(E_R - E) - i\Gamma/2}\right|^2 = \frac{4\pi}{k^2}(2\ell + 1)\frac{\Gamma^2/4}{(E - E_R)^2 + \Gamma^2/4} \tag{7}$$

This equation is referred to as the Breit-Wigner formula and describes the width of any resonance state produced through a scattering process. The value Γ is defined such that it is the full width at half the maximum value. It is also related to the mean lifetime τ through the equation $\Gamma = \hbar/\tau$.

The expression given in equation 7, corresponds to the scattering of two spin zero particles. The addition of the initial particles having spin, introduces one more factor into the cross section, in addition to making the transformation of ℓ the orbital angular momentum to J the total angular momentum:

$$\sigma_{\text{el}} = \frac{4\pi}{k^2}\frac{(2J + 1)\Gamma^2/4}{(2s_a + 1)(2s_b + 1)\left[(E - E_R)^2 + \Gamma^2/4\right]} \tag{8}$$