Lecture 11 (Part 1)
Physics 4213/5213

1 Introduction

This lecture introduces the violation of parity in the weak interactions. The lecture starts by describing the τ, θ puzzle and how it lead to the hypothesis of parity violation. This is followed by the description of the Wu parity violation experiment and how the only conclusion that can be drawn is that parity is not conserved in weak interactions. Finally the neutrino is introduced and shown to not transform to anything meaningful under both parity and charge conjugation. This leads to the introduction of the combined symmetry of charge conjugation and parity CP.

2 Parity Violations in Weak Interactions

The first strong evidence for parity violation came the observation of the decay modes of two particles that were identical in every way, except that they decayed to states with different parity. The two particles were referred to as the θ and τ; at the time they were known to have identical masses, charges, lifetimes, production probabilities (cross sections) and spins.

To see that they have different parities, consider their decay modes;

$$\begin{aligned} \theta^+ &\rightarrow \pi^+ + \pi^0 \\ \tau^+ &\rightarrow \pi^+ + \pi^+ + \pi^- \end{aligned} \tag{1}$$

Starting with the τ, the parity can be determined by taking the angular momentum of the two π^+ in their center of mass frame and then add in the orbital angular momentum of the π^- relative to the center of mass of the two π^+. The relative orbital angular momentum of the two π^+ is denoted by $\vec{L}$ and that for the π^- by $\vec{\ell}$. The spin of the τ is zero (this was determined by looking at the angular distribution of the pions), the orbital angular momentum is given by $\vec{L}+\vec{\ell}=0$. The last expression implies that $L=\ell$ and that the parity is $(-1)^3(-1)^{L+\ell} = -(-1)^{2L} = -1$, that is the parity is odd and opposite that of the θ.

Given that the θ has the same lifetime (along with all other properties) as the τ, the spin is assumed to be the same as that of the τ. Since the decay is to two pions and they each have an odd intrinsic parity, the parity of the θ is $(-1)^2(-1)^0 = +1$ where the first term is the intrinsic parity of the two pions and the second is due to the zero angular momentum. Therefore the two particles have opposite parity. Assuming that the two are different particle obviously solves the problem, but it is hard to see that there are two identical particles with different parity. The other possibility was proposed by Lee and Yang, that parity is not conserved in weak interactions; the decay of these particles was known to be weak due to the lifetime of 10^{-8} s. What Yang and Lee found, was that there had never been an experiment that showed parity was conserved in weak interactions. They proposed their own, which was carried out by Wu.

2.1 The Experiment

The next step in the building of a model for weak interactions is the incorporation of parity violation; parity violation in weak decays was first proposed by T. D. Lee and C. N. Yang. Before proceeding to incorporate parity violation in the weak interaction, the experiment that proved that in fact this conjecture was correct is described; the experiment was carried out by C. S. Wu. The experiment involved the decay of ^{60}Co $\rightarrow$ ^{60}Ni. The underlying process is a neutron decaying to a proton plus electron and an anti-neutrino. The spin of the ^{60}Co was aligned using a magnetic field; note that in order to have as many ^{60}Co spins aligned as possible, the ^{60}Co was kept at a temperature of 0.01 K. The spin of ^{60}Co is five while that of ^{60}Ni is four, therefore a change of one unit of angular momentum occurs, further both have the same parity (positive).

When the angular distribution of the electron was observed, it was found to be asymmetric; the electron's momentum is more likely to be opposite the direction of the spin of the ^{60}Co. Since the spin of the atomic nucleus decreases by one unit the total angular momentum of the electron plus anti-neutrino must equal one. It was known that the relative angular momentum of the two leptons was zero, therefore the only angular momentum the leptons have is due to their spins. This then implies that spin of the electron is anti-parallel to its momentum and for the anti-neutrino it is parallel. If the parity operator is applied to this system, the momenta are inverted while the spin remains the same. This implies that if parity is conserved, the momentum of the electron should be just as likely in the direction of the ^{60}Co spin as in the opposite direction. This was clearly not seen in the experiment, therefore parity violation was shown to exist for this system.

3 The Helicity Operator

To describe the spin of a particle relative to its momentum, the helicity operator is introduced. This operator projects out the spin of a particle onto the momentum axis; it can also be looked at as the momentum being projected onto the spin axis. This operator is given by:

$$\Lambda = \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}, \qquad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \tag{2}$$

where $\vec{\sigma}$ are the Pauli matrices. This operator has eigenvalues of ±1 corresponding to states with their spins either aligned or anti-aligned to the direction of the momentum.

As an example of this operators use, consider the decay of ^{60}Co and calculate the angular distribution of the electron. The electron is emitted in a plane with momentum:

$$\vec{p} = |\vec{p}| \left(\hat{\imath}\sin\theta + \hat{k}\cos\theta \right) \tag{3}$$

where the angle θ is relative to the z-axis which is in the direction of the spin. The helicity operator can now be expanded out as:

$$\Lambda = \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|} = \begin{pmatrix} \sigma_z\cos\theta + \sigma_x\sin\theta & 0 \\ 0 & \sigma_z\cos\theta + \sigma_x\sin\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ \sin\theta & -\cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & \sin\theta & -\cos\theta \end{pmatrix} \tag{4}$$

The eigenvalues as stated earlier are ±1; this can be arrived at by diagonalizing the matrix above. The eigenvectors are determined to be:

$$\chi_\theta^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \\ \sin\frac{\theta}{2} \end{pmatrix}, \qquad \chi_\theta^- = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \\ -\sin\frac{\theta}{2} \\ \cos\frac{\theta}{2} \end{pmatrix} \tag{5}$$

Since the spin of the ^{60}Co is in the positive z direction, it defines positive helicity. The electron as stated has negative helicity, but is emitted at an angle θ, therefore the angular distribution is given by the magnitude squared of overlap of the two wave-functions:

$$w(\theta) = |\langle \chi^{+}_{\theta=0} | \chi^{-}_{\theta} \rangle|^2 = \sin^2 \frac{\theta}{2} = \frac{1}{2}(1 - \cos\theta) \tag{6}$$

The angular distribution gives a maximum at $\theta = 180^\circ$ and is zero at $\theta = 0^\circ$.

4 The Neutrino

The next puzzle in weak interactions is the neutrino. It is an approximately massless particle of spin $\frac{1}{2}$ with its spin always anti-aligned to its momentum; left handed helicity. The anti-neutrino on the other hand is right handed. This leads to a problem when the neutrino is examined under parity and charge conjugation. Under a parity transformation, the momentum is inverted but the spin remains the same giving a right handed neutrino. This state does not exist and in fact this implies that parity is not a good symmetry for describing the neutrino.

Next examine the neutrino under charge conjugation. Under this operation, the lepton number is inverted giving an anti-neutrino. But the spin and momentum haven't changed and are still anti-aligned. This state also does not exist, the anti-neutrino always has its spin aligned with its momentum. This says that charge conjugation is not a good symmetry for describing the neutrino. So the conclusion to this investigation, is that neither parity nor charge conjugation are good symmetries for describing the neutrino. Further, since the neutrino has only weak interactions, it can be deduced that the weak interactions cannot be properly described by either parity or charge conjugation—these symmetries are not conserved by weak interactions.

An interesting thing happens when C and P are combined. As stated above, a neutrino under a parity transformation gives a neutrino with momentum and spin aligned (this state does not exist). Applying charge conjugation to this state, gives an anti-neutrino with momentum and spin aligned, which is a state that does exist. Therefore, even though the separate symmetries where not valid for the neutrino, the combined symmetry is valid and at this point is assumed to be conserved.