Physics 4213/5213
Lecture 8 (Part 1)

1 Angular Momentum Wave-function

To be able to determine quantities beyond simply the probability that a given system is in a specific angular momentum state, the angular momentum wave-functions are needed. In this section, only particles of spin zero will be considered and only those that have a relative angular momenta about some axis. The wave function for these systems are given by solving the Schrodinger equation in radial coordinates—see any book on quantum mechanics that solves the hydrogen atom for the explicit form of the wave-function. To simplify things and concentrate only on the angular portion of the wave-function, consider systems composed of spin zero particles. These particles are described by a scalar wave-function (no spinors, vectors etc.), where the wave-function is composed of a function that depends only the radial coordinate and one that depends only on the angular coordinates (momentum):

$$\Phi(\vec{r}) = \psi(|\vec{\mathbf{r}}|)Y_\ell^m(\theta,\phi), \tag{1}$$

where

$$Y_\ell^m(\theta,\phi) = \sqrt{\frac{(2\ell+1)(\ell-m)!}{4\pi(\ell+m)!}}P_\ell^m(\theta)e^{im\phi}. \tag{2}$$

the first few angular momentum states:

$Y_\ell^m(\theta)$	ℓ	m
$1/\sqrt{4\pi}$	0	0
$\sqrt{3/4\pi}\cos\theta$	1	0
$-\sqrt{3/8\pi}\sin\theta(e^{i\phi})$	1	1
$\sqrt{5/16\pi}(3\cos^2\theta-1)$	2	0
$-\sqrt{15/8\pi}\sin\theta\cos\theta(e^{i\phi})$	2	1
$\sqrt{15/32\pi}\sin^2\theta(e^{2i\phi})$	2	2

2 The Spin of the Δ

One method of measuring the spin of the Δ, is to measure the cross section of πp scattering in the region of the resonance. Then fit the resonance to the partial wave cross section:

$$\sigma_{\text{el}} = \frac{4\pi}{k^2}\frac{(2J+1)\Gamma^2/4}{(2s_a+1)(2s_b+1)\left[(E-E_R)^2+\Gamma^2/4\right]} \tag{3}$$

From the data everything is known directly, except for the spin (J) of the resonance—s are the spins of the incident particles, E_R the energy of the peak, Γ the width of the resonance at half the peak value. The main drawback to this method, is that the total rate has to be measured, therefore any loss of acceptance in the detector has to be very well understood, also the incident beam rates must be accurately known to normalize the total rate and determine the total cross section.

An alternate method, in fact a cross check, for determining the spin of the Δ, that is not as prone to the problems described above, is to measure the angular distribution of the decay products of the Δ. Since there are only two particles and these must come out back to back (in the rest frame), the only variable that is available to measure angles with respect to, is the direction of the proton spin. Therefore, to measure the angular distribution, the protons spins must all be aligned in the same direction.

Recall, that the reaction of interest is $\pi + p \rightarrow \Delta \rightarrow \pi + p$. With the spin of the protons aligned along the axis of the incident pions (z-axis), the z-component of the relative angular momentum is $m_z = 0$—this comes from $(\vec{r} \times \vec{p})_z = 0$. From this point the problem will be worked knowing that the spin of the Δ is 3/2 and the angular distribution of the outgoing pion will be determined. From the information already given, m_z for the Δ is 1/2, therefore the spin state of the Δ is $|3/2, 1/2\rangle$. There are two possibilities for the relative angular momentum 1 and 2—this comes from the total spin being in the range $\ell + 1/2 \rightarrow |\ell - 1/2|$. Typically, the smaller orbital angular momentum state makes the largest contribution to the resonance. In this case, the $|3/2, 1/2\rangle$ state is given by:

$$|3/2, 1/2\rangle = \sqrt{\frac{1}{3}}|1, 1\rangle|1/2, -1/2\rangle + \sqrt{\frac{2}{3}}|1, 0\rangle|1/2, 1/2\rangle \tag{4}$$

The angular distribution is given by the magnitude square of the amplitude:

$$I(\theta) \propto \frac{1}{3}(Y_1^1)^2 + \frac{2}{3}(Y_1^0)^2 = 1 + 3\cos^2\theta \tag{5}$$

If the orbital angular momentum is $\ell = 2$ the the amplitude is given by:

$$|3/2, 1/2\rangle = \sqrt{\frac{3}{5}}|2, 1\rangle|1/2, -1/2\rangle - \sqrt{\frac{2}{5}}|2, 0\rangle|1/2, 1/2\rangle \tag{6}$$

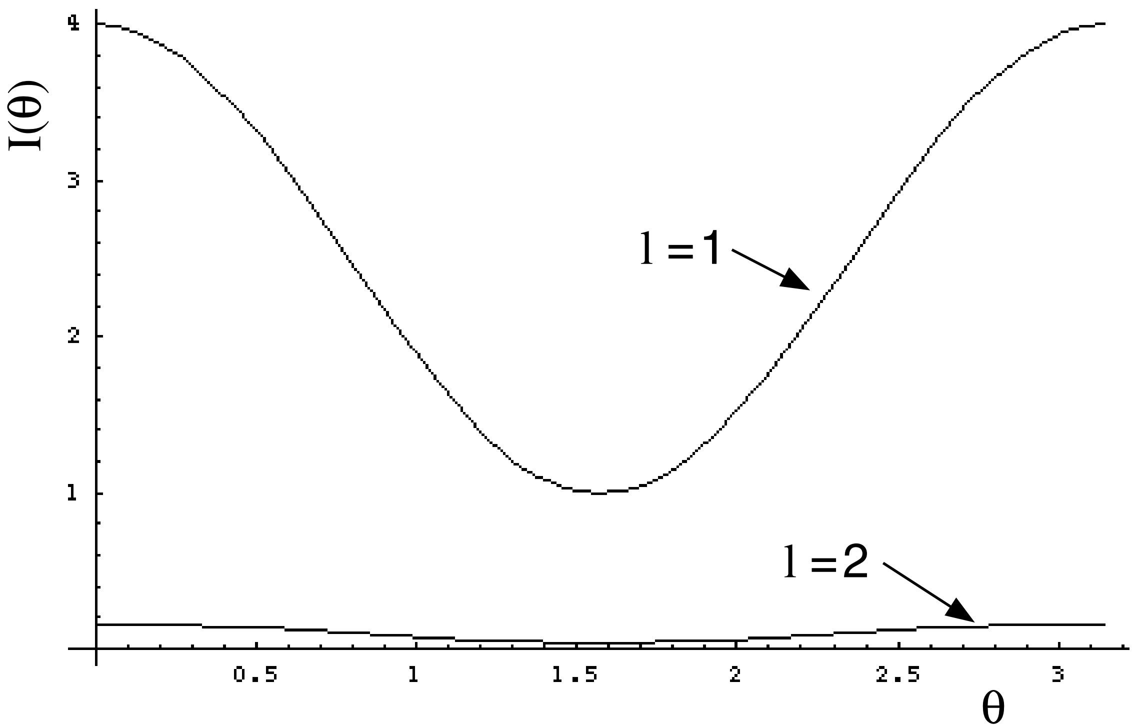

Figure 1: The angular distributions for $\ell = 1$ and $\ell = 2$ orbital angular momentum states.