Physics 4213/5213
Lecture 5 (Part 2)

1 Introduction

So far electromagnetic and strong interactions have been covered. The only remaining force that has not been introduce is the weak interaction. Both the strong and electromagnetic interactions are responsible for scattering particles, forming bound states and causing bond states to decay. The weak interaction, on the other hand is responsible for the decay of point particles—quarks and leptons. This interaction also causes both leptons and quarks to change flavor (type) something that neither of the other two interactions can do. Another interesting point, is that in the weak decay of hadrons, the hadron changes charge by one unit.

This set of lectures will start by looking at weak interactions of leptons. This will be followed by a discussion of how quarks behave under the weak interaction.

2 Weak Interactions of Leptons

As has already been stated the electron is stable since it is the lightest charged particle. The muon is known to not decay in a mode that does not conserve lepton number; $\mu^- \rightarrow e^- + \bar{\nu}_\mathrm{e} + \nu_\mu$: muon and electron lepton number are the same on both sides of the equation. Notice that in the final state an electron and an electron type anti-neutrino are produced to conserve electric charge and electron lepton number while, a muon type neutrino is create to conserve muon lepton number.

To construct a Feynman diagram for this process, two things need to be conserved at each vertex, one is charge and the second is lepton number. To conserve lepton number, the incoming lepton must have the same lepton number as the outgoing lepton. This at first glance appears to violate charge conservation, but if the carrier of the force carries a charge this problem is solved. Figure 1 shows the fundamental vertex along with its application to muon decay. As an aside this can be extended to the tau lepton also.

Before proceeding, the fundamental diagram also describes scattering processes which shows that in weak interactions the scattered lepton changes charge—these are referred to charge current interactions. A second type of interaction also occurs, those that do not change charge, which allows neutrinos to scatter off other particles without changing charge (see fig. 2). These later processes are referred to as neutral current interactions.

3 The Weak Gauge Bosons

Two carriers of the weak force have been introduced, the $W^\pm$ and the Z^0. These two particles are responsible for the transmission of the weak force. These particles like the gluon carry the charge of the interaction. In fact the $W^\pm$ also carries the charge of the electromagnetic force. Therefore, these particles also interact with each other and in the case of the $W^\pm$ with the photon (see fig. 3). This later statement indicates that the electromagnetic force and the weak force may be related to each other. This in fact is the case as will be shown later in the semester.

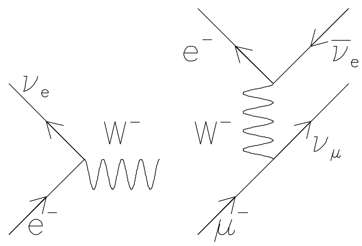

Figure 1: This figure show the fundamental charge current vertex for leptons. e^- and ν_{e} represent any lepton and its neutrino.

The other difference between the weak bosons, the photon and gluons, is that the weak bosons are massive. In fact they have a mass of 80 GeV for the $W^\pm$ and 90 GeV for the Z^0. The fact that they are massive indicates that the weak interaction may be connected with the origin of mass. This in fact appears to be the case.

4 The Weak Coupling Constant

For both the strong and electromagnetic interaction, the Feynman diagrams have a square root of the coupling constant (α) associated with each vertex—recall that the Feynman diagram represents the amplitude. This leads to a factor of the coupling constant for each vertex in the cross section (the cross section is the magnitude square of the amplitude; $\sigma \propto \alpha^n$ for n vertices). Therefore in the case of weak interactions the same is done. The coupling constant for weak interactions is $\alpha_w \approx 1/29$.

Before proceeding further, the Fermi coupling constant will be introduced. This is a left over from the original theory of weak interactions, but it appears quite often. In the original Fermi theory, the interaction was treated as a contact interaction (some small but finite range was assumed for the force). This of course makes the assumption that the carrier of the weak force is massive. In hindsight, this can be seen by applying the uncertainty principle $\Delta E c \Delta t \approx \hbar c$ where ΔE represents

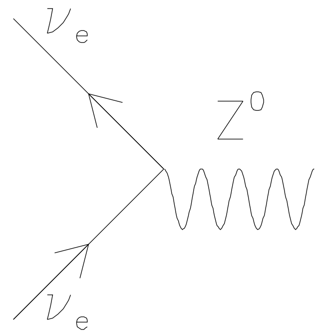

Figure 2: This figure shows the neutral current interaction.

the mass of the exchange particle and $c\Delta t$ its range. For the currently measured mass of the W, the range is $\approx 2.5 \times 10^{-16}$ cm, which can be considered as a point interaction (see fig 4).

