Physics 4213/5213
Lecture 13

1 Introduction

That the neutrino associated with the electron and that associated with the muon are different, required direct experimental verification. The first evidence came from the non-observation of the muon decay through the channel $\mu^- \to e^- + \gamma$—branching ratio (probability) $< 4.9 \times 10^{-11}$. This channel satisfies, energy-momentum conservation, charge conservation, plus lepton number conservation, assuming that only the total number of leptons needs to be conserved. This method is indirect, in that there maybe some other unknown reason (other than specific flavor lepton number being conserved) for the reaction not occurring.

In 1961 a direct measurement of the muon type neutrino was attempted. The goal of the experiment was to show that muon-type neutrinos, in charged current reactions, only produce muons and not electrons—note in 1961, only two leptons were known. The experiment required a beam of ν_μ, that was intense enough to allow for the possibility of some number interacting and being detected—note neutrinos have a very low probability of interacting with matter. The ν_μ are produced from pion decay $\pi^+ \to \mu^+ + \nu_\mu$—branching ratio $(99.98782 \pm 0.00014)\%$ while $BR(\pi^+ \to e^+ + \nu_e) = (1.218 \pm 0.014) \times 10^{-4}$. Finally, the pions are produced through proton nucleon interactions.

This lecture will describe the experiment, along with an introduction to cross sections and the probability of interaction plus decay probabilities.

2 Pion Production

As stated in the introduction, the pions are produced through proton nucleon interactions $p + N \to p + N + n\pi$ where n is the number of charged pions produced and must be a multiple of three—note, for every two charged pions produced a neutral pion is also produced; in these reactions to conserve charge, $\pi^+\pi^-$ pairs must be produced. To simplify the calculations, it will be assumed that both the neutron and proton have the same mass and that the mass is 1 GeV. The experiment performed used 15 GeV protons incident on a beryllium target (collection of protons and neutrons).

To determine how many pions are produced, calculate the center of mass energy

$$s = (P_1 + P_2) = 2m_p^2 + 2E_p^i m_p \qquad \Rightarrow \sqrt{s} = 5.7\,\text{GeV} \tag{1}$$

Assuming that all particles are produced at rest in the center of mass frame (equal momenta in the lab frame), this energy is required to produce the two protons plus the n pions. Since the pion mass is 140 MeV (note the neutral pion has a mass of 135 MeV) and pions are produced in triples, there can be a maximum of 9 pions produced or three of each charge species. Since this configuration is not likely (all particles produced at rest in the center of mass frame), the number of pions is more likely to be 3 or 6 meaning that there will be 1 to 2 π^+ per collision.

The next question that requires an answer is, given a beam of protons what is the probability or how many will interaction in the beryllium. Consider one incident proton from a beam of cross sectional area A, which is less than the cross sectional area of the target. If there are N nucleons scattered over the surface of an infinitesimally thin target over the cross sectional area A, then the probability of the proton interacting is $P = N\sigma/A$. This is rewritten in terms of a volume density, since the target will have a finite thickness;

$$P = \frac{N\sigma}{A\ell}\ell \tag{2}$$

where ℓ is the thickness of the target. Next convert the number density into a mass density since this is what is measurable and given in tables:

$$P = N_A \rho\sigma\ell \tag{3}$$

where N_A is the number of particles per moles—note that one mole is equal to a mass equal to the atomic number of the element in grams; in this case the atomic number is divided into the density to get the number of beryllium atoms and then multiplied through to get the number of nucleons. Finally the number of protons that interact is given by multiplying equation 3 by the number of protons incident on the target

$$N_{\rm int} = N_p N_A \rho\sigma\ell \tag{4}$$

For the case of 15 GeV protons incident on beryllium, $\sigma = 50\,\text{mb}$—$1\,\text{mb} = 1\times10^{-27}\,\text{cm}^2$. The probability for a single proton to interact in 7.6 cm thick beryllium is $P = 0.41$—the density of Be is $1.848\,\text{g/cm}^3$.

