Lecture 10 (Part 1)
Physics 4213/5213

1 Introduction

When the neutron was discovered Heisenberg noticed that its mass was almost identical to that of the proton. In a quest to gain a deeper understanding of this situation, he proposed that the proton and neutron are in fact the same particle and that their mass difference is due to the electromagnetic interaction. Therefore he proposed a new symmetry, that is only obeyed by the strong interaction. This symmetry like, parity and charge conjugation is an intrinsic symmetry of the particles that can not be altered without transforming the particle to a new particle. This lecture will examine this symmetry and its consequences.

2 Electromagnetic Splitting

The neutron and the proton are seen to have a mass difference of 1.3 MeV or given as a percentage, a difference of 0.14%; all other properties appear identical. The two charge pions have the same mass while the neutral pion is about 5 MeV lighter, again all other properties appear to be identical. Based on these statements it might be assumed that the mass difference can be attributed to the electromagnetic interaction and that if it were switched off the particles in the two systems might have the same mass.

In essence what is being said, is that the proton and neutron are two states of the same particle and the three pions are three different states of the same particle. One way of looking at this is to take a positively charged particle of spin 1/2 and place it in a magnetic field. If the spin is aligned with the magnetic field the energy is given by $-\mu B$ where μ is the magnetic dipole moment of the particle. The energy of the particle is μB if the spin is anti-aligned with the field. Each of the two states have a different energies, that is the degeneracy of the system is broken by the magnetic field.

Likewise, the neutron and proton are identical particles as long as there is no electromagnetic interaction. Once the electromagnetic interaction is switched on the energy of the two states is different and the mass degeneracy disappears. Of course it should be stressed, that unlike the spin 1/2 particle in the external magnetic field, the electromagnetic interaction can not be switched off.

3 Isospin

In analogy with the magnetic field example, a spin (isospin) is introduced for the nucleon. This spin is defined in an abstract isospin space—this is not a space-time property of the nucleon, but an intrinsic property of the nucleon. One problem that might be noticed with this is that the proton is lighter than the neutron. This would not be expected since the electric field of the proton adds to the inertia of the proton, therefore its mass would be expected to be larger than that of the proton. This difficulty is over looked, with the assumption that it can later be explained.

Isospin can be treated like regular spin (angular momentum) in how it is added etc. In the case of the nucleon, since there are two states, the isospin assigned is $I = 1/2$ with the proton having

$I_3 = 1/2$ and the neutron having $I_3 = -1/2$. (Remember that the degeneracy for a given spin is $2S+1$.) The pion on the other hand has three possible states, these are assigned to $I = 1$ with the I_3 assignments given by $(+1, 0, -1)$ for (π^+, π^0, π^-).

With the I_3 assignments given above, the electric charge can be related to I_3 through the relation $Q = I_3 + (1/2)A$ where A is the baryon number. For the time being only particles with no strangeness, charm, bottom or top will be looked at. Based on this, electromagnetic interactions are expected to conserve I_3 but not isospinsince it is what electromagnetism that breaks the mass symmetry.

4 Consequences of Isospin—Two Nucleon State

The deuteron is know to be a bound state of a proton and a neutron with a spin of one and a relative orbital angular momentum of zero between neutron and proton. Since these are now identical particles under the strong interaction, the total wave-function must be anti-symmetric (the proton and neutron are fermions). The wave-function is given by:

$$\psi(\text{total}) = \phi(\text{space})\alpha(\text{spin})\chi(isospin) \tag{1}$$

From the statements given above, the space part is even under particle exchange, $(-1)^{\ell=0} = 1$, under spin the wave function is also symmetric (both spins aligned). This implies that the isospin part must be anti-symmetric and therefore the isospin is zero—the symmetric states of the isospin wave-function are:

$$\begin{aligned} \chi(1,1) &= |\frac{1}{2},\frac{1}{2};\frac{1}{2},\frac{1}{2}\rangle \\ \chi(1,0) &= \frac{1}{\sqrt{2}}\left[|\frac{1}{2},\frac{1}{2};\frac{1}{2},-\frac{1}{2}\rangle + |\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2}\rangle\right] \\ \chi(1,-1) &= |\frac{1}{2},-\frac{1}{2};\frac{1}{2},-\frac{1}{2}\rangle \end{aligned} \tag{2}$$

while the anti-symmetric state is:

$$\chi(0,0) = \frac{1}{\sqrt{2}}\left[|\frac{1}{2},\frac{1}{2};\frac{1}{2},-\frac{1}{2}\rangle - |\frac{1}{2},-\frac{1}{2};\frac{1}{2},\frac{1}{2}\rangle\right] \tag{3}$$

Based on the isospin assignment of the deuteron, the following reactions are examined:

$$p + p \to d + \pi^+ \qquad p + n \to d + \pi^0 \tag{4}$$

In the first reaction the protons are in an isospin state of 1; since $I_3 = 1$ the only possible state of isospin is one:

$$|pp\rangle = |\frac{1}{2},\frac{1}{2}\rangle|\frac{1}{2},\frac{1}{2}\rangle = |1,1\rangle. \tag{5}$$

The left hand side of the first reaction also has $I_3 = 1$, so again the only possible isospin state that the deuteron and pion can be in is $I = 1$:

$$|d\pi^+\rangle = |0,0\rangle|1,1\rangle = |1,1\rangle. \tag{6}$$

For the second reaction, $I_3 = 0$ on both sides, with the possible states of isospin being $I = 1$ and $I = 0$. This implies the isospin wave-function for the proton and neutron is composed of equal amounts of $I = 1$ and $I = 0$—this can also be arrived at by looking at the Clebsch-Gordon coefficients:

$$|pn\rangle = |\frac{1}{2}, \frac{1}{2}\rangle|\frac{1}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{2}}\left[|1, 0\rangle + |0, 0\rangle\right] \tag{7}$$

The left hand side has isospin $I = 1$ as in the first reaction:

$$|d\pi^0\rangle = |0, 0\rangle|1, 0\rangle = |1, 0\rangle. \tag{8}$$

Since this is a strong reaction, only the isospin one term on the right hand side can contribute. Given this information, it is deduced that reaction two is half as likely as reaction one.