Physics 4213/5213
Lecture 9 (Part 1)

1 Introduction

In the previous two lectures, the parity operator was introduced along with its application to particle physics. Its importance being that it is conserved in both strong and electromagnetic interactions. This in fact places limits on the mathematical form of both these forces; it should be pointed out that weak interactions do not conserve parity and thereby limiting the mathematical form this force can have.

In this lecture charge conjugation is introduced. This operator transforms particles into anti-particles.

2 Charge Conjugation

A second discrete symmetry that is seen in particle physics is charge conjugation. The charge conjugation operator transforms particles into anti-particles—notice that only states with zero net internal quantum number are eigenstates of this operator. This operator does not affect the space-time properties of the state, such as velocity, angular momentum, etc. or the mass of the particle. Further this symmetry is only conserved in strong and electromagnetic interactions, as is parity. (For those systems—interactions—that conserve charge conjugation, the mass and lifetime or reaction rates must be the same for the charge conjugate state.)

The eigenvalues of charge conjugation are arrived at in the same manner as are those for parity. Apply the operator twice and the original state should return. This implies that the eigenvalues are ± 1. The rest of this lecture will be devoted to determining how to arrive at the eigenvalues for simple systems.

Another property of charge conjugation conservation is that, any reaction that is allowed is also allowed if particles and anti-particles are interchanged. Further it imposes certain restrictions on the kinematics of the particles. Consider the following reaction and its charge conjugate:

$$p + \bar{p} \rightarrow \pi^+ + \pi^- + \pi^0 \tag{1}$$

$$\stackrel{C}{\Longrightarrow} \bar{p} + p \rightarrow \pi^- + \pi^+ + \pi^0 \tag{2}$$

since the charge conjugation operator does not change the space-time properties of the particles, the π^+ and π^- must have the same energy and momentum distribution. This has been measured experimentally and gives a limit of < 0.01 on a C violating amplitude for strong interactions.

3 Charge Conjugation of the Photon

The charge conjugation of the photon can be deduced from the properties of the electromagnetic field by inverting the charges—application of the charge conjugation operator. (C applied to a charge inverts its sign.) Applying this operator to a current, changes its sign since it depends on the charge. Therefore, the field written in terms of the potentials gives the following:

$$\Phi \approx \frac{q}{r} \xrightarrow{C} -\frac{q}{r} \qquad \vec{A} \approx \frac{\vec{J}}{r} \xrightarrow{C} -\frac{\vec{J}}{r} \tag{3}$$

This is similar to parity, except that in this case the scalar part of the potential also changes sign. In four vector notation this is gives $A^\mu \xrightarrow{C} -A^\mu$—this should be compared to what happens under a parity transformation. From this result the charge conjugation of the photon is taken as negative, therefore the parity and charge conjugation have the same value -1. Since the strong interaction is similar in form to the electromagnetic, the gluon has a -1 charge conjugation.

4 Charge Conjugation of a Neutral System of Bosons

Start by considering the decay of the π^0 and the η. Both these particles are neutral and decay electromagnetically to two photons. Since charge conjugation is conserved in electromagnetic interaction and the charge conjugation of each photon is -1, the charge conjugation of the π^0 and the η are therefore $+1$. Another set of simple systems are the ρ^0 and the ω, these decay to three photons, therefore these mesons have a charge conjugation of -1.

At this point, this does not appear to be a particularly useful symmetry. All that charge conjugation tell us, is how many photons a neutral particle can decay to. Further, only neutral particles are eigenstates of charge conjugation—remember by neutral here we mean no net charge or no net baryon number.

What if a system is composed of two charged particles with zero net charge, what would its charge conjugation be? Since this system is neutral it should be an eigenstate of charge parity. The system can be made even simpler by requiring the particles have zero spin. As another example, consider two charged pions in a state of relative orbital angular momentum $|\pi^+\pi^-; L\rangle$. Applying the charge conjugation operator on this system, does not affect the angular momentum or any other space-time property but it does change the relative positions of the particles. The π^+ is now where the π^- was and the π^- where the π^+ was. This is equivalent to the operation of the parity operator, therefore the eigenvalues of the charge conjugation operator are:

$$C|\pi^+\pi^-; L\rangle = (-1)^\ell |\pi^-\pi^+; L\rangle \tag{4}$$

An example is the $\rho^0 \to \pi^+ + \pi^-$, according to our previous conclusion it has a charge conjugation of -1. For this to hold in the $\rho^0 \to \pi^+ + \pi^-$ decay, the two pions must be in a state of relative orbital angular momentum of at least 1—any odd value will work. If this is the case the ρ^0 must have an angular momentum of 1. This is in fact what has been measured.

What about a state of two neutral pions? Since these are identical spin zero particles, they have to be in a symmetric state. That is they are only allowed to be in states of even angular momentum. Therefore, the charge conjugation will always be equal to $+1$, meaning that the ρ^0 cannot decay through this mode.

5 Charge Conjugation of a Neutral System of Fermions

Finally, the charge conjugation of a fermion and anti-fermion system are considered. This gives the allowed values of charge conjugation for neutral mesons in the quark model; $|f\bar{f}; S, L\rangle$. Applying the charge conjugation operator again inverts the relative positions of the particles, therefore a term of $(-1)^\ell$ is needed. The spin is affected as follows, a symmetric state remains the same; $S = 1$

$$|1,1\rangle = |1/2,1/2\rangle_1|1/2,1/2\rangle \tag{5}$$

$$|1,0\rangle = \frac{1}{\sqrt{2}}[|1/2,1/2\rangle_1|1/2,-1/2\rangle + |1/2,-1/2\rangle_1|1/2,1/2\rangle] \tag{6}$$

$$|1,-1\rangle = |1/2,-1/2\rangle_1|1/2,-1/2\rangle \tag{7}$$

An anti-symmetric ($S = 0$) state is inverted,

$$|0,0\rangle = \frac{1}{\sqrt{2}}[|1/2,1/2\rangle_1|1/2,-1/2\rangle - |1/2,-1/2\rangle_1|1/2,1/2\rangle] \tag{8}$$

therefore introducing a -1. The charge conjugation therefore introduces a factor of $(-1)^{S+1}$. An additional -1 is introduced from field theory—this term is due to the anti-commuting property of the fermion wave functions. Putting everything together gives:

$$C|f\bar{f};S,L\rangle = (-1)^{\ell+S}|f\bar{f};S,L\rangle \tag{9}$$

Given that mesons are composed of quarks and anti-quarks the charge conjugation of the meson can easily be calculated once the spin and relative angular momentum of the quarks is known. For the mesons discussed in this lecture the parity and charge conjugation are given in the table below. Notice that these agree with the previous assignments.

Particle	Spin	L	P	C
π^0	0	0	-1	+1
η	0	0	-1	+1
ρ^0	1	0	-1	-1
ω	1	0	-1	-1