Next, consider the quantity that represents the W in the Feynman diagram. In the case of the massless photon and gluon, this was given by $1/q^2$, where q^2 represents the momentum transfered between the real particles at the vertex. For a massive particle, this quantity is $\propto 1/(m^2 - q^2)$, where m represents the mass of the W. For low energies this factor is $1/M_W^2$. Therefore, for a two vertex interaction, the amplitude is proportional to α_w/m_W^2, which is the definition of the Fermi coupling constant, its value is given by

$$G_F/(\hbar c)^3 = 1.166 \times 10^{-5}\,\text{GeV}^{-2} \tag{1}$$

Notice that this coupling constant has dimensions of inverse energy square unlike all previous coupling constant definitions. Also notice that any cross section or decay rate is proportion to G_F^2.

As a final comment on the Fermi coupling constant, cross sections and decay rates, an estimate of the cross section can be derived just by dimensional analysis. The cross section is given as an area, which is inversely proportional to an energy squared (area $\propto$ energy^2). Given that the cross section has to depend on the Fermi coupling constant squared, a factor of the energy squared has to be introduced to get the dimensions right. This quantity should be a relativistic invariant, with dimensions of energy squared. For neutrino scattering $\nu_e + e^- \rightarrow \nu_e + e^-$ the relativistic invariant is the center of mass energy squared s—notice that there are no other independent quantities

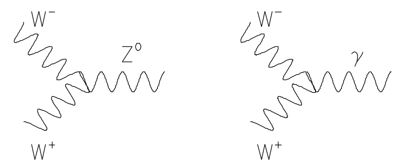

Figure 3: This shows the multi-gauge boson couplings for the weak and electromagnetic boson.

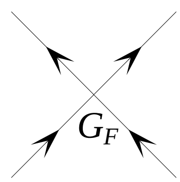

Figure 4: Effective weak interaction coupling at low energy.

available. The cross section is therefore proportional to:

$$\sigma(\nu_e + e^- \rightarrow \nu_e + e^-) \propto G_F^2 s \tag{2}$$

If the cross section is given in terms of the lab frame energy of the neutrino, the cross section is then proportional to

$$\sigma(\nu_e + e^- \rightarrow \nu_e + e^-) \propto G_F^2 E_{\nu_e} \tag{3}$$

This is exactly what is found doing an exact calculation.

5 The Decay of the Neutron and Lambda

Before looking into the weak interaction of quarks, the decay of the neutron is examined. The neutron decays through the channel $n \rightarrow p + e^- + \bar{\nu}_\mathrm{e}$. It is clear that this decay is associated with a weak charged decay since there is an electron and a neutrino. This can be associated with the W vertex.

Since both the neutron and proton are composed of quarks, it can be assumed that the quarks are the objects that actually participate in the interaction. First notice that the proton and neutron differ by one unit of charge and one quark flavor $p = (uud)$ and $n = (udd)$. Since the u and d quarks differ by one unit of charge as does the electron and neutrino, it can be assumed that the W is responsible for the transformation of a d to a u-quark and therefore for the decay of the neutron.

Now consider the decay of the $\Lambda \rightarrow p + \pi^-$ which causes the strangeness of the system to change—recall the Λ has a strangeness of -1. In terms of quarks the $\Lambda = (uds)$, the $p = (uud)$ and the $\pi^- = (d\bar{u})$. In this case, the system starts with uds and ends with $uddu\bar{u}$. Since the quark anti-quark pair cancel each other out, what appears to have happened is the s has transformed itself into a d-quark. But for this to be a charged current and also take the $u\bar{u}$ into account, the real process is the $s \rightarrow u + W^-$ and the W gives $W^- \rightarrow d\bar{u}$. This process implies that quarks can make transitions from one family to another—leptons can not do this. Recall that the quarks and leptons are grouped into families as follows:

$$\begin{pmatrix} u \\ d \end{pmatrix} \qquad \begin{pmatrix} e^- \\ \nu_e \end{pmatrix}$$
$$\begin{pmatrix} c \\ s \end{pmatrix} \qquad \begin{pmatrix} \mu^- \\ \nu_\mu \end{pmatrix}$$
$$\begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{pmatrix} \tau^- \\ \nu_\tau \end{pmatrix}$$