Lecture 11 (Part 2)
Physics 4213/5213

1 Introduction

The combined symmetry of CP has been introduced since, this was found to be a good symmetry for the neutrino, while the individual symmetries C and P were found to not be good symmetries. Therefore, the assumption is made that CP is a better symmetry for weak interactions than the two individually and that it is a conserved quantity. This assumption will be used to study the K^0–$\bar{K}^0$ system.

2 The Neutral Kaon

The neutral kaons, K^0 and $\bar{K}^0$, both decay into two pions and three pions through the weak interaction. Since they decay to the same states, a valid question to ask, is how can they be distinguished. Both these particles contain a strange quark—$K^0(\bar{s}d)$ and $\bar{K}^0(\bar{d}s)$—being particle anti-particle they have opposite strangeness, implying that they should be produced in different reactions. These reactions correspond to:

$$\begin{aligned} \pi^- + p &\to \Lambda + K^0 \\ \pi^- + p &\to \bar{\Lambda} + \bar{K}^0 + n + n \end{aligned} \tag{1}$$

The two neutrons in the second reaction are needed to conserve baryon number. Because these processes involve the strong interactions, strangeness must be conserved—the strong interaction is blind to quark flavor and therefore cannot change it.

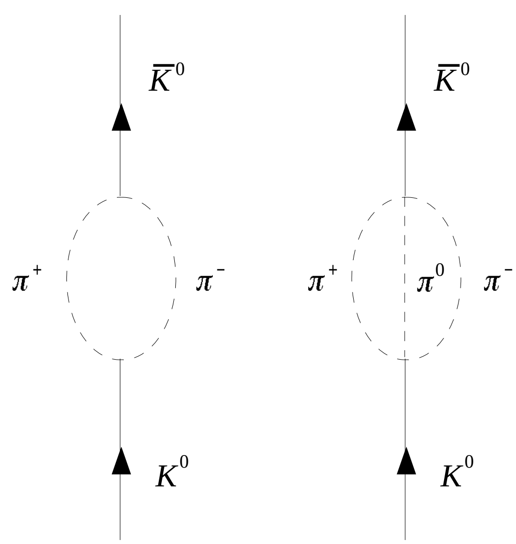

Figure 1: Transition from K^0 to $\bar{K}^0$ through virtual pions.

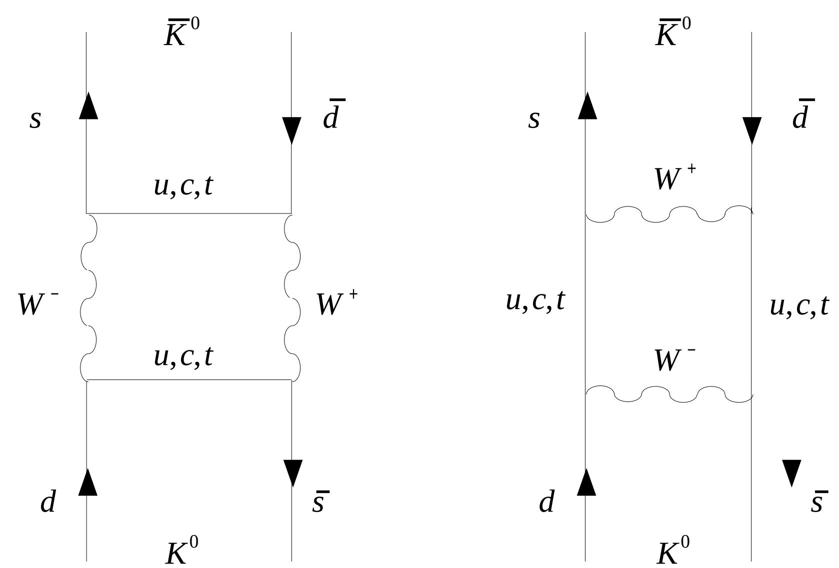

Figure 2: Kaon oscillations as view from the quark model and the theory of weak interactions.

Given that they are distinguishable and decay into the same states, they might be able to mix. That is a K^0 can transform itself into a $\bar{K}^0$ through the exchange of virtual pions (see fig 1). The mixing must come about through the weak interaction, since there is a change of strangeness ($\Delta S = 2$). Therefore, in the quark model and using the exchange of the W, the Feynman diagram that corresponds to this process is (see fig 2).

3 CP of the Kaon—The Quark Model

The parity and charge conjugation quantum numbers of the two kaons can be determined from the quark model. The kaons are the lightest neutral mesons containing a strange quark. This implies that they are spin zero mesons, with the quarks having zero relative orbital angular momentum. The parity is then given by $(-1)^{\ell+1} = -1$ for both of the kaons.

As far as charge conjugation goes, neither of the two are eigen-states. The reason for this is that the flavor of the meson (this refers to the strangeness in this case) changes under the charge conjugation operator:

$$\begin{aligned} C|K^0\rangle &= |\bar{K}^0\rangle \\ C|\bar{K}^0\rangle &= |K^0\rangle \end{aligned} \tag{2}$$

Therefore neither of the two kaons is an eigen-state of CP.

4 CP of the Pions

Start with the decays $K^0 \rightarrow \pi^0 + \pi^0$ and $\bar{K}^0 \rightarrow \pi^0 + \pi^0$; here nothing will be assumed about the spin of the kaon. Since the system is composed of identical bosons, the system must be symmetric under particle exchange. This implies that the parity of the system must be even since parity and particle exchange are equivalent operations; the intrinsic parity of each pion is -1, therefore the combined intrinsic parity of the two pions is $+1$. Since the system is even under parity, the angular momentum must be even. If the angular momentum has an even value greater than or equal to two, then the decay $K^0 \to \pi^0 + \gamma$ should occur; note this could never be in a state of zero total spin, since by taking the z-axis as the photon direction $s_z = \pm 1$. This decay has never been seen, therefore it ia assumed that the spin of the K^0 and similarly for the $\bar{K}^0$ must be zero.

