Lecture 10 (Part 2)
Physics 4213/5213

1 Δ Decays and Production

In addition to the three pions and the nucleons (neutron and proton), other isospin multiplets exist. These are states that are identical except that they have slightly different masses and differ in their charge. One such additional state consists of the four Δ's. These are baryons with an isospin of 3/2. They are labeled as follows:

$$\begin{aligned} \Delta^{++} &= |\frac{3}{2},\frac{3}{2}\rangle \qquad & \Delta^{+} &= |\frac{3}{2},\frac{1}{2}\rangle \\ \Delta^{0} &= |\frac{3}{2},-\frac{1}{2}\rangle \qquad & \Delta^{-} &= |\frac{3}{2},-\frac{3}{2}\rangle \end{aligned} \tag{1}$$

These states have a mass of about 1230 MeV, therefore the only possible decay states are to a nucleon and a pion. This allows baryon number to be conserved and also isospin;$1/2 + 1 = 3/2$. The possible decay states are given by expanding out the states:

$$\begin{aligned} \Delta^{++} &= |1,1\rangle|\frac{1}{2},\frac{1}{2}\rangle && = \pi^+ p \\ \Delta^{+} &= \sqrt{\frac{1}{3}}|1,1\rangle|\frac{1}{2},-\frac{1}{2}\rangle + \sqrt{\frac{2}{3}}|1,0\rangle|\frac{1}{2},\frac{1}{2}\rangle && = \sqrt{\frac{1}{3}}\left[\pi^+ + n\right] + \sqrt{\frac{2}{3}}\left[\pi^0 + p\right] \\ \Delta^{0} &= \sqrt{\frac{2}{3}}|1,0\rangle|\frac{1}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}|1,-1\rangle|\frac{1}{2},\frac{1}{2}\rangle && = \sqrt{\frac{2}{3}}\left[\pi^0 + n\right] + \sqrt{\frac{1}{3}}\left[\pi^- + p\right] \\ \Delta^{-} &= |1,-1\rangle|\frac{1}{2},-\frac{1}{2}\rangle && = \pi^- n \end{aligned} \tag{2}$$

The Δ^{++} and the Δ^- have only one possible decay mode, in fact these also define possible production modes. The Δ^+ and the Δ^0 have two possible decay and production modes. For the Δ^+, the mode $\Delta^+ \to p + \pi^0$ is twice as likely as the $\Delta^+ \to n + \pi^+$. The decay of the Δ^0 is twice as likely to proceed through the decay $\Delta^0 \to n + \pi^0$ as through the mode $\Delta^0 \to p + \pi^-$.

2 Pion Nucleon Interactions

A second example of the use of isospin conservation is in pion nucleon scattering; for certain energies these are connected with the Δ. There are six possible cases, only three will be considered here. These are the three that have a pion and a proton in the initial state:

$$\begin{aligned} &\pi^+ + p \to \pi^+ + p && \text{elastic scattering} \\ &\pi^- + p \to \pi^- + p && \text{elastic scattering} \\ &\pi^- + p \to \pi^0 + n && \text{charge exchange} \end{aligned} \tag{3}$$

These processes can be written in terms of their isospin states as follows:

$$\begin{aligned}
&\pi^+ + p \rightarrow \pi^+ + p \qquad && \psi_i = \psi_f = |1,1;\frac{1}{2},\frac{1}{2}\rangle = |\frac{3}{2},\frac{3}{2}\rangle \\
&\pi^- + p \rightarrow \pi^- + p \qquad && \psi_i = \psi_f = |1,-1;\frac{1}{2},\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle \\
&\pi^- + p \rightarrow \pi^0 + n \qquad && \psi_i = |1,-1;\frac{1}{2},\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle \\
& && \psi_f = |1,0;\frac{1}{2},-\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|\frac{3}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}|\frac{1}{2},-\frac{1}{2}\rangle
\end{aligned} \tag{4}$$

To determine the ratio of the various cross sections, the transition probability from initial to final state, has to be determined. These transition probabilities, since they are strong interactions that conserve isospin, connect only those states that have the same value of isospin. These transition probabilities are given by:

$$M_1 = \langle\psi_f\left(\frac{1}{2}\right)|H_1|\psi_i\left(\frac{1}{2}\right)\rangle \qquad M_3 = \langle\psi_f\left(\frac{3}{2}\right)|H_3|\psi_i\left(\frac{3}{2}\right)\rangle \tag{5}$$

The cross section for each of these processes is proportional to the square of the magnitude for the transition from a given initial state to a given final state:

$$\sigma = K|\langle\psi_f|H|\psi_i\rangle|^2 \tag{6}$$

where K is a constant of proportionality.

