Physics 4213/5213
Lecture 12

1 Introduction

This lecture takes a look at two phenomena, that are connected to the phenomena of mixing in the K^0 and $\bar{K}^0$ system. Both these phenomena involve directly observing the mixture of the two states. In the first case, the phenomena of regeneration, a beam with no K_S can be converted to a beam with K_S by introducing a target. The second case looks at strangeness oscillations, were the net strangeness of the beam varies with time.

2 Regeneration

A phenomena that occurs due to mixing, is regeneration of the K_S component of the beam long after it has decayed away. This is done by placing a block of material in the path of the remaining beam. Since the K_L beam is composed of both K^0 and $\bar{K}^0$ and the probability to interact with the material in the block is different for the two, the ratio of K^0 and $\bar{K}^0$ will be different after the beam leaves the block. The possible strong interactions for the K^0 and $\bar{K}^0$ are:

$$\begin{aligned} \bar{K}^0 + p &\rightarrow \Lambda + \pi^+ \\ &\rightarrow \Sigma^+ + \pi^0 \\ \bar{K}^0 + n &\rightarrow K^- + p \\ K^0 + p &\rightarrow K^+ + n \end{aligned} \tag{1}$$

Notice, the K^0 can only produce anti-baryons containing strange anti-quarks. This requires considerably more energy in order to conserve baryon number in the reaction, since two additional baryons need to be created.

After the K_L beam has traversed the target the wave-function is given by:

$$|\psi\rangle = \left[f|K^0\rangle + \bar{f}|\bar{K}^0\rangle\right] \tag{2}$$

where f and $\bar{f}$ are the fraction of the components that did not interact. This can be rewritten in terms of the K_S and K_L as:

$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}}\left[f\left(|K_S\rangle + |K_L\rangle\right) + \bar{f}\left(|K_L\rangle - |K_S\rangle\right)\right] \\ &= \frac{1}{\sqrt{2}}\left[\left(f - \bar{f}\right)|K_S\rangle + \left(f + \bar{f}\right)|K_L\rangle\right] \end{aligned} \tag{3}$$

Therefore, as long as the reaction rates are different the K_S can be regenerated.

3 CPT

The combined symmetry CPT is satisfied for all interactions. This being proved in Quantum Field Theory. The assumptions that go into the proof (in simplified form) are:

1. The Lagrangian must be a Lorentz scalar;
2. Integral spin particles must obey Bose statistics and half integral spin particles must obey Fermi statistics.

Two consequence of this theorem, stated without proof, are that a particle and its anti-particle must have the same mass and the same lifetime—partial decay rates do not have to be the same and can be different if CP is violated.

4 Strangeness Oscillations

The first thing to notice is that the states K_S and K_L are not anti-particles of each other. Therefore, their decay rates and their masses can be different. Secondly and more important, is that in the production experiment, because of the larger strength of the strong interaction, the particle that is produced is the K^0 or the $\bar{K}^0$. These are a mixture of the weak interaction states:

$$|K^0\rangle = \frac{1}{\sqrt{2}}\left[|K_S\rangle + |K_L\rangle\right] \qquad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}\left[|K_L\rangle - |K_S\rangle\right] \tag{4}$$

Given that the K_S and K_L have different lifetimes, after some distance (time) the will be predominately K_L.

But more interesting than this, is the question of whether the K^0 can transform itself into a $\bar{K}^0$. This can be explored by looking at the time dependence of the wave-functions. This time dependence is given by the normal phase factor from the Schrodinger equation plus a factor to account for the lifetime:

$$a_\alpha(t) = e^{-im_\alpha t}e^{-t/2\tau_\alpha} \tag{5}$$

The particles are taken in their rest frame and again the masses and lifetimes are assumed different since the K_S and K_L are not anti-particles of each other. Assuming that the initially produced particle is a K^0, the phase factors are added to the wave-function:

$$\begin{aligned}
|\psi(t)\rangle &= \frac{1}{\sqrt{2}}\left[e^{-im_1 t}e^{-t/2\tau_1}|K_S\rangle + e^{-im_2 t}e^{-t/2\tau_2}|K_L\rangle\right] \\
&= \frac{1}{2}\left[e^{-im_1 t}e^{-t/2\tau_1}\left(|K^0\rangle - |\bar{K}^0\rangle\right) + e^{-im_2 t}e^{-t/2\tau_2}\left(|K^0\rangle + |\bar{K}^0\rangle\right)\right] \\
&= \frac{1}{2}\left[\left(e^{-im_1 t}e^{-t/2\tau_1} + e^{-im_2 t}e^{-t/2\tau_2}\right)|K^0\rangle - \left(e^{-im_1 t}e^{-t/2\tau_1} - e^{-im_2 t}e^{-t/2\tau_2}\right)|\bar{K}^0\rangle\right]
\end{aligned} \tag{6}$$

Notice that at $t = 0$ the state is pure K^0, but at later times there is a mixture of the two. The probability of finding either particle is given by the following expression:

$$\begin{aligned}
|\langle K^0|\psi(t)\rangle|^2 &= \frac{1}{4}\left[e^{-t/\tau_1} + e^{-t/\tau_2} + 2e^{-(t/2)(1/\tau_1+1/\tau_2)}\cos\Delta mt\right] \\
|\langle \bar{K}^0|\psi(t)\rangle|^2 &= \frac{1}{4}\left[e^{-t/\tau_1} + e^{-t/\tau_2} - 2e^{-(t/2)(1/\tau_1+1/\tau_2)}\cos\Delta mt\right]
\end{aligned} \tag{7}$$

Figure 1 shows the time dependence of the probability of finding either the K^0 or the $\bar{K}^0$.

From equation 7, unless there is a mass difference there will be no oscillation, the two would simply decay away. Experimentally an oscillation is observed. The experiment is done by placing a target at varying distances from the point of production and looking for reactions that indicate a K^0 or a $\bar{K}^0$ interacted. The typical method of doing the experiment, is to look for evidence of the $\bar{K}^0$ through the reaction:

$$\begin{aligned} \bar{K}^0 + p &\rightarrow \Lambda + \pi^+ \\ &\rightarrow \Sigma^+ + \pi^0 \end{aligned} \tag{8}$$

By looking at the period of oscillation, the mass difference between the K_S and K_L can be determined and is found to be 3.5×10^{-6} eV.

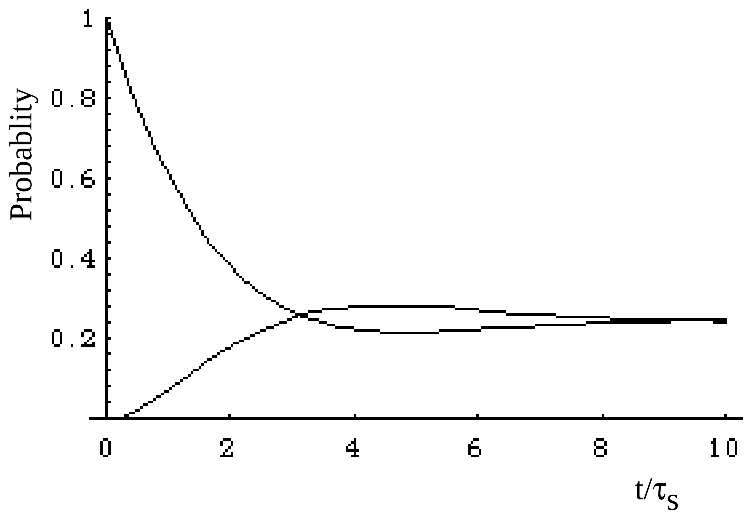

Figure 1: Plot of equations 7 for a value of $\Delta m\tau = 0.5$. This shows how the amount of K^0 and $\bar{K}^0$ vary with time.