# Physics 4213/5213
# Lecture 9 (Part 2)

## 1 Introduction

So far two discrete symmetries have been discussed, parity and charge conjugation. Parity refers to a rotation of the space coordinates by 180°. Further, every particle in its rest frame is also an eigenstate of the parity operator—the photon is also an eigenstate of the parity operator even though it has no rest frame. The charge conjugation operator converts a particle into its anti-particle. Because of this, only particle states that have zero internal quantum numbers—particles that are their own anti-particles—are eigenstates of the charge conjugation operator. Both these symmetries are conserved by the strong and electromagnetic interaction. This lecture explores a new discrete symmetry. This is time reversal, where the direction of time is inverted.

## 2 Time Reversal

As in the case of parity and charge conjugation, the action of the time reversal operator on the space-time coordinates is examined first. Since this operator reverses the direction of time, it has no effect on the space coordinates. On the time coordinate, it reverses the sign:

$$\left.\begin{array}{r}\vec{x} \xrightarrow{T} \vec{x} \\ t \xrightarrow{T} -t\end{array}\right\} \Rightarrow x^{\mu} \xrightarrow{T} -x_{\mu} \tag{1}$$

On the energy and momentum the effect is the opposite of the coordinates, the energy remains constant and the momentum reverses sign:

$$\left.\begin{array}{r}\vec{p} \xrightarrow{T} -\vec{p} \\ E \xrightarrow{T} E\end{array}\right\} \Rightarrow p^{\mu} \xrightarrow{T} p_{\mu} \tag{2}$$

Finally, the affect of the time reversal operator on angular momentum—this includes spin—is to reverse the sign. This comes about from the definition of angular momentum as a displacement vector multiplied by a momentum ($\vec{L} = \vec{r} \times \vec{p}$):

$$\vec{J} \xrightarrow{T} -\vec{J} \tag{3}$$

The time reversal operator has no effect on mass, charge, baryon number or lepton number. Its only effect is on the time coordinate. In fact, as opposed to parity where every particle is a eigenstate, and charge conjugation, where every neutral particle—this includes no baryon or lepton number—is an eigenstate, no particle or combination of particles is an eigenstate of the time reversal operator. This at first may indicate that its usefulness is limited, but in fact it provides constraints on possible theories. One consequence can be seen by applying the time reversal operator on a reaction and assuming that the interaction is invariant to this symmetry:

$$\begin{aligned} & a(p_a, s_a) + b(p_b, s_b) \to c(p_c, s_c) + d(p_d, s_d) \\ & \xrightarrow{T} c(-p_c, -s_c) + d(-p_d, -s_d) \to a(-p_a, -s_a) + b(-p_b, -s_b) \end{aligned} \tag{4}$$

Notice the change in sign of both the momentum and angular momentum. The change in sign of the momentum is of no consequence, since the measurement can always be looked at in the center of mass frame. Further if parity is conserved and applied to the reaction, the sign of the momentum goes back to its original value (flip the coordinates around):

$$\begin{aligned} & c(-p_c,-s_c)+d(-p_d,-s_d)\rightarrow a(-p_a,-s_a)+b(-p_b,-s_b) \\ \xrightarrow{P}\; & c(p_c,-s_c)+d(p_d,-s_d)\rightarrow a(p_a,-s_a)+b(p_b,-s_b) \end{aligned} \tag{5}$$

The spin (angular momenta) does not change. This can be taken care of since, in the typical reaction (experiment) the spin of the incident particles are random, therefore they are averaged over. In this case one can write:

$$a(p_a)+b(p_b)\leftrightarrow c(p_c)+d(p_d)\Rightarrow\langle|M_{fi}|^2\rangle=\langle|M_{if}|^2\rangle \tag{6}$$

where

$$\langle|M_{fi}|^2\rangle=\frac{1}{(2S_{i_1}+1)(2S_{i_2}+1)(2S_{f_1}+1)(2S_{f_2}+1)}\sum_{S_i}\sum_{S_f}|M_{fi}|^2 \tag{7}$$

and $M_{fi}$ is the amplitude of going from an initial state $i$ to a final state $f$. This leads to the conclusion that a reaction and its inverse are equally likely. This is referred to as the principle of detailed balance. If the charge conjugation operator is applied to the reaction, then also the anti-particle version of the reaction will work.

In the strong and electromagnetic interactions no evidence exists that time reversal invariance is violated. In the weak interaction, it has been shown to be violated experimentally.

# 3 The Spin of the $\pi^{\pm}$

The principle of detailed balance can be used to determine the spin of the $\pi^{\pm}$ through the reactions:

$$\begin{aligned} & p+p\rightarrow\pi^{+}+d \\ & d+\pi^{+}\rightarrow p+p \end{aligned} \tag{8}$$

The probability (differential cross section) for the reaction to occur is given by:

$$\frac{d\sigma}{d\cos\theta}=\text{constants}\times\text{phase space}\times|M_{fi}|^2 \tag{9}$$

where $M_{fi}$ corresponds to the transition amplitude $\langle\psi_f|H|\psi_i\rangle$ and the phase space gives density of final states. Using the principle of detailed balance $\langle|M_{if}|^2\rangle=\langle|M_{fi}|^2\rangle$.

If the spins in the initial and final states are not measured, then the amplitudes above are averaged over the spins of the initial particles and summed over the spins of the final particles. Since detailed balance requires that the average be taken over both initial and final states the amplitude squared (matrix element) takes on the form

$$|M_{fi}|^2\rightarrow(2S_{f_1}+1)(2S_{f_2}+1)\langle|M_{fi}|^2\rangle \tag{10}$$

The factors out in front of the amplitude are given by:

$$\text{phase space} \times \text{constants} = \frac{p_f^2}{2\pi v_i v_f}. \tag{11}$$

The differential cross sections for the reaction and its time reverse can then be written as:

$$\frac{d\sigma(p+p \to \pi^+ + d)}{d\cos\theta} = \frac{(2S_{\pi^+}+1)(2S_d+1)}{2\pi}\frac{p_\pi^2}{v_i v_f}\langle|M_{if}|^2\rangle \tag{12}$$

and as

$$\frac{d\sigma(\pi^+ + d \to p+p)}{d\cos\theta} = \frac{(2S_p+1)(2S_p+1)}{2\pi}\frac{p_p^2}{v_i v_f}\langle|M_{fi}|^2\rangle. \tag{13}$$

With the known values of the proton spin (1/2) and the deuteron (1), the ratio of the two differential cross sections is:

$$\frac{d\sigma(p+p \to \pi^+ + d)/d\cos\theta}{d\sigma(\pi^+ + d \to p+p)/d\cos\theta} = 3\left(\frac{p_\pi}{p_p}\right)^2(2S_\pi+1). \tag{14}$$

The experimental value of $2S_\pi + 1 = 1.0 \pm 0.01$.