Since the first reaction is a pure $I = 3/2$ transition, the cross section is given by:

$$\sigma_a = KM_3^2 \tag{7}$$

Notice when the decay of the Δ was discussed, the transitions involved like states so that the exact strength of the transition canceled in the ratio. Now look at the second process, it involves initial and final states that are identical, even though they are not pure isospin states. This cross section is given by:

$$\sigma_b = K|\frac{1}{3}M_3 + \frac{2}{3}M_1|^2 \tag{8}$$

Finally, the third cross section is given by:

$$\sigma_c = K|\frac{\sqrt{2}}{3}M_3 - \frac{\sqrt{2}}{3}M_1|^2 \tag{9}$$

The ratio of these cross sections is not particularly illuminating, since they involve quantities that can not be directly calculated. But it should be pointed out that in specific cases where experiment can provide measurements and they correspond to almost pure isospin states, these can be used to verify isospin invariance. One such corresponds to pion proton scattering where the energy is sufficient to produce a Δ—notice that π^+p has an isospin component of $I = 3/2$ as does π^-p. This component leads to the production of the Δ assuming that sufficient energy is available.

The typical pion proton production experiment involves protons at rest with an incident pion beam. To determine the energy of the incident pion use the definition of the rest (invariant) mass of an object:

$$M^2 = \left(\sum_i P_i\right)^2 \tag{10}$$

where the sum is over all particles that are involved in the interaction that creates the particle. For pion proton scattering to just create the Δ, the incident pion energy is given by:

$$M^2 = (E_\pi + m_p)^2 - p_\pi^2 = (E_\pi + m_p)^2 - \left(\sqrt{E_\pi - m_\pi}\right)^2 \Rightarrow E = 327\,\text{MeV} \tag{11}$$

At the pion energy given above, the $I = 3/2$ amplitude dominates; $M_3 >> M_1$. Therefore it is expected that the cross sections are given by:

$$\sigma_a = KM_3^2, \qquad \sigma_b = \frac{1}{9}KM_3^2, \qquad \sigma_c = \frac{2}{9}KM_3^2 \tag{12}$$

Notice that the cross sections for b and c both correspond to $\pi^- + p \rightarrow \Delta^0$ but with different possible decay modes.

3 Strangeness and Isospin

As a last example of isospin, systems with strangeness will be considered. These systems involve the use of the strong interaction to create the particles and electromagnetic and weak interactions for their decay. In strong interactions isospin is conserved, while in electromagnetic only the third component of isospin is conserved and in weak decays isospin is not conserved. Start with the production of a pair of strange particles $\pi^- + p \rightarrow \Sigma^0 + K^0$:

$$\begin{aligned} \pi^- p = |1,-1\rangle|\frac{1}{2},\frac{1}{2}\rangle &= \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle \\ \Sigma^0 K^0 = |1,0\rangle|\frac{1}{2},-\frac{1}{2}\rangle &= \sqrt{\frac{2}{3}}|\frac{3}{2},-\frac{1}{2}\rangle + \sqrt{\frac{1}{3}}|\frac{1}{2},-\frac{1}{2}\rangle \end{aligned} \tag{13}$$

As an example of an electromagnetic decay of a strange baryon, take the case of $\Sigma^0 \rightarrow \Lambda + \gamma$— notice that mass differnece between the Σ^0 and the Λ is $\approx 77\,\text{MeV}$ less than the mass of the π. The Λ has an isospin of zero as does the photon, therefore the decay conserves the third component of isospin but not isospin:

$$\Sigma^0 = |1,0\rangle, \qquad \Lambda + \gamma = |0,0\rangle \Rightarrow \begin{cases} I^{\Sigma^0} \neq I^{\Lambda\gamma} \\ I_3^{\Sigma^0} = I_3^{\Lambda\gamma} \end{cases} \tag{14}$$

Next look at the decay of the Λ; $\Lambda \rightarrow p + \pi^-$:

$$\Lambda = |0,0\rangle, p\pi^- = |1,-1\rangle|\frac{1}{2},\frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|\frac{3}{2},-\frac{1}{2}\rangle - \sqrt{\frac{2}{3}}|\frac{1}{2},-\frac{1}{2}\rangle \Rightarrow \begin{cases} I^{\Lambda} \neq I^{p\pi^-} \\ I_3^{\Lambda} \neq I_3^{p\pi^-} \end{cases} \tag{15}$$