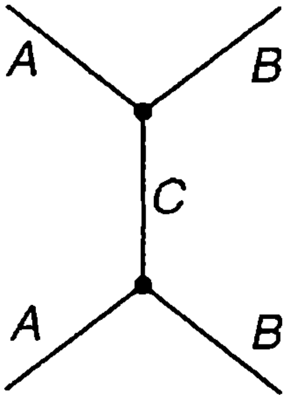
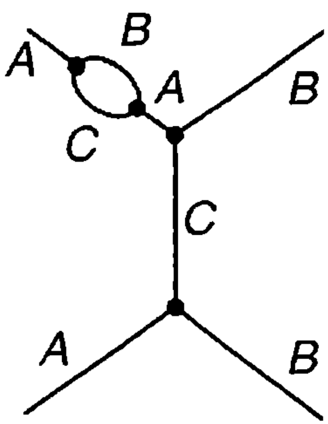
# HIGHER ORDER DIAGRAMS

# Higher order diagram

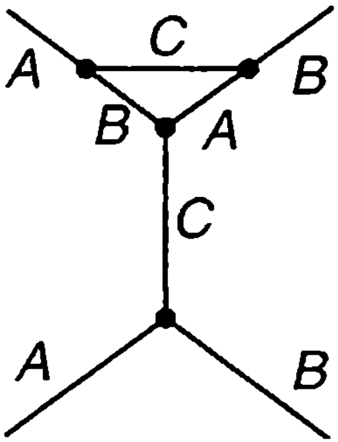
- A + A -> B + B

  1 two vertex diagram
  8 eight vertex diagrams

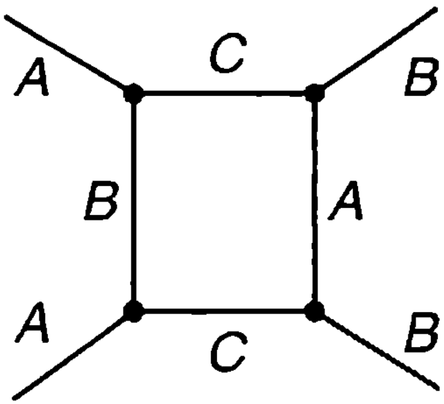



5 loops diagrams




2 triangles




1 box

# Consider one loop

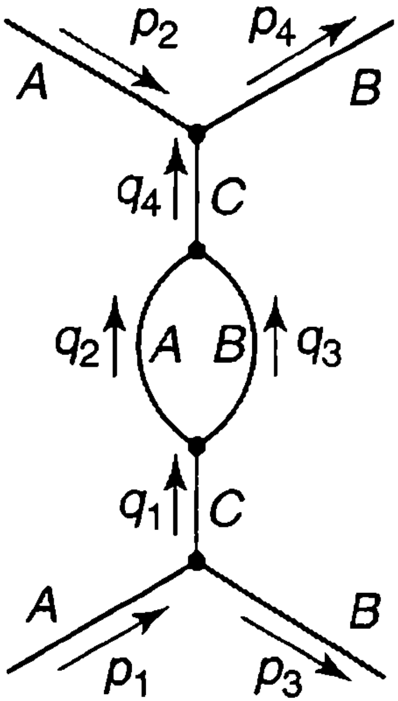
Rule 3

Rule 2

$$= \int (ig)^4 \quad * \frac{i}{(q_1^2 - m_1^2c^2)} \frac{i}{(q_2^2 - m_2^2c^2)} \frac{i}{(q_3^2 - m_3^2c^2)} \frac{i}{(q_4^2 - m_4^2c^2)}$$

$$* (2\pi)^4\delta^4(p_1 - q_1 - p_3) * (2\pi)^4\delta^4(q_1 - q_2 - q_3) * (2\pi)^4\delta^4(q_2 + q_3 - q_4)$$

$$* (2\pi)^4\delta^4(q_4 + p_2 - p_4) * \frac{d^4q_1 d^4q_2 d^4q_3 d^4q_4}{(2\pi)^{16}}$$




Rule 4

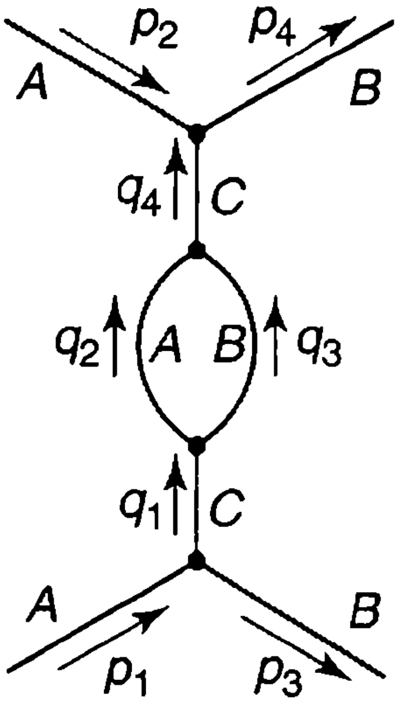
Rule 5

# Consider one loop

$$= \int (ig)^4 \quad * \frac{i}{(q_1^2 - m_1^2c^2)} \frac{i}{(q_2^2 - m_2^2c^2)} \frac{i}{(q_3^2 - m_3^2c^2)} \frac{i}{(q_4^2 - m_4^2c^2)}$$