The charge conjugation of the two π^0's is given by the intrinsic charge conjugation of the two pions and is therefore $+1$. This leads to $CP = +1$ for the two pion system. The parity of the two charged pions is given by $(-1)^\ell$ as is the charge conjugation (since the charge conjugation inverts the signs of the particles, this being equivalent to rotating the system by π). This leads to $CP = (-1)^{2\ell} = +1$.

Next, determine CP for the decays $K^0 \to \pi^0 + \pi^0 + \pi^0$ and $\bar{K}^0 \to \pi^0 + \pi^0 + \pi^0$. The charge conjugation is again just given by the intrinsic charge conjugation of the three pions $(+1)$. The parity is given by the intrinsic parity of the three pions $(-1)^3 = -1$ and the spatial part is derived by knowing that the K^0 has spin zero. In this case, the relative angular momentum about the common center of mass of two of π^0 is given by ℓ. The relative orbital angular momentum about the common center of mass of the $2\pi^0$ system and the other π^0 is L. Since the total angular momentum is in the range $L = \ell \to |L - \ell|$, implying $\ell = L$. Therefore the parity is given by $(-1)(+1) = -1$, which finally gives $CP = -1$. Again similar arguments can be used to determine CP for the decay modes $K^0 \to \pi^+ + \pi^- + \pi^0$ and $\bar{K}^0 \to \pi^+ + \pi^- + \pi^0$, except that here the fact that the relative orbital angular momentum between the various pions is zero must be used (this being an experimental fact).

Pion State	P	C	CP
$\pi^+\pi^-$	$(-1)^\ell$	$(-1)^\ell$	1
$\pi^0\pi^0$	1	1	1
$\pi^+\pi^-\pi^0$	-1	$(-1)^0$	-1
$\pi^0\pi^0\pi^0$	-1	1	-1

5 Eigen-states of CP

Earlier in this lecture, it was pointed out that the two kaons can mix, that is oscillate between the two. It has also been pointed out that the kaons are not states with a definite CP but do have a definite strangeness. Further, based on the CP properties of the neutrino it is assumed that even though C and P are not conserved, the combined operation CP might be conserved. Also, in weak interactions strangeness is not conserved, strangeness is a property of strong interactions. Therefore, from all this information it might be inferred that the weak interaction states are states with a definite CP while the strong interaction states are states with definite strangeness. The CP states can be written as:

$$\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\bar{K}^0\rangle\right] \Rightarrow CP = +1 \\ |K_2\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\bar{K}^0\rangle\right] \Rightarrow CP = -1 \end{aligned} \tag{3}$$

where the eigen-value of CP on the kaons is -1 since the parity is odd (-1). Given these states, the decay modes are:

$$\begin{aligned} K_1 &\rightarrow \pi^+\pi^- \qquad & K_1 &\rightarrow \pi^0\pi^0 \\ K_2 &\rightarrow \pi^+\pi^-\pi^0 \qquad & K_2 &\rightarrow \pi^0\pi^0\pi^0 \end{aligned} \tag{4}$$

Given that the K_1 decays to two pions and the K_2 decays to three pions, it might be expected that the K_1 has a shorter lifetime, it being easier to create two rather than three pions. In fact this is what is observed. The lifetime for the K_1 is 0.9×10^{-10} s, while that for the K_2 has a lifetime of 0.5×10^{-7} s; the K_1 is usually referred to as the K_S (K-short) and the K_2 as the K_L (K-long).

6 CP Violations

In 1964 Christenson, **Cronin**, **Fitch** and Turley, discovered that on occasion the K_L decays to two pion—a definite violation of CP:

$$\frac{\text{Amplitude}\,(K_L \rightarrow \pi^+\pi^-)}{\text{Amplitude}\,(K_S \rightarrow \pi^+\pi^-)} = 2 \times 10^{-3} \tag{5}$$

This violation can have two sources, one is that the K_L is really not an eigenstate of CP as is assumed and is really:

$$|K_L\rangle = \left(\frac{1}{\sqrt{1+\epsilon^2}}\right)\left[|K_2\rangle + \epsilon|K_1\rangle\right] \tag{6}$$

where ϵ is a small parameter. The second is that CP is not conserved—direct CP violation K_2 decaying to two pions. Which of the two is valid is at present not known. Experiments in this area are being conducted but these are not simple experiments. Currently the errors on the measurements are not small enough to distinguish between the two cases.

As a final comment on CP violation, a second mode is also seen. This mode is as follows:

$$\begin{aligned} K_L &\rightarrow \pi^+ + e^- + \bar{\nu}_e \\ &\rightarrow \pi^- + e^+ + \nu_e. \end{aligned} \tag{7}$$

Note that the two equations are CP conjugates of each other, which implies that the decay rates should be the same. In fact the decay rates are different and allow one to distinguish matter from anti-matter:

$$\frac{\Gamma(\pi^+ + e^- + \bar{\nu}_e) - \Gamma(\pi^- + e^+ + \nu_e)}{\Gamma(\pi^+ + e^- + \bar{\nu}_e) + \Gamma(\pi^- + e^+ + \nu_e)} = 3.3 \times 10^{-3}. \tag{8}$$

This has been stated as a necessary condition for the universe to have a matter anti-matter asymmetry.