Physics 4213/5213
Lecture 4 (Part 1)

1 Classification Scheme and Symmetry

The classification scheme for the mesons and baryons given in the previous lecture, is based on the observation that the masses of those hadrons in a given group have approximately the same masses; recall that the spins of all the particles in a given group are the same. This near similarity in masses is taken as a sign that maybe all the particles have the same masses but that the charge and whatever strangeness is causes a small shift in the observed rest mass—the charge (electromagnetic interaction) changes the rest energy of a particle causing its mass to change.

As an example of the similarity of the masses, consider the group of baryons that contain the proton. Start by looking at each row since strangeness is constant by row. The first row contains the proton and neutron, which have a 0.13% mass difference, the second row has a 0.66% mass difference between the various Σ states—the Λ is excluded for now since later it will be shown to have different quantum numbers than the Σ. Finally the last row has a mass difference of 0.48% mass difference between the two Ξ states. Finally the mass difference between the smallest and largest mass particles is 34%, with mass increasing with strangeness.

	Charge			Mass Difference	
strangeness	Negative	Neutral	Positive	$\Delta S = 0$	$\Delta S = 1$
0		$n(939.)$	$p(938.)$	0.13%	
-1		$\Lambda^0(1115.)$			17.%
-1	$\Sigma^-(1197.)$	$\Sigma^0(1192.)$	$\Sigma^+(1189.)$	0.66%	23.%
-2	$\Xi^-(1321.)$	$\Xi^0(1314.)$		0.48%	10.%

Table 1: List of the spin 1/2 baryon octet. The numbers in parenthesis are the mass of the given particle in MeV. The mass difference are the average difference for a given row and between rows of different strangeness.

The similarity in masses is also evident for other groupings of baryons and mesons with the same total spin. This similarity is what lead the to classification scheme in the first place.

2 The Quark Model

Given the geometrical structures used in the previous lecture to build up hadron multiplets and since these can be built from triangles, there may be a simple underlying structure to the hadron spectroscopy seen. Figure 1 shows such a structure—this structure, as will be seen later in the semester, comes from the similarity of the proton, neutron and Λ masses and the symmetry principle this leads to. If the vertices of the triangle are taken as real particles these can be used to build both the baryons (assuming that these particles carry fractional charges and are fermions) and the mesons. These in fact are the quarks that are believed to build all hadrons. The rules are as follows, all baryons are composed of three quarks and the mesons are composed of a quark and an antiquark. Table 2 gives the composition of a few of the baryons and mesons already mentioned.

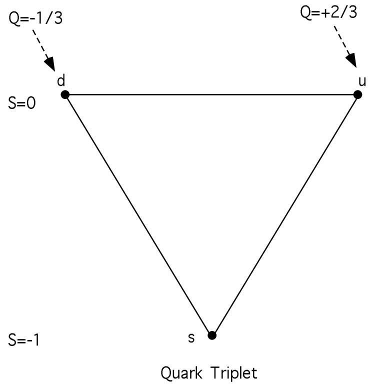

Figure 1: Quark classification scheme.

Hadron	Composition
p	uud
n	ddu
Δ^{++}	uuu
Δ^{+}	uud
Δ^{0}	udd
Δ^{-}	ddd
Λ	uds
Σ^{+}	uus
Σ^{0}	uds
Σ^{-}	dds
π^{+}	$u\bar{d}$
π^{0}	$1\sqrt{2}(u\bar{u} - d\bar{d})$
π^{-}	$u\bar{d}$
K^{0}	$d\bar{s}$
K^{+}	$u\bar{s}$

Table 2: Listing of the quark composition of a few hadrons.