$$* (2\pi)^4\delta^4(p_1 - q_1 - p_3) * (2\pi)^4\delta^4(q_1 - q_2 - q_3) * (2\pi)^4\delta^4(q_2 + q_3 - q_4)$$

$$* (2\pi)^4\delta^4(q_4 + p_2 - p_4) * \frac{d^4q_1 d^4q_2 d^4q_3 d^4q_4}{(2\pi)^{16}}$$




Use 1st Dirac and let $q_1 = p_1 - p_3$
And 4th $q_4 = p_4 - p_2$

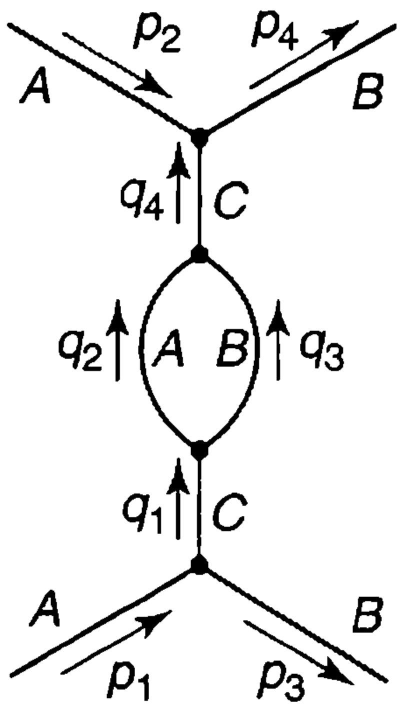
$$\frac{g^4}{[(p_1 - p_3)^2 - m_C^2c^2][(p_4 - p_2)^2 - m_C^2c^2]}$$

$$\times \int \frac{\delta^4(p_1 - p_3 - q_2 - q_3)\,\delta^4(q_2 + q_3 - p_4 + p_2)}{(q_2^2 - m_A^2c^2)(q_3^2 - m_B^2c^2)}\, d^4q_2\, d^4q_3$$

# Consider one loop

$$\frac{g^4}{[(p_1 - p_3)^2 - m_C^2c^2][(p_4 - p_2)^2 - m_C^2c^2]}$$

$$\times \int \frac{\delta^4(p_1 - p_3 - q_2 - q_3)\,\delta^4(q_2 + q_3 - p_4 + p_2)}{(q_2^2 - m_A^2c^2)(q_3^2 - m_B^2c^2)}\, \mathrm{d}^4q_2\, \mathrm{d}^4q_3$$




Use first Dirac to let $q_2 = p_1 - p_3 - q_3$

Use rule 6 to cancel remaining dirac

$$\mathcal{M} = i\left(\frac{g}{2\pi}\right)^4 \frac{1}{[(p_1 - p_3)^2 - m_C^2c^2]^2}$$

$$\int \frac{1}{[(p_1 - p_3 - q)^2 - m_A^2c^2](q^2 - m_B^2c^2)}\, \mathrm{d}^4q$$

# Solving the Integral

$$\mathcal{M} = i\left(\frac{g}{2\pi}\right)^4 \frac{1}{[(p_1 - p_3)^2 - m_C^2 c^2]^2} \int \frac{1}{[(p_1 - p_3 - q)^2 - m_A^2 c^2](q^2 - m_B^2 c^2)} \mathrm{d}^4 q$$

Write: $d^4 q = q^3 \, \mathrm{d}q \, \mathrm{d}\Omega'$

As q approaches infinity, we get $\int^{\infty} \frac{1}{q^4} q^3 \, dq = \ln q|^{\infty} = \infty$

# Renormalize

introduce $$\frac{-M^2c^2}{(q^2 - M^2c^2)}$$ Which approaches 1 as $M \to \infty.$

In the final answer, all M dependent terms appear as addition to the mass and to the coupling constant.

$$m_{\text{physical}} = m + \delta m; \quad g_{\text{physical}} = g + \delta g$$

# Conclusions

$$m_{\text{physical}} = m + \delta m; \quad g_{\text{physical}} = g + \delta g$$

$$\delta m \; and \; \delta g \; are \; infinite \; as \; M \rightarrow \infty$$

$$so \; m \; and \; g \; contain \; compensating \; infinities$$

The loop M-independent contributions are perfectly calculable.
Hooft showed that all gauge theories are renormalizable.