# Physics 4213/5213
# Lecture 8 (Part 2)

## 1 Parity

This lecture introduces one of several discreet symmetries that are used in particle physics. This symmetry is parity, which refers to the operation of transforming the spatial coordinates by 180°. For the space-time coordinates this leads to:

$$\vec{x} \xrightarrow{P} -\vec{x} \quad t \to t \Longrightarrow x^\mu \xrightarrow{P} x_\mu. \tag{1}$$

This in fact defines a vector, it is an object that transforms under the Lorentz transformation like the coordinate and under parity as given in equation 1. Given that energy and momentum form a 4-vector under Lorentz transformations, these would be expected to have the same transformation properties as the coordinates. This can be easily seen since the 4-momentum is given by $p^\mu = mu^\mu$, where the 4-velocity is given in terms of the derivatives of the space-time coordinates:

$$p^\mu = mu^\mu = m\frac{dx^\mu}{d\tau} \Rightarrow p^\mu \xrightarrow{P} p_\mu \tag{2}$$

On the other hand, the angular momentum under a parity transformation does not change direction. The reason for this is that the angular momentum is the cross product of the coordinates and the momentum $\vec{L} = \vec{r} \times \vec{p}$ and each of these change sign under a parity transformation. Therefore, the transformation of the angular momentum is:

$$\vec{L} \xrightarrow{P} \vec{L}. \tag{3}$$

This quantity is referred to as a pseudo-vector, in that in three dimensions it transforms as a vector under rotations and translation, but not under parity—note also that the angular momentum does not form a 4-vector, it is part of a more complicated object.

Notice that objects like mass and charge, which are Lorentz scalars (objects that have the same value in all Lorentz frames), do not change sign under a parity transformation. There are also objects that do transform like Lorentz scalars but change sign under a parity transformation, these are called pseudo-scalars–these are important in defining the wave-function of a certain class of particles.

### 1.1 Parity of States with Definite Angular Momentum States

In this section, only particles of spin zero will be considered and only those that have a relative angular momenta about some axis. The wave function for these systems are given by solving the Schrodinger equation in radial coordinates—see any book on quantum mechanics that solves the hydrogen atom for the explicit form of the wave-function. To simplify things and concentrate only on the angular portion of the wave-function, consider systems composed of spin zero particles. These particles are described by a scalar wave-function (no spinors, vectors etc.), where the wave-function is composed of a function that depends only the radial coordinate and one that depends only on the angular coordinates (momentum):

$$\Phi(\vec{r}) = \psi(\mathbf{r})Y_\ell^m(\theta, \phi), \tag{4}$$

where

$$Y_\ell^m(\theta,\phi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}} P_\ell^m(\theta) e^{im\phi}. \tag{5}$$

The parity of the wave-function is then just the parity of the angular momentum piece, since the radial wave-function has even parity, which is given by:

$$\theta \xrightarrow{P} \pi-\theta, \quad \phi \xrightarrow{P} \phi+\pi, \qquad Y_\ell^m(\theta,\phi) \xrightarrow{P} (-1)^\ell Y_\ell^m(\theta,\phi). \tag{6}$$

The fact that the parity depends on whether the angular momentum is even or odd, can be seen from the first few angular momentum states:

| $P_m^\ell(\theta)$ | $\ell$ | $m$ |
|---|---|---|
| $1/\sqrt{4\pi}$ | 0 | 0 |
| $-\sqrt{3/8\pi}\sin\theta$ | 1 | 1 |
| $\sqrt{3/4\pi}\cos\theta$ | 1 | 0 |
| $\sqrt{15/32\pi}\sin^2\theta$ | 2 | 2 |
| $-\sqrt{15/8\pi}\sin\theta\cos\theta$ | 2 | 1 |
| $\sqrt{5/16\pi}(3cos^2\theta-1)$ | 2 | 0 |

## 1.2 Parity of States with Definite Momentum

States of definite momentum in one dimension are given by:

$$\psi(x) = e^{ip_x x}. \tag{7}$$

If the parity operator is applied on this state, then the wave-function changes:

$$Pe^{ip_x x} = e^{-ip_x x} \tag{8}$$

implying that states of definite momentum, are not eigenfunctions of parity. Therefore, when the parity of a system is given, it is assumed to be in its rest frame.