2.1 The Quark Masses

The quark masses are listed in table 3, with two values for each quark. Since quarks do not exist as free particles, the definition of their masses is not unique. There are two methods at arriving at these masses, the first is through the use of potential models, where a potential is selected and the quark masses are then calculated so that the mass spectrum of the observed hadrons is reproduced. This is not simply adding together the quark masses to get a hadron mass. Recall that the total energy of a system is given by $E = mc^2$. If the system is at rest, then the total energy is equal to the mass—there is no lack of generality in taking the system at rest since the calculations can be carried out in the rest frame of the object and a transformation to any other frame can be done afterwards. Now take a bound system of two particles. The total energy is given by the sum of the rest mass energy of the particles plus any kinetic energy plus the binding energy—for the system to form a bound state the sum of kinetic plus potential energy must be negative. Therefore, the total energy of the system is given by $E = m_1 + m_2 - V(r)$ where $V(r)$ includes both kinetic and potential energies. The mass of the bound state is given by $m_{12} = m_1 + m_2 - V(r) < m_1 + m_2$.

Quark	Effective mass	Bare mass	Charge
d	480 MeV	5 MeV	−1/3
u	300 MeV	2 MeV	2/3
s	500 MeV	100 MeV	−1/3
c	1500 MeV	1000 MeV	2/3
b	4800 MeV	4100 MeV	−1/3
t	180000 MeV	180000 MeV	2/3

Table 3: Listing of the quark masses and charges.

The other method of obtaining the quark masses is through scattering experiments. In this case an object of known mass (such as an electron or a muon) is scattered off a hadron. By measuring the recoil of the electron and the quark, the mass of the unknown quark can be measured. This is of course more easily said than done since the quark does not appear as a free particle but as a jet of hadrons—this is due to the quarks not being able to exist as free particles. This method requires calculating the differential cross section (angular scattering distribution) and fitting for the mass. As can be seen in the table 3 the two methods give very different results for the light quarks.

2.2 Building the Meson Structure

To show that in fact the hadron structure proposed can be arrived at from the quark structure proposed, the meson structure will be built—the baryons are not as trivially built and require some group theory—*some discussion of this will follow the introduction of isospin.* First start with the fact that mesons are composed of quarks and antiquarks. Use a single quark triangle and with each quark (vertex) associate an antiquark triangle (see fig 2). From this structure build up the $q\bar{q}$ states. The physics is introduced into the structure by associating mesons of the same spin, with states of the same strangeness and charge as the quark states. This pattern is repeated for all known meson spins.

3 Problems with The Quark Model and The Introduction of Color

The quark model as introduced so far has a major problem in that no model for how quarks interact, other than electromagnetic, has been introduced. This model also has to be able to explain why quarks are not seen as free particles. Two pieces of information will help select a model for how quarks interact with each other. The first of these is the problem with the lightest spin $\frac{3}{2}$ baryons.

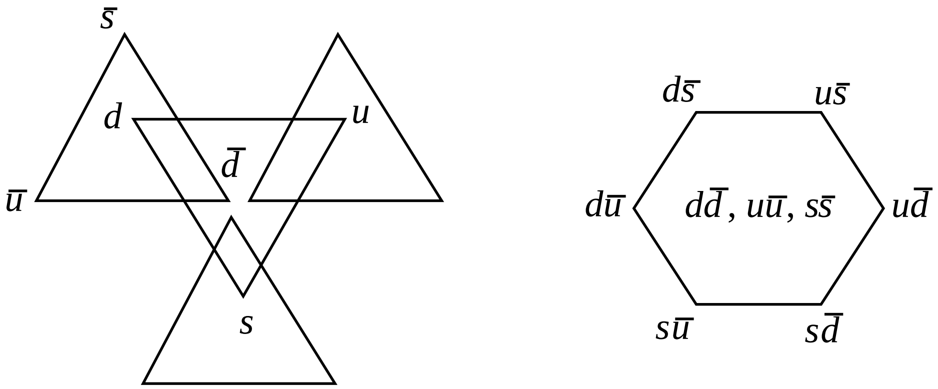

Figure 2: The creation of the meson octet by combining the basic quark and antiquark triplets.

