Physics 4213/5213
Lecture 10 (Part 3)

1 Introduction

This lecture introduces the quark model in a qualitative way. It does so by expanding the idea of isospin to include particles with strangeness. This model does not introduce quarks as real particles, it simply introduces them as mathematical constructs. These states could be interpreted as particles but it is up to experiment to prove whether that interpretation is correct or not.

2 The Quark Model

To start with, consider the following three isospin multiplets:

Particle	Isospin	Spin
η	0	0
p, n	1/2	1/2
π^+, π^0, π^-	1	0
Δ^{++}, Δ^+, Δ^0, Δ^-	3/2	3/2

The fundamental representation of isospin—that value from which all others can be built—from the table above is $I = 1/2$; even though $I = 0$ is smaller it can not form the fundamental representation since the other values can not be built from it. From the $I = 1/2$ representation the η is given by:

$$\eta = |0,0\rangle = \sqrt{\frac{1}{2}}\left[\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle\right]. \tag{1}$$

Next build up the pions:

$$\begin{aligned}
\pi^+ = |1,1\rangle &= \left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle \\
\pi^0 = |1,0\rangle &= \sqrt{\frac{1}{2}}\left[\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle + \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle\right] \\
\pi^- = |1,-1\rangle &= \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle
\end{aligned} \tag{2}$$

Finally, build up the Δ's:

$$\begin{aligned}
\Delta^{++} = \left|\frac{3}{2},\frac{3}{2}\right\rangle &= \left|\frac{1}{2},\frac{1}{2}\right\rangle|1,1\rangle \\
\Delta^{+} = \left|\frac{3}{2},\frac{1}{2}\right\rangle &= \sqrt{\frac{1}{3}}|1,1\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}|1,0\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle \\
\Delta^{0} = \left|\frac{3}{2},-\frac{1}{2}\right\rangle &= \sqrt{\frac{2}{3}}|1,0\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \sqrt{\frac{1}{3}}|1,-1\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle \\
\Delta^{-} = \left|\frac{3}{2},-\frac{3}{2}\right\rangle &= \left|\frac{1}{2},-\frac{1}{2}\right\rangle|1,-1\rangle
\end{aligned} \tag{3}$$

These can be further reduced such that it is completely written in terms of $I = 1/2$ states (this will not be done to save space, but it simply amounts to using the pion states found in equation 2).

Notice that each of the states given above are written in terms of nucleons (protons and neutrons). The mesons are written in terms of two nucleons and the Δ in terms of three. This model could imply that the mesons and baryons are composite particles, obviously not composed of nucleons since the charges don't work. Also in the model given above, the proton and neutron do not exactly fit in. To expand this idea further, notice that the Δ's are composed of the following isospin states:

Particle	I^1	I^2	I^3
Δ^{++}	1/2, 1/2	1/2, 1/2	1/2, 1/2
Δ^{+}	1/2, 1/2	1/2, 1/2	1/2, -1/2
Δ^{0}	1/2, 1/2	1/2, -1/2	1/2, -1/2
Δ^{-}	1/2, -1/2	1/2, -1/2	1/2, -1/2

To get the charges to come out correctly, the isospin $I_3 = 1/2$ state must have a charge of 2/3 while the $I_3 = -1/2$ state must correspond to a charge of $-1/3$. Further, if these are to describe states from which to build the Δ they must be fermions with spin 1/2. For ease of labeling states, these are given by:

Particle	I_3	Charge	Spin
u	1/2	2/3	1/2
d	-1/2	-1/3	1/2

From these assignments, it is easy to see that the Δ's and the nucleons are composed of the following states:

Particle	Composition
Δ^{++}	uuu
Δ^{+}	uud
Δ^{0}	udd
Δ^{-}	ddd
p	uud
n	udd

where the only difference between the Δ's and the nucleons are their spin; Δ's are $S = 3/2$, while the nucleons are $S = 1/2$ states.

Does this model fit the mesons. Starting with the η it should be composed of ud which does not give the right charge. In fact, even for the pions this does not work. But, there is still the possibility of charge conjugate states. In this case the $\bar{u} = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ and the $\bar{d} = -\left|\frac{1}{2}, \frac{1}{2}\right\rangle$. In this case the η is composed of a $u\bar{u}$ and $d\bar{d}$ and the pions can be similarly constructed:

Particle	Composition
η	$u\bar{u}$, $d\bar{d}$
π^{+}	$u\bar{d}$
π^{0}	$u\bar{u}$, $d\bar{d}$
π^{-}	$d\bar{u}$

This model appears to imply that the various non-strange particles are (could be) composed of more fundamental particles. Now examine what has to be added to the model to get the strange particles. Starting with the Λ, which has an isospin of zero, it can be composed of isospin states as follows:

$$\Lambda = |0,0\rangle = \sqrt{\frac{1}{2}}\left[\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle - \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle\right]. \tag{4}$$

This is composed of only two states, which is different from the other baryons. Also, strangeness has not been taken into account and the charge and spin come out wrong. If a new state is added to this system with $I = 0$ but with a value of strangeness of -1 and a charge of $-1/3$ and a spin of $1/2$, then the Λ can be built from three states and explain where strangeness comes from. To see if this works for the Σ, recall that it is a state of $I = 1$, expand out the Σ in terms of isospin:

$$\Sigma^+ = |1,1\rangle = \left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},\frac{1}{2}\right\rangle \tag{5}$$

$$\Sigma^0 = |1,0\rangle = \sqrt{\frac{1}{2}}\left[\left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle + \left|\frac{1}{2},\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle\right] \tag{6}$$

$$\Sigma^- = |1,-1\rangle = \left|\frac{1}{2},-\frac{1}{2}\right\rangle\left|\frac{1}{2},-\frac{1}{2}\right\rangle \tag{7}$$

and again multiply each with the $I = 0$, $S = -1$ and $Q = -1/3$ state and the observed particles are retrieved. The remaining baryons are given by:

Particle	Composition	Isospin	Strangeness	Spin
n, p	udd, uud	1/2	0	1/2
Σ^-, Σ^0, Σ^+	dds, uds, uus	1	-1	1/2
Λ^0	uds	0	-1	1/2
Ξ^-, Ξ^0	dss, uss	1/2	-2	1/2
Δ^-, Δ^0, Δ^+, Δ^{++}	ddd, udd, uud, uuu	3/2	0	3/2
Σ^{*-}, Σ^{*0}, Σ^{*+}	dds, uds, uud	1	-1	3/2
Ξ^{*-}, Ξ^{*0}	dss, uss	1/2	-2	3/2
Ω^-	sss	0	-3	3/2

Finally, the strange mesons are considered. The lightest of these are the kaons. The K^+ and K^0 form an isospin doublet $I = 1/2$ as do the K^- and the $\bar{K}^0$. Each of these can be built from an isospin 1/2 state plus the state containing strangeness. These states (the lowest mass mesons) are given as:

Particle	Composition	Isospin	Strangeness	Spin
K^0, K^+	$d\bar{s}$, $u\bar{s}$	1/2	+1	0
π^-, π^0, π^+	$d\bar{u}$, $(u\bar{u} - d\bar{d})$, $u\bar{d}$	1	0	0
η	$((u\bar{u} + d\bar{d}) - 2s\bar{s})$	0	0	0
K^-, $\bar{K}^0$	$s\bar{u}$, $s\bar{d}$	1/2	-1	0

As a final comment, the relation between charge and isospin for particles with strangeness is $Q = I_3 + (A + S)/2$ where A is the baryon number and S is the strangeness.