3 Production of ν_μ

As stated in the introduction, the production of ν_μ comes from the decay of the pions; $\pi^+ \to \mu + \nu_\mu$. The pion lifetime is 2.6×10^{-8} sec, which corresponds to a half-life, that is not every pion decays after this time. The probability as a function of time for any pion to decay is derived by assuming that the probability is independent of how long it has lived. In this case (assume N pions) the number of pions left after any interval dt is independent of any previous interval $-dN = \lambda N dt$—the number that are lost (decay) is proportional to the number in the time interval. Solving for $N(t)$ gives;

$$\frac{dN}{N} = -\lambda dt \qquad \Rightarrow \qquad N(t) = N_0 e^{-t/\tau} \tag{5}$$

In the experiment, a gap of 21 m was left for the pions to decay in. To determine what fraction of pions decay in this space, an assumption of their momentum has to be made. Assume that 6 pions are produced and that all particles in the final state share equal momentum. Since all the particles have to carry 15 GeV of momentum, each particle must carry 1.9 GeV under our assumption. Transforming the life-time to the lab frame from the rest frame gives

$$\tau_{\rm lab} = \tau_{\rm rest}\gamma = \tau_{\rm rest}\frac{E}{m_\pi} = 3.5\times10^{-7}\,\text{sec} \tag{6}$$

Given that for this momentum, the pion is traveling at the speed of light, the pion travels 105 m and the probability of a single pion decaying in 21 m is

$$P = 1 - e^{-x/x_0} = 1 - e^{-21/105} = .18 \tag{7}$$

Taking pions of 1 *GeV increases the probability to* 0.31.

4 Neutrino Interactions

Earlier in the semester, it was shown that the cross section of a neutrino interacting with an electron (same for a quark) is proportional to the energy of the neutrino ($\sigma_{\nu\text{-}q} \propto E_\nu$). The exact form, with the proportionality constant is $\sigma_{\nu p} = 0.67 \times 10^{-38} E\,\text{cm}^2$ where the energy is in GeV—note that this is 12 orders of magnitude smaller than the proton-proton cross section for an energy of 15 GeV.

To make an estimate of the ν energy, first determine the energy in the pion rest frame. Start by using energy conservation and realizing that the energy and momentum are equal for the neutrino;

$$m_\pi = E_\mu + E_\nu \qquad \Rightarrow \qquad m_\pi = \sqrt{E_\nu^2 + m_\mu^2} + E_\nu \qquad \Rightarrow \qquad E_\nu = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \tag{8}$$

were momentum conservation is used in the second expression. Inserting the masses into the equation above gives an energy for the ν_μ of 30 MeV. Next assume that the neutrino is emitted in the same direction as the pion was traveling, also assume that the pion momentum is 2.5 GeV. The neutrino momentum is given by;

$$\begin{pmatrix} E_{\text{lab}} \\ p_{\text{lab}} \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E_{\text{rest}} \\ p_{\text{rest}} \end{pmatrix} \qquad \gamma \approx \gamma\beta = \frac{E_\pi}{m_\pi} = 13.57 \qquad \Rightarrow \qquad E_\nu = .814\,\text{GeV} \tag{9}$$

Based on this neutrino energy $\sigma_\nu = 5 \times 10^{-39}\,\text{cm}^2$, which is 13 orders of magnitude smaller than the proton cross section.

For the experiment, a 13.5 m thick iron sheild ($\rho = 7.87\,\text{g/cm}^3$) was placed after the decay path. The probability for a neutrino to interact in this sheild was 4.4×10^{-11} while for a hadron the probability is 1. Beyond this sheild, 90 inches of aluminum ($\rho = 2.7\,\text{g/cm}^3$) was placed to provide a means for the neutrinos to interact and create muons; probability of neutrino interacting is 1.8×10^{-12}. (The proton beam intesity was $\approx 10^{16}\,\text{pulse}^{-1}$; pulse every 1.2 sec.) The muons were detected by ionization detectors placed between the aluminum plates.