In this collection there are three baryons that are composed of three identical quarks:

$$\begin{aligned}\Delta^{++} &\Rightarrow \{uuu\}\\ \Delta^{-} &\Rightarrow \{ddd\}\\ \Omega^{-} &\Rightarrow \{sss\}.\end{aligned}$$

Since these are the lightest baryons, the three quarks are assumed to be in the same state. This is forbidden for Fermions, since they must form an anti-symmetric state. To get around this problem, each quark must be given three additional degrees of freedom.

A second problem was also observed. When electrons and positrons collide, they can annihilate each other forming a virtual photon that can create muon, anti-muon pairs and quark anti-quark pairs. Recall that the probability of producing either type of pairs is proportional to the fine structure constant to the fourth power. For the quarks, the fine structure constant has to be modified since they have fractional charges, with the modifications given by:

$$\alpha\left[\frac{2}{3}\right] = \left(\frac{2}{3}\right)^2 \alpha_{em} \qquad \alpha\left[\frac{1}{3}\right] = \left(\frac{1}{3}\right)^2 \alpha_{em} \tag{1}$$

The ratio of hadrons to muons in electron-positron scattering should then be the sum of the quark charges squared (this is where the hadrons come from):

$$\frac{\sigma(e^+ + e^- \to \text{hadrons})}{\sigma(e^+ + e^- \to \mu^+ + \mu^-)} = \sum_{i=1}^{n_q} Q_i^2, \tag{2}$$

where n_q is the total number of quarks with masses less than or equal to the total amount of energy available to produce particles—note that quarks are produced in pairs so that the energy available for particle production must be able to produce the pair to be included in the sum above. It turns out when compared to experiment, that the ratio is a factor of three too small. This indicates that each quark has three difference species or degrees of freedom.

A third problem comes from the electromagnetic decay of the neutral pion; $\pi^0 \to \gamma\gamma$. This decay can be described in terms of a quark loop that radiates two photons as shown in figure 3; the

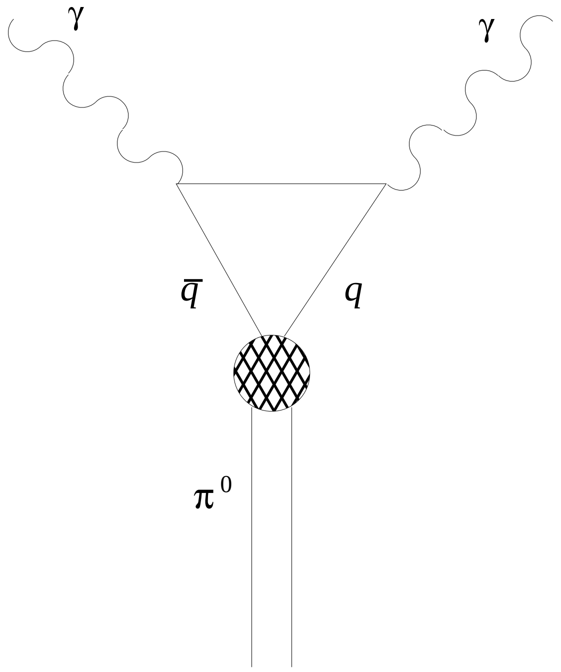

Figure 3: Feynman diagram that describes the electromagnetic decay of the π^0.

blob at the vertex introduces our ignorance of what actually happens at the vertex. The calculated decay rate depends primarily on the triangle above the vertex, but the decay rate is found to be a factor of three too small. By increasing the number of each species of quarks by three, the problem can be overcome, since a diagram for each species has to be included.

Combining the three problems given above and that they both require increasing the number of degrees of freedom of each quark to three, plus needing a strong interaction charge to bind the quarks together inside hadrons, the quarks are given three strong charges. These charges are given the name of color charges and are specified as red, green and blue; anti-quarks will carry anti-colors.