## 1.3 Wave Functions for Particles

All elementary particles are described by wave-functions that describe the spin and parity of the particle. These are given by:

| Type of wave-function | Spin | Parity |
|---|---|---|
| Scalar | 0 | +1 |
| Pseudo-Scalar | 0 | -1 |
| Spinor (Particle) | 1/2 | +1 |
| Spinor (Anti-Particle) | 1/2 | -1 |
| Vector | 1 | -1 |
| Pseudo-Vector | 1 | +1 |

## 2 Parity of Hadrons

As an example of determining the parity of a system of particles, consider the bound state of particles and anti-particles. In particular, consider the composition of mesons as quark anti-quark states. First of all, the fermion anti-fermion pair have a parity given by $P_{q\bar{q}} = -1 \times +1 = -1$. next the angular momentum needs to be taken into account; $P_\ell = (-1)^\ell$. The parity of any meson is therefore given by $P_m = -1(-1)^\ell = (-1)^{\ell+1}$.

As a more complicated example, consider baryons. The lowest lying states, are those that have spin 1/2 with no relative orbital angular momentum. These states have a parity of $P_{qqq}(1/2) = +1$. These include particles such as the proton, neutron and the lambda. The next highest mass states are the spin 3/2 baryons. These have the same parity as the previous case, since these also have zero net relative angular momentum; $P_{qqq}(3/2) = +1$; this includes particles like the $\Delta$'s. Beyond these two simple cases, the parity requires some work to calculate.

## 3 Parity of a Single Photon

The photon is described by a vector potential and a scalar potential or most easily in terms of the 4-vector $A^\mu$. The scalar potential under a parity transformation is even:

$$\Phi = \frac{q}{r} \xrightarrow{P} \frac{q}{r} \tag{9}$$

The vector potential depends on the current density, which is proportional to the velocity. Therefore it transforms under parity as:

$$\vec{A} = \frac{\vec{J}}{r} \xrightarrow{P} -\frac{\vec{J}}{r} \tag{10}$$

The scalar potential transforms like the time and the vector potential like the coordinates, the photon therefore has a negative intrinsic parity: $A^\mu \xrightarrow{P} A_\mu$. Because the theory of strong and weak interactions are similar to that of electromagnetism, the gluon and the three weak bosons also have odd (-1) intrinsic parity.

## 4 Parity of Photon in Interactions

In interactions, photons are emitted or absorbed in various relative angular momentum states. It turns out that the orbital and spin angular momenta of the photons cannot be separated out in any unambiguous way; the only meaningful quantity is the total angular momentum $\vec{J} = \vec{L} + \vec{S}$. For this reason, the parity of the photon can also not be written as the product of the spin and orbital angular momentum as can be done for massive particles. Consider the case of $J = 1$, this can be composed of $L = 1$, $S = 1$ or $L = 0$, $S = 1$. The first case corresponds to even (+1) parity while the second case corresponds to odd $(-1)$ parity; each value of total angular momentum $J$ can have either value of parity.

The question now arises as to what the two different parities correspond to. Recall from the study of classical electromagnetic fields, that radiation field can be expressed as an expansion. The first term in the expansion corresponds to electric dipole radiation, while the second term corresponds to magnetic dipole radiation. Both terms are described by a vector type quantity; the electric dipole moment $\vec{p} = q\vec{x}$ a vector and the magnetic dipole moment $\vec{m} = \frac{1}{2}\int \vec{r} \times \vec{j} dV'$ a pseudo-vector, where $j$ is the current density. Since the two are described by vector type quantities, both correspond to a total angular momentum of $J = 1$.

The simplest means to describe the two is to consider the form of the dipole moments. In the case of electric dipole moment, it depends on the separation. When a photon is emitted, the separation must change. This can be thought of classically as a change in orbital angular momentum. Therefore, the electric dipole transition correspond in a change of orbital angular momentum in the initial system. The magnetic dipole momentum is associated with the direction of angular momentum of the system (the spin). Therefore, a change in the direction of the spin of the system corresponds to the emission of a photon. Each emits a photon with total angular momentum 1, but with different parity.

Higher order moments are described by tensors of higher rank and therefore correspond to larger values of angular momentum. The association with orbital angular momentum continues to correspond to electric transitions, while spin flips continue to be associated with magnetic transitions. The parity of each of these states is given by $E_J = (-1)^J$ and $M_J = (-1)^{J